



- 1 A stone is dropped from rest from a bridge 20 m above a river. The stone is modelled as a particle moving freely under gravity.

- Find the time taken for the stone to reach the river.
- Find the speed of the stone as it hits the water.

Take $g = 9.8 \text{ m s}^{-2}$.

(4 marks)

Take down as positive

$$u = 0, a = 9.8, s = 20$$

$$a) s = ut + \frac{1}{2}at^2$$

$$20 = 0 + 4.9t^2$$

$$t^2 = \frac{20}{4.9}$$

$$t = 2.0203...$$

$$\boxed{2.02 \text{ s}}$$

$$b) v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 9.8 \times 20$$

$$= 392$$

$$v = 19.79...$$

$$\boxed{19.8 \text{ m s}^{-1}}$$

2

A ball is thrown vertically upwards from ground level with speed 14.7 m s^{-1} . The ball is modelled as a particle moving freely under gravity.

- Find the greatest height reached by the ball.
- Find the total time the ball is in the air.

Take $g = 9.8 \text{ m s}^{-2}$.

(4 marks)

Take up as positive: $u = 14.7$, $a = -9.8$

a) At the greatest height $v = 0$

$$0 = 14.7^2 - 2 \times 9.8 \times s$$

$$19.6s = 216.09$$

$$s = 11.025$$

$$\boxed{11.0 \text{ m}}$$

b) Back to the ground: $s = 0$

$$0 = 14.7t - 4.9t^2$$

$$0 = 4.9t(3 - t)$$

$$t = 3 \text{ s}$$

($t = 0$ is the start)

$$\boxed{3 \text{ s}}$$

3

A ball is projected vertically upwards with speed 21 m s^{-1} from a point P . The ball is modelled as a particle moving freely under gravity.

For the instant 3 s after projection,

- find the speed of the ball and state whether it is moving up or down,
- find the height of the ball above P .

Take $g = 9.8 \text{ m s}^{-2}$.

(4 marks)

Take up as positive: $u = 21$, $a = -9.8$, $t = 3$

a) $v = u + at$

$$\begin{aligned} v &= 21 - 9.8 \times 3 \\ &= -8.4 \end{aligned}$$

negative, so moving down

8.4 m s^{-1} downwards

b) $s = ut + \frac{1}{2}at^2$

$$\begin{aligned} s &= 21 \times 3 - 4.9 \times 3^2 \\ &= 63 - 44.1 \\ &= 18.9 \text{ m} \end{aligned}$$

18.9 m above P

- 4 A stone is thrown vertically downwards with speed 5 m s^{-1} from the top of a cliff. It hits the sea 3 s later. The stone is modelled as a particle moving freely under gravity.

- Find the height of the cliff above the sea.
- Find the speed of the stone as it hits the sea.

Take $g = 9.8 \text{ m s}^{-2}$.

(4 marks)

Take down as positive: $u = 5$, $a = 9.8$, $t = 3$

$$a) s = ut + \frac{1}{2}at^2$$

$$s = 5 \times 3 + 4.9 \times 3^2$$

$$= 15 + 44.1$$

$$= 59.1 \text{ m}$$

$$\boxed{59.1 \text{ m}}$$

$$b) v = u + at$$

$$v = 5 + 9.8 \times 3$$

$$= 34.4 \text{ m s}^{-1}$$

$$\boxed{34.4 \text{ m s}^{-1}}$$

- 5 A small ball is projected vertically upwards from a point A. The greatest height of the ball above A is 8.1 m. The ball is modelled as a particle moving freely under gravity.

- Find the speed of projection.
- Find the time taken for the ball to return to A.

Take $g = 9.8 \text{ m s}^{-2}$.

(4 marks)

Take up as positive: $v = 0$ at the top, $s = 8.1$, $a = -9.8$

$$a) v^2 = u^2 + 2as$$

$$0 = u^2 - 2 \times 9.8 \times 8.1$$

$$u^2 = 158.76$$

$$u = 12.6 \text{ m s}^{-1}$$

$$\boxed{12.6 \text{ m s}^{-1}}$$

$$b) \text{Back at A: } s = 0$$

$$0 = 12.6t - 4.9t^2$$

$$t = \frac{12.6}{4.9}$$

$$= 2.571...$$

$$\boxed{2.57 \text{ s}}$$

6

A ball is projected vertically upwards from ground level with speed 19.6 m s^{-1} . The ball is modelled as a particle moving freely under gravity.

