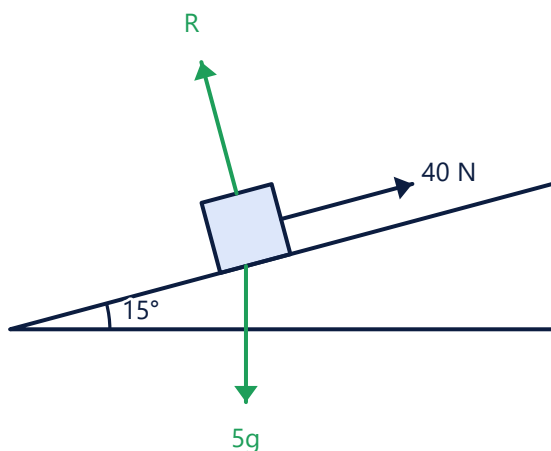




- 1 A box of mass 5 kg is pulled up a line of greatest slope of a smooth ramp inclined at 15° to the horizontal. The pulling force has magnitude 40 N and acts parallel to the ramp.

The box is modelled as a particle.

- Find the acceleration of the box.
- Find the magnitude of the normal reaction of the ramp on the box.



(4 marks)

a) $F = ma$ up the ramp

$$40 - 5g \sin 15^\circ = 5a$$

$$40 - 12.682... = 5a$$

$$27.317... = 5a$$

$$a = 5.4635...$$

$$a = 5.46 \text{ m s}^{-2}$$

b) No motion perpendicular to the ramp

$$R = 5g \cos 15^\circ$$

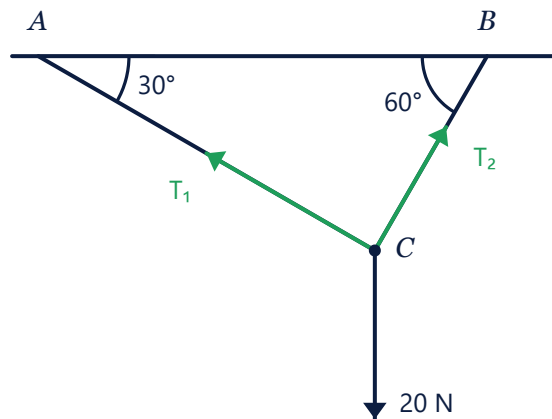
$$R = 47.330...$$

$$R = 47.3 \text{ N}$$

2

A particle C of weight 20 N hangs in equilibrium from two light inextensible strings AC and BC . The points A and B are on a horizontal ceiling. The string AC makes an angle of 30° with the ceiling and the string BC makes an angle of 60° with the ceiling, as shown in the diagram.

Find the tension in each string.



(5 marks)

Let the tensions be T_1 in AC and T_2 in BC

Resolve horizontally

$$T_1 \cos 30^\circ = T_2 \cos 60^\circ$$

$$\frac{\sqrt{3}}{2} T_1 = \frac{1}{2} T_2$$

$$T_2 = \sqrt{3} T_1$$

Resolve vertically

$$T_1 \sin 30^\circ + T_2 \sin 60^\circ = 20$$

$$\frac{1}{2} T_1 + \sqrt{3} T_1 \times \frac{\sqrt{3}}{2} = 20$$

$$\frac{1}{2} T_1 + \frac{3}{2} T_1 = 20$$

$$2T_1 = 20$$

$$T_1 = 10$$

$$T_2 = 10\sqrt{3} = 17.320\dots$$

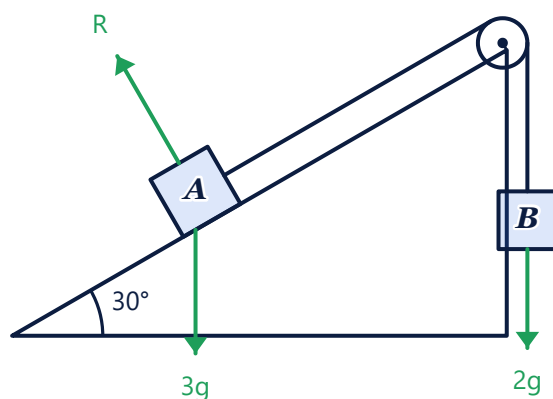
$$\boxed{AC: 10\text{ N}, BC: 10\sqrt{3}\text{ N (17.3 N)}}$$

3

A particle A of mass 3 kg rests on a smooth plane inclined at 30° to the horizontal. It is attached to one end of a light inextensible string which passes over a small smooth pulley fixed at the top of the plane. The other end of the string is attached to a particle B of mass 2 kg , which hangs freely below the pulley. The part of the string from A to the pulley is parallel to a line of greatest slope, as shown in the diagram.

