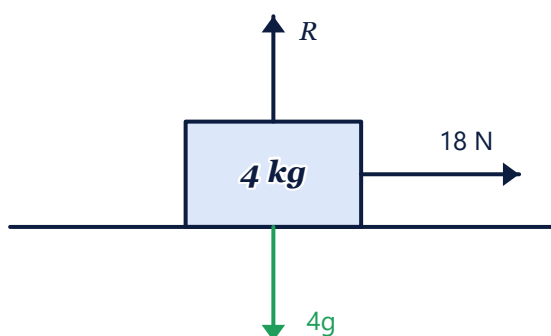




1 A box of mass 4 kg lies on a smooth horizontal floor. The box is pulled along the floor by a constant horizontal force of magnitude 18 N, as shown in the diagram. The box is modelled as a particle.

- Find the acceleration of the box.
- Find the magnitude of the normal reaction, R N, between the box and the floor.
- The box starts from rest. Find the time it takes to reach a speed of 9 m s^{-1} .



(5 marks)

a) Resolve horizontally and use $F = ma$

$$18 = 4a$$

$$\div 4 \quad a = 4.5 \quad \div 4$$

$$a = 4.5 \text{ m s}^{-2}$$

b) Resolve vertically: no vertical acceleration

$$R = 4g$$

$$R = 4 \times 9.8$$

$$R = 39.2$$

$$R = 39.2 \text{ N}$$

c) $u = 0$, $v = 9$, $a = 4.5$, use $v = u + at$

$$9 = 0 + 4.5t$$

$$\div 4.5 \quad t = 2 \quad \div 4.5$$

$$t = 2 \text{ s}$$

2

Two forces $\mathbf{F}_1 = (3\mathbf{i} - 2\mathbf{j})$ N and $\mathbf{F}_2 = (-5\mathbf{i} + 6\mathbf{j})$ N act on a particle P of mass 0.5 kg. No other forces act on P .

- Find the resultant force acting on P .
- Find the acceleration of P , giving your answer in the form $(a\mathbf{i} + b\mathbf{j}) \text{ m s}^{-2}$.
- Find the magnitude of the acceleration of P .

(5 marks)

a) Add the i parts and the j parts

$$\mathbf{R} = (3\mathbf{i} - 2\mathbf{j}) + (-5\mathbf{i} + 6\mathbf{j})$$

$$\mathbf{R} = (3 - 5)\mathbf{i} + (-2 + 6)\mathbf{j}$$

$$\mathbf{R} = (-2\mathbf{i} + 4\mathbf{j}) \text{ N}$$

b) $\mathbf{F} = m\mathbf{a}$

$$-2\mathbf{i} + 4\mathbf{j} = 0.5\mathbf{a}$$

$$\mathbf{a} = \frac{-2\mathbf{i} + 4\mathbf{j}}{0.5}$$

$$\mathbf{a} = (-4\mathbf{i} + 8\mathbf{j}) \text{ m s}^{-2}$$

c) Magnitude by Pythagoras

$$|\mathbf{a}| = \sqrt{(-4)^2 + 8^2}$$

$$|\mathbf{a}| = \sqrt{80}$$

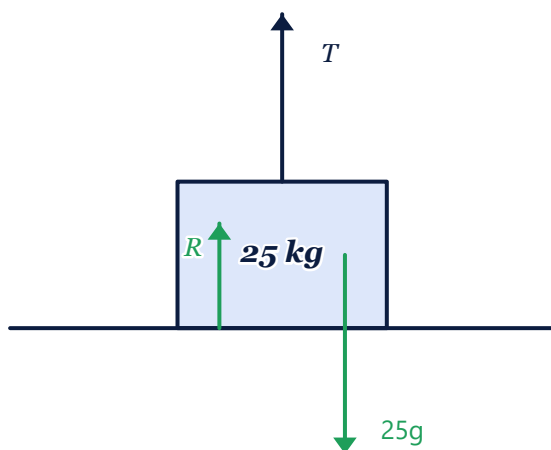
$$|\mathbf{a}| = 4\sqrt{5} = 8.944\dots$$

$$8.94 \text{ m s}^{-2}$$

3

A crate of mass 25 kg rests on a horizontal floor. A light rope is attached to the top of the crate and the rope is held vertical, as shown in the diagram. The tension in the rope is T N.