Find the length of time for which the ball is at least 14.7 m above the ground.

Take $g = 9.8 \text{ m s}^{-2}$.

(5 marks)

Take up as positive: $u = 19.6$, $a = -9.8$

Find when $s = 14.7$

$$14.7 = 19.6t - 4.9t^2$$

$$4.9t^2 - 19.6t + 14.7 = 0$$

Divide by 4.9

$$t^2 - 4t + 3 = 0$$

$$(t - 1)(t - 3) = 0$$

$$t = 1 \text{ or } t = 3$$

Above 14.7 m between the two times

$$\text{time} = 3 - 1$$

$$= 2 \text{ s}$$

2 s

7

A girl throws a ball vertically upwards with speed 12 m s^{-1} . The ball leaves her hand 2 m above the ground. The ball is modelled as a particle moving freely under gravity.

- Find the time taken for the ball to hit the ground.
- Find the speed of the ball as it hits the ground.

Take $g = 9.8 \text{ m s}^{-2}$.

(6 marks)

Take up as positive: $u = 12$, $a = -9.8$

ground is 2 m below the hand: $s = -2$

$$a) s = ut + \frac{1}{2}at^2$$

$$-2 = 12t - 4.9t^2$$

$$4.9t^2 - 12t - 2 = 0$$

Quadratic formula

$$t = \frac{12 \pm \sqrt{(144 + 39.2)}}{9.8}$$

$$= \frac{12 \pm \sqrt{183.2}}{9.8}$$

$$t = 2.6056...$$

(reject the negative root)

$$\boxed{2.61 \text{ s}}$$

$$b) v^2 = u^2 + 2as$$

$$v^2 = 12^2 + 2 \times (-9.8) \times (-2)$$

$$= 183.2$$

$$v = 13.535...$$

$$\boxed{13.5 \text{ m s}^{-1}}$$

8

A stone is released from rest at the top of a well and falls freely to the water at the bottom. In the last second of its fall the stone travels 24.5 m.

- Find the total time for which the stone falls.
- Find the depth of the water below the point of release.

Take $g = 9.8 \text{ m s}^{-2}$.

(6 marks)

Take down as positive, $u = 0$: $s = 4.9t^2$

Let the whole fall take T seconds

$$\text{distance in } T \text{ seconds} = 4.9T^2$$

$$\text{distance in } T - 1 \text{ seconds} = 4.9(T - 1)^2$$

a) The difference is the last second

$$4.9T^2 - 4.9(T - 1)^2 = 24.5$$

$$4.9(T^2 - T^2 + 2T - 1) = 24.5$$

$$4.9(2T - 1) = 24.5$$

$$2T - 1 = 5$$

$$T = 3$$

3 s

b) Total distance

$$s = 4.9 \times 3^2$$

$$= 44.1 \text{ m}$$

44.1 m

9

A small stone is projected vertically upwards with speed 14 m s^{-1} from the top of a tower. The top of the tower is 40 m above horizontal ground. The stone falls past the tower and lands on the ground. The stone is modelled as a particle moving freely under gravity.

- Find the greatest height of the stone above the ground.
- Find the time from projection until the stone hits the ground.
- State one limitation of the model that could affect your answer to part (b), and say how.

Take $g = 9.8 \text{ m s}^{-2}$.

(8 marks)

Take up as positive: $u = 14$, $a = -9.8$

a) At the top $v = 0$

$$0 = 14^2 - 2 \times 9.8 \times s$$

$$s = \frac{196}{19.6}$$

$$= 10 \text{ m}$$

$$\text{height above ground} = 40 + 10$$

$$= 50 \text{ m}$$

50 m

b) Ground is 40 m below the start: $s = -40$

$$-40 = 14t - 4.9t^2$$

$$4.9t^2 - 14t - 40 = 0$$

Quadratic formula

$$t = \frac{14 \pm \sqrt{(196 + 784)}}{9.8}$$

$$= \frac{14 \pm \sqrt{980}}{9.8}$$

$$t = 4.6229...$$

(reject the negative root)

4.62 s

c) Limitation

Air resistance is ignored

In reality it slows the stone, so the time would be different
(or: the stone is not really a particle, so spin is ignored)

10

A ball is dropped from rest from a height of 2.5 m above a horizontal floor. It hits the floor and rebounds vertically upwards with half the speed it had just before impact. The ball is modelled as a particle moving freely under gravity, and the time in contact with the floor is ignored.