The system is released from rest with the string taut.

- Find the acceleration of the particles.
- Find the tension in the string.



(6 marks)

a) The string is inextensible, so A and B have the same acceleration a
 $F = ma$ for B (down positive)

$$2g - T = 2a$$

$F = ma$ for A (up the plane positive)

$$T - 3g \sin 30^\circ = 3a$$

Add the equations to remove T

$$2g - 1.5g = 5a$$

$$0.5g = 5a$$

$$a = 0.1g = 0.98$$

$$a = 0.98 \text{ m s}^{-2}$$

b) Substitute into the equation for B

$$2g - T = 2(0.98)$$

$$T = 19.6 - 1.96$$

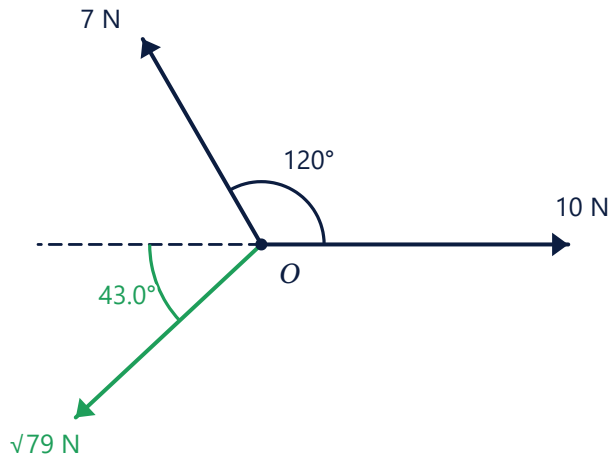
$$T = 17.64$$

$$T = 17.6 \text{ N}$$

4

Three horizontal forces act on a particle O , which is in equilibrium. One force has magnitude 10 N and acts due east. A second force has magnitude 7 N and acts in a direction making an angle of 120° with the first force, measured anticlockwise (so it points north of west). The third force has magnitude P newtons.

- Find the value of P , giving your answer as an exact surd.
- Find the angle that the third force makes with due west.



(6 marks)

Take east as positive x and north as positive y

Resolve the 7 N force

$$x: 7 \cos 120^\circ = -3.5$$

$$y: 7 \sin 120^\circ = \frac{7\sqrt{3}}{2} = 6.0621\dots$$

a) In equilibrium, the third force cancels the other two

$$x: 10 - 3.5 + X = 0, \text{ so } X = -6.5$$

$$y: 6.0621\dots + Y = 0, \text{ so } Y = -6.0621\dots$$

$$P^2 = 6.5^2 + \left(\frac{7\sqrt{3}}{2}\right)^2$$

$$P^2 = 42.25 + 36.75 = 79$$

$$P = \sqrt{79} (= 8.89)$$

b) Both components are negative: the force points south of west

$$\tan \theta = \frac{6.0621\dots}{6.5}$$

$$\theta = 43.003\dots^\circ$$

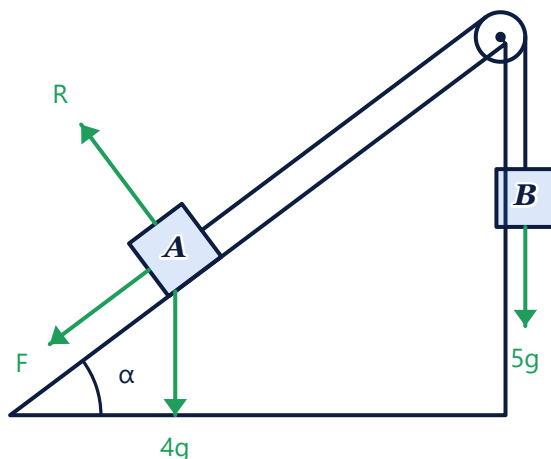
$$43.0^\circ \text{ south of west}$$

5

A rough plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$. A particle A of mass 4 kg is on the plane and is attached to a light inextensible string. The string passes over a small smooth pulley at the top of the plane, and a particle B of mass 5 kg hangs freely on the other end. The coefficient of friction between A and the plane is 0.25

The system is released from rest with the string taut and A moves up the plane.

- Find the acceleration of A .
- Find the tension in the string.
- State how you have used the fact that the string is inextensible.