- When $T = 80$ the crate stays at rest on the floor. Find the normal reaction between the crate and the floor.
- The tension is increased to 300 N and the crate rises vertically. Find the acceleration of the crate.
- The crate starts from rest. Find the speed of the crate when it has risen 1.5 m.



(6 marks)

a) Resolve vertically: up = down (at rest)

$$\begin{array}{rcl}
 R + 80 & = & 25g \\
 R + 80 & = & 245 \\
 -80 & & -80 \\
 R & = & 165
 \end{array}$$

$$R = 165 \text{ N}$$

b) Off the floor, $R = 0$. Resolve vertically, up positive, $F = ma$

$$\begin{array}{rcl}
 300 - 25g & = & 25a \\
 300 - 245 & = & 25a \\
 55 & = & 25a \\
 \div 25 & & \div 25 \\
 a & = & 2.2
 \end{array}$$

$$a = 2.2 \text{ m s}^{-2} \text{ upwards}$$

c) $u = 0$, $a = 2.2$, $s = 1.5$, use $v^2 = u^2 + 2as$

$$v^2 = 0^2 + 2(2.2)(1.5)$$

$$v^2 = 6.6$$

$$v = 2.569...$$

$$v = 2.57 \text{ m s}^{-1}$$

4

A car of mass 1200 kg starts from rest on a straight horizontal road. The engine of the car produces a constant driving force of 3000 N and the total resistance to the motion of the car is modelled as a constant force of magnitude 840 N.

- Find the acceleration of the car.
- Find the speed of the car after 8 seconds.

After 8 seconds the engine is switched off. The resistance to motion is unchanged.

- Find the distance the car travels from the moment the engine is switched off until it comes to rest.

(7 marks)

a) Resolve in the direction of motion, $F = ma$

$$3000 - 840 = 1200a$$

$$2160 = 1200a$$

$\div 1200$

$$a = 1.8$$

$\div 1200$

$$a = 1.8 \text{ m s}^{-2}$$

b) $u = 0$, $a = 1.8$, $t = 8$, use $v = u + at$

$$v = 0 + 1.8 \times 8$$

$$v = 14.4$$

$$v = 14.4 \text{ m s}^{-1}$$

c) Engine off: only the resistance acts horizontally

$$-840 = 1200a$$

$$a = -0.7$$

$u = 14.4$, $v = 0$, $a = -0.7$, use $v^2 = u^2 + 2as$

$$0 = 14.4^2 + 2(-0.7)s$$

$$1.4s = 207.36$$

$$s = 148.11...$$

$$148 \text{ m}$$

5

Two forces $\mathbf{F}_1 = (2\mathbf{i} + 3\mathbf{j})$ N and $\mathbf{F}_2 = (c\mathbf{i} - 5\mathbf{j})$ N, where c is a constant, act on a particle P of mass 1.5 kg. No other forces act on P .

The acceleration of P is in the direction of the vector $(2\mathbf{i} - \mathbf{j})$.

- Find the value of c .
- Find the magnitude of the acceleration of P .
- Find the angle between the direction of the acceleration of P and the vector $\mathbf{i}$.

(7 marks)

a) Find the resultant force

$$\mathbf{R} = (2\mathbf{i} + 3\mathbf{j}) + (c\mathbf{i} - 5\mathbf{j})$$

$$\mathbf{R} = (2 + c)\mathbf{i} - 2\mathbf{j}$$

$\mathbf{F} = m\mathbf{a}$, so $\mathbf{R}$ is in the same direction as $\mathbf{a}$