- Find the speed of the ball just before it hits the floor.
- Find the greatest height reached by the ball after its first bounce.
- Find the time from release until the ball reaches this greatest height.

Take $g = 9.8 \text{ m s}^{-2}$.

(7 marks)

a) Fall: take down as positive, $u = 0$, $s = 2.5$

$$v^2 = 0 + 2 \times 9.8 \times 2.5$$

$$= 49$$

$$v = 7 \text{ m s}^{-1}$$

$$\boxed{7 \text{ m s}^{-1}}$$

b) Rebound: take up as positive, $u = 3.5$, $v = 0$

$$0 = 3.5^2 - 2 \times 9.8 \times h$$

$$h = \frac{12.25}{19.6}$$

$$= 0.625 \text{ m}$$

$$\boxed{0.625 \text{ m}}$$

c) Time for each stage: $v = u + at$

$$\text{fall: } 7 = 0 + 9.8t_1$$

$$t_1 = 0.7143 \text{ s}$$

$$\text{rise: } 0 = 3.5 - 9.8t_2$$

$$t_2 = 0.3571 \text{ s}$$

$$\text{total} = 0.7143 + 0.3571$$

$$= 1.071 \text{ s}$$

$$\boxed{1.07 \text{ s}}$$

11

A ball A is dropped from rest from a window 30 m above the ground. At the same instant a second ball B is projected vertically upwards from the ground directly below the window with speed 15 m s^{-1} . The balls move in the same vertical line and are modelled as particles moving freely under gravity.

- Find the time at which the balls collide.
- Find the height above the ground at which they collide.
- Find the velocity of B just before the collision, and state whether B is moving up or down.

Take $g = 9.8 \text{ m s}^{-2}$.

(7 marks)

After t seconds

$$A \text{ has fallen} = 4.9t^2$$

$$B \text{ has risen} = 15t - 4.9t^2$$

a) They meet when the distances add to 30

$$4.9t^2 + 15t - 4.9t^2 = 30$$

$$15t = 30$$

$$t = 2 \text{ s}$$

$$\boxed{2 \text{ s}}$$

b) Height of B

$$h = 15 \times 2 - 4.9 \times 2^2$$

$$= 30 - 19.6$$

$$= 10.4 \text{ m}$$

$$\boxed{10.4 \text{ m}}$$

c) B : take up as positive, $v = u + at$

$$v = 15 - 9.8 \times 2$$

$$= -4.6$$

negative, so moving down

$$\boxed{4.6 \text{ m s}^{-1} \text{ downwards}}$$

12

A cricket ball is hit vertically upwards from a point 1.5 m above the ground. It hits the ground 3.2 s later. The ball is modelled as a particle moving freely under gravity.

- Find the speed with which the ball was hit.
- Find the greatest height of the ball above the ground.
- Find the length of time for which the ball is more than 10 m above the ground.
- State one way in which the model could be refined.

Take $g = 9.8 \text{ m s}^{-2}$.

(9 marks)

Take up as positive, $a = -9.8$

the ground is 1.5 m below the start

a) $s = -1.5$ when $t = 3.2$

$$-1.5 = 3.2u - 4.9 \times 3.2^2$$

$$-1.5 = 3.2u - 50.176$$

$$3.2u = 48.676$$

$$u = 15.211\dots$$

$$\boxed{15.2 \text{ m s}^{-1}}$$

b) At the top $v = 0$

$$0 = 15.211^2 - 2 \times 9.8 \times s$$

$$s = \frac{231.38}{19.6}$$

$$= 11.805 \text{ m}$$

$$\text{height} = 1.5 + 11.805$$

$$= 13.305 \text{ m}$$

$$\boxed{13.3 \text{ m}}$$

c) 10 m above the ground is 8.5 m above the start

$$8.5 = 15.211t - 4.9t^2$$

$$4.9t^2 - 15.211t + 8.5 = 0$$

Quadratic formula

$$t = \frac{15.211 \pm \sqrt{(231.38 - 166.6)}}{9.8}$$

$$t = 0.7308 \text{ or } t = 2.3734$$

$$\text{time} = 2.3734 - 0.7308$$

$$= 1.6426 \text{ s}$$

1.64 s

d) Refinement

Include air resistance in the model