(8 marks)

$$\tan \alpha = \frac{3}{4} : \sin \alpha = \frac{3}{5}, \cos \alpha = \frac{4}{5}$$

a) A : resolve perpendicular to the plane

$$R = 4g \cos \alpha = 3.2g$$

A moves up, so friction acts down the plane

$$F = 0.25 \times 3.2g = 0.8g$$

$F = ma$ for A (up the plane)

$$T - 4g\left(\frac{3}{5}\right) - 0.8g = 4a$$

$$T - 2.4g - 0.8g = 4a$$

$F = ma$ for B (down)

$$5g - T = 5a$$

Add to remove T

$$5g - 3.2g = 9a$$

$$1.8g = 9a$$

$$a = 0.2g = 1.96$$

$$a = 1.96 \text{ m s}^{-2}$$

b) Substitute into the equation for B

$$T = 5g - 5(1.96)$$

$$T = 49 - 9.8 = 39.2$$

$$T = 39.2 \text{ N}$$

c) Where the model was used

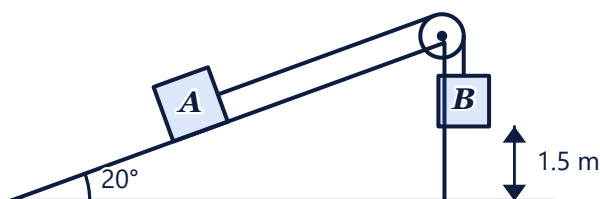
A and B have the same acceleration

6

A particle A of mass 2 kg is on a rough plane inclined at 20° to the horizontal. The coefficient of friction between A and the plane is 0.2 . A is attached to a light inextensible string which passes over a small smooth pulley at the top of the plane. A particle B of mass 3 kg hangs on the other end of the string, 1.5 m above the ground.

The system is released from rest with the string taut. B hits the ground and does not rebound. In the subsequent motion A does not reach the pulley.

- Find the acceleration of B before it hits the ground.
- Find the speed of B as it hits the ground.
- Find the further distance that A moves up the plane before it comes to rest.



(9 marks)

a) A : $R = 2g \cos 20^\circ$, moving up so friction acts down the plane

$$F = 0.2 \times 2g \cos 20^\circ = 3.6835\dots$$

$F = ma$ for A (up the plane)

$$T - 2g \sin 20^\circ - 3.6835\dots = 2a$$

$F = ma$ for B (down)

$$3g - T = 3a$$

Add to remove T

$$3g - 2g \sin 20^\circ - 3.6835\dots = 5a$$

$$29.4 - 6.7035\dots - 3.6835\dots = 5a$$

$$19.012\dots = 5a$$

$$a = 3.8025\dots$$

$$a = 3.80 \text{ m s}^{-2}$$

b) B : $u = 0$, $a = 3.8025\dots$, $s = 1.5$

$$v^2 = 0 + 2(3.8025\dots)(1.5)$$

$$v^2 = 11.407\dots$$

$$v = 3.3775\dots$$

$$3.38 \text{ m s}^{-1}$$

c) B has landed, so the string is slack: $T = 0$

$F = ma$ for A, down the plane positive

$$2g \sin 20^\circ + 3.6835... = 2d$$

$$d = 5.1935...$$

$$u^2 = 11.407..., v = 0$$

$$0 = 11.407... - 2(5.1935...)s$$

$$s = \frac{11.407...}{10.387...}$$

$$s = 1.0982...$$

$$\boxed{1.10 \text{ m}}$$

7

A particle A of mass 4 kg rests on a smooth plane inclined at an angle θ to the horizontal. A is attached to a light inextensible string which passes over a small smooth pulley at the top of the plane. A particle B of mass 2.5 kg hangs freely on the other end of the string. The system is in equilibrium.

a. Find the size of the angle θ .

B is replaced by a particle of mass 3.5 kg and the system is released from rest.

b. Find the acceleration of A and the tension in the string.

(7 marks)

a) B in equilibrium

$$T = 2.5g$$

A in equilibrium: resolve parallel to the plane

$$T = 4g \sin \theta$$

$$4g \sin \theta = 2.5g$$

$$\sin \theta = 0.625$$

$$\theta = 38.682\dots^\circ$$

$$\theta = 38.7^\circ$$

b) $F = ma$ for the 3.5 kg particle (down)

$$3.5g - T = 3.5a$$

$F = ma$ for A (up the plane), and $4g \sin \theta = 2.5g$

$$T - 2.5g = 4a$$

Add to remove T

$$g = 7.5a$$

$$a = 1.3066\dots$$

$$T = 3.5(9.8 - 1.3066\dots)$$

$$T = 29.726\dots$$

$$a = 1.31\text{ m s}^{-2}, T = 29.7\text{ N}$$

8

A car of mass 1200 kg tows a trailer of mass 400 kg up a straight road inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{20}$. The tow bar is light and parallel to the road. The engine produces a driving force of 2400 N. The resistances to motion are 300 N on the car and 100 N on the trailer.

- Find the acceleration of the car.
- Find the tension in the tow bar.