$\mathbf{R}$ is a multiple of $(2\mathbf{i} - \mathbf{j})$: compare the ratios

$$\begin{array}{rcl} \frac{2+c}{2} & = & \frac{-2}{-1} \\ 2+c & = & 4 \\ -2 & & -2 \\ c & = & 2 \end{array}$$

$$\boxed{c = 2}$$

b) Magnitude of the resultant

$$\mathbf{R} = 4\mathbf{i} - 2\mathbf{j}$$

$$|\mathbf{R}| = \sqrt{4^2 + 2^2}$$

$$|\mathbf{R}| = \sqrt{20}$$

$\mathbf{F} = m\mathbf{a}$ with magnitudes

$$\sqrt{20} = 1.5a$$

$$a = 2.981\dots$$

$$\boxed{2.98 \text{ m s}^{-2}}$$

c) Sketch 4 across, 2 down: use $\tan$

$$\tan \theta = \frac{2}{4}$$

$$\theta = 26.56\dots^\circ$$

$$\boxed{26.6^\circ \text{ below } \mathbf{i}}$$

6

A particle P of mass 2 kg is at rest in equilibrium under the action of three forces $\mathbf{F}_1 = (6\mathbf{i} - 2\mathbf{j})$ N, $\mathbf{F}_2 = (-3\mathbf{i} + 5\mathbf{j})$ N and $\mathbf{F}_3$.

a. Find $\mathbf{F}_3$ in the form $(a\mathbf{i} + b\mathbf{j})$ N.

The force $\mathbf{F}_3$ is now removed. Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ continue to act on P .

b. Find the magnitude of the acceleration of P .

c. Find the distance travelled by P in the first 4 seconds after $\mathbf{F}_3$ is removed.

(7 marks)

a) Equilibrium: the forces add to zero

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$$

$$(6\mathbf{i} - 2\mathbf{j}) + (-3\mathbf{i} + 5\mathbf{j}) + \mathbf{F}_3 = \mathbf{0}$$

$$3\mathbf{i} + 3\mathbf{j} + \mathbf{F}_3 = \mathbf{0}$$

$$\boxed{\mathbf{F}_3 = (-3\mathbf{i} - 3\mathbf{j}) \text{ N}}$$

b) The resultant is now $\mathbf{F}_1 + \mathbf{F}_2$

$$\mathbf{R} = 3\mathbf{i} + 3\mathbf{j}$$

$$|\mathbf{R}| = \sqrt{3^2 + 3^2}$$

$$|\mathbf{R}| = 3\sqrt{2}$$

$\mathbf{F} = m\mathbf{a}$ with magnitudes

$$3\sqrt{2} = 2a$$

$$a = \frac{3\sqrt{2}}{2} = 2.121\dots$$

$$\boxed{2.12 \text{ m s}^{-2}}$$

c) From rest in a straight line: $u = 0$, $t = 4$, use $s = ut + \frac{1}{2}at^2$

$$s = 0 + \frac{1}{2} \times \frac{3\sqrt{2}}{2} \times 4^2$$

$$s = 12\sqrt{2}$$

$$s = 16.97\dots$$

$$\boxed{17.0 \text{ m}}$$

7

Nadia pulls a sledge of mass 15 kg along horizontal snow using a horizontal rope. The tension in the rope is 60 N and the resistance to motion is a constant force of magnitude F N, as shown in the diagram.

The sledge starts from rest and reaches a speed of 3 m s^{-1} after moving 5 m.

- Find the acceleration of the sledge.
- Find the value of F .

When the speed of the sledge is 3 m s^{-1} , Nadia lets go of the rope.

- Find how much further the sledge travels before it comes to rest.
- State one modelling assumption you have made about the rope.



(8 marks)

a) $u = 0, v = 3, s = 5$, use $v^2 = u^2 + 2as$

$$3^2 = 0^2 + 2a(5)$$

$$9 = 10a$$

$$\div 10 \quad a = 0.9 \quad \div 10$$

$$a = 0.9 \text{ m s}^{-2}$$

b) Resolve horizontally, $F = ma$

$$60 - F = 15 \times 0.9$$

$$60 - F = 13.5$$

$$F = 46.5$$

$$F = 46.5$$

c) Rope released: only the resistance acts

$$-46.5 = 15a$$

$$a = -3.1$$

$u = 3, v = 0, a = -3.1$, use $v^2 = u^2 + 2as$

$$0 = 3^2 + 2(-3.1)s$$

$$6.2s = 9$$

$$s = 1.451...$$

$$1.45 \text{ m}$$

d) Modelling assumption

The rope is light and inextensible,
so the tension is 60 N all along it

8

A small stone of mass 0.4 kg is released from rest at a height of 1.6 m above the surface of a pond. The stone falls vertically. Air resistance may be ignored.

a. Show that the stone hits the surface of the water with speed 5.6 m s^{-1} .

In the water the stone continues to move vertically downwards and the water exerts a constant resistance of magnitude 2.6 N on the stone. The pond is 3 m deep.

b. Find the speed of the stone when it reaches the bottom of the pond.

c. Suggest one way in which the model for the resistance of the water could be refined.

(7 marks)

a) Free fall: down positive, $u = 0$, $a = 9.8$, $s = 1.6$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2(9.8)(1.6)$$

$$v^2 = 31.36$$

$$v = 5.6$$

$$v = 5.6 \text{ m s}^{-1} \text{ as required}$$

b) In the water: resolve vertically, down positive

$$0.4g - 2.6 = 0.4a$$

$$3.92 - 2.6 = 0.4a$$

$$1.32 = 0.4a$$

$$\div 0.4 \quad a = 3.3 \quad \div 0.4$$

$u = 5.6$, $a = 3.3$, $s = 3$, use $v^2 = u^2 + 2as$

$$v^2 = 5.6^2 + 2(3.3)(3)$$

$$v^2 = 31.36 + 19.8$$

$$v^2 = 51.16$$

$$v = 7.152...$$

$$7.15 \text{ m s}^{-1}$$

c) Refinement

Let the resistance increase with the speed
of the stone, instead of being constant

9

A van of mass m kg moves along a straight horizontal road. The engine produces a constant driving force of 2400 N and the resistance to motion is a constant 600 N. The speed of the van increases from 8 m s^{-1} to 20 m s^{-1} in 15 seconds.

a. Show that $m = 2250$

When the van is moving at 20 m s^{-1} the engine is switched off and the brakes are applied. The van comes to rest in a distance of 50 m. The total force resisting the motion while the van brakes is modelled as a constant force of magnitude R N.

b. Find the value of R .

c. Find the time it takes the van to stop.

(7 marks)

a) $u = 8, v = 20, t = 15$, use $v = u + at$

$$\begin{array}{rcl} 20 & = & 8 + 15a \\ -8 & & -8 \\ 12 & = & 15a \\ \div 15 & & \div 15 \\ a & = & 0.8 \end{array}$$

Resolve horizontally, $F = ma$

$$\begin{array}{rcl} 2400 - 600 & = & m \times 0.8 \\ 1800 & = & 0.8m \\ \div 0.8 & & \div 0.8 \\ m & = & 2250 \end{array}$$

$m = 2250$ as required

b) $u = 20, v = 0, s = 50$, use $v^2 = u^2 + 2as$

$$0 = 20^2 + 2a(50)$$

$$-400 = 100a$$

$$a = -4$$

$F = ma$: the only horizontal force is R , backwards

$$-R = 2250 \times (-4)$$

$$R = 9000$$

$$R = 9000$$

c) Use $v = u + at$

$$0 = 20 - 4t$$

$$t = 5$$

$$5 \text{ s}$$

10

A hot-air balloon, together with its basket and passengers, has a total mass of 450 kg. The balloon is moving vertically downwards with a constant acceleration of 0.4 m s^{-2} . The air exerts a constant vertical upward force of magnitude $L \text{ N}$ on the balloon. Air resistance is ignored.

a. Find the value of L .

To slow the descent, the crew throw out a bag of sand of mass $m \text{ kg}$. The balloon then moves with a constant upward acceleration of 0.3 m s^{-2} . The upward force on the balloon is still $L \text{ N}$.

b. Find the value of m .

c. State how you have used the modelling assumption that the balloon is a particle.

(7 marks)

a) Accelerating downwards: resolve vertically, down positive

$$450g - L = 450 \times 0.4$$

$$4410 - L = 180$$

$$L = 4230$$

$$\boxed{L = 4230}$$

b) New mass is $(450 - m)$. Resolve vertically, up positive

$$L - (450 - m)g = (450 - m)(0.3)$$

$$4230 = (450 - m)(9.8 + 0.3)$$

$$4230 = 10.1(450 - m)$$

$\div 10.1$

$$450 - m = 418.81...$$

$\div 10.1$

$$m = 31.18...$$

$$\boxed{m = 31.2}$$

c) Particle model

All the forces act at one point,

so the balloon does not turn and its size is ignored

11

Two forces $(3\mathbf{i} + 2\mathbf{j})$ N and $(\lambda\mathbf{i} - 8\mathbf{j})$ N, where λ is a constant, act on a particle Q of mass 2 kg. No other forces act on Q . The resultant of the two forces has magnitude 10 N.

a. Find the two possible values of λ .