When the speed is 12 m s^{-1} the tow bar breaks. The resistance on the trailer is unchanged.

- Find the distance the trailer moves up the road after the tow bar breaks before it comes to rest.

(8 marks)

a) Whole system up the road: weight component $1600g \times \frac{1}{20}$

$$2400 - 300 - 100 - 1600g \times \frac{1}{20} = 1600a$$

$$2000 - 784 = 1600a$$

$$a = \frac{1216}{1600}$$

$$a = 0.76 \text{ m s}^{-2}$$

b) Trailer alone, up the road

$$T - 100 - 400g \times \frac{1}{20} = 400 \times 0.76$$

$$T - 100 - 196 = 304$$

$$T = 600 \text{ N}$$

c) Trailer alone, no tension: down the road positive

$$100 + 196 = 400d$$

$$d = 0.74$$

$$u = 12, v = 0$$

$$0 = 12^2 - 2(0.74)s$$

$$s = \frac{144}{1.48}$$

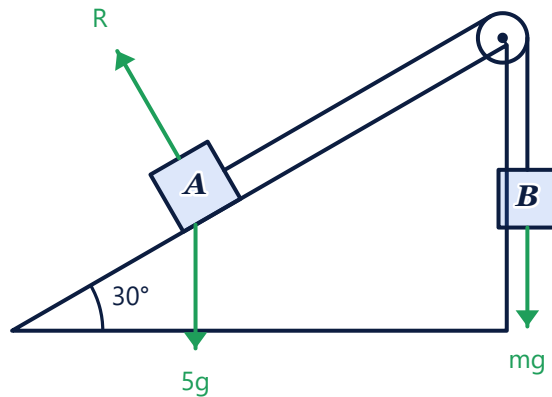
$$s = 97.297...$$

$$97.3 \text{ m}$$

9

A particle A of mass 5 kg is on a rough plane inclined at 30° to the horizontal. The coefficient of friction between A and the plane is 0.4 . A is attached to a light inextensible string which passes over a small smooth pulley at the top of the plane. A particle B of mass $m\text{ kg}$ hangs freely on the other end of the string.

Given that the system is in equilibrium, find the range of possible values of m .



(8 marks)

B in equilibrium

$$T = mg$$

A : resolve perpendicular to the plane

$$R = 5g \cos 30^\circ$$

$$F_{\text{max}} = 0.4 \times 5g \cos 30^\circ = 2g \cos 30^\circ$$

Least m : A about to slide DOWN, friction acts UP the plane

$$mg + 2g \cos 30^\circ = 5g \sin 30^\circ$$

$$m = 2.5 - 2 \cos 30^\circ$$

$$m = 2.5 - \sqrt{3} = 0.76794\dots$$

Greatest m : A about to slide UP, friction acts DOWN the plane

$$mg = 5g \sin 30^\circ + 2g \cos 30^\circ$$

$$m = 2.5 + \sqrt{3} = 4.2320\dots$$

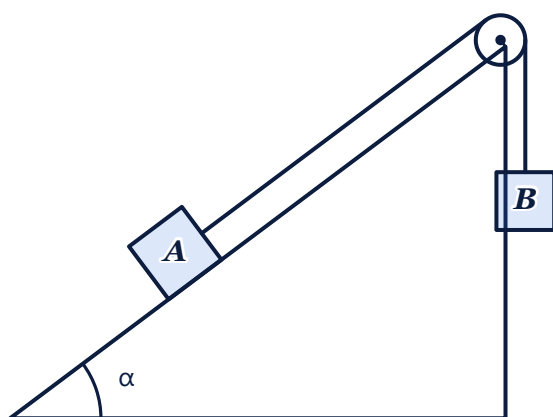
$$\boxed{0.768 \leq m \leq 4.23}$$

10

A rough plane is inclined to the horizontal at an angle α , where $\sin \alpha = \frac{3}{5}$. A particle A of mass $2m$ is on the plane and is attached to a light inextensible string which passes over a small smooth pulley P at the top of the plane. A particle B of mass $3m$ hangs freely on the other end of the string. The part of the string from A to P is parallel to a line of greatest slope. The coefficient of friction between A and the plane is $\frac{1}{2}$.

The system is released from rest with the string taut.

- Show that the acceleration of B is $\frac{1}{5}g$.
- Find, in terms of m and g , the tension in the string.
- Find, in terms of m and g , the magnitude of the force exerted on the pulley by the string, giving your answer as an exact surd.



(10 marks)

$$\sin \alpha = \frac{3}{5}, \text{ so } \cos \alpha = \frac{4}{5}$$

a) A : resolve perpendicular to the plane

$$R = 2mg \times \frac{4}{5} = \frac{8}{5}mg$