Given that $\lambda > 0$,

b. find the magnitude of the acceleration of Q ,

c. find the angle between the acceleration of Q and the vector $\mathbf{i}$.

Q starts from rest.

d. Find the speed of Q after 3 seconds and the distance it has travelled in that time.

(9 marks)

a) Find the resultant in terms of λ

$$\mathbf{R} = (3 + \lambda)\mathbf{i} + (2 - 8)\mathbf{j}$$

$$\mathbf{R} = (3 + \lambda)\mathbf{i} - 6\mathbf{j}$$

Use $|\mathbf{R}| = 10$ with Pythagoras

$$(3 + \lambda)^2 + (-6)^2 = 10^2$$

$$(3 + \lambda)^2 + 36 = 100$$

$$(3 + \lambda)^2 = 64$$

$$3 + \lambda = \pm 8$$

$$\lambda = 5 \text{ or } \lambda = -11$$

b) $\lambda = 5$, so $\mathbf{R} = 8\mathbf{i} - 6\mathbf{j}$ and $|\mathbf{R}| = 10$

$$\begin{array}{rcl} 10 & = & 2a \\ \div 2 & & \div 2 \\ a & = & 5 \end{array}$$

$$5 \text{ m s}^{-2}$$

c) 8 across, 6 down: use $\tan$

$$\tan \theta = \frac{6}{8}$$

$$\theta = 36.86\dots^\circ$$

$$36.9^\circ \text{ below } \mathbf{i}$$

d) From rest, constant force: straight line along $\mathbf{R}$

$$u = 0, a = 5, t = 3$$

$$v = u + at$$

$$v = 0 + 5 \times 3 = 15$$

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} \times 5 \times 3^2 = 22.5$$

15 m s^{-1} and 22.5 m

12

A crane lifts a crate of mass 300 kg vertically using a light inextensible cable. The crate starts from rest on the ground. It accelerates uniformly at 0.5 m s^{-2} for 4 seconds, then moves at constant speed for 10 seconds, then decelerates uniformly, coming to rest after a further 3 seconds.

- Find the tension in the cable during each of the three stages of the motion.
- Find the height of the crate above the ground when it comes to rest.
- The cable will break if the tension in it exceeds 3200 N. Find the greatest upward acceleration the crane can give the crate.

(9 marks)

a) Stage 1: resolve vertically, up positive, $F = ma$

$$T_1 - 300g = 300 \times 0.5$$

$$T_1 - 2940 = 150$$

$$T_1 = 3090$$

Speed at the end of stage 1: $v = u + at$

$$v = 0 + 0.5 \times 4 = 2$$

Stage 2: constant speed, so the forces balance

$$T_2 = 300g = 2940$$

Stage 3: $u = 2$, $v = 0$, $t = 3$

$$0 = 2 + 3a$$

$$a = -\frac{2}{3}$$

$$T_3 - 300g = 300 \times \left(-\frac{2}{3}\right)$$

$$T_3 = 2940 - 200 = 2740$$

$$\boxed{3090 \text{ N}, 2940 \text{ N}, 2740 \text{ N}}$$

b) Distance in each stage: $s = \frac{1}{2}(u + v)t$

$$s_1 = \frac{1}{2}(0 + 2)(4) = 4$$

$$s_2 = 2 \times 10 = 20$$

$$s_3 = \frac{1}{2}(2 + 0)(3) = 3$$

$$\text{height} = 4 + 20 + 3$$

$$\boxed{27 \text{ m}}$$

c) Greatest tension 3200 N gives the greatest acceleration

$$3200 - 300g = 300a$$

$$260 = 300a$$

$$a = 0.8666\ldots$$

$$0.867 \text{ m s}^{-2}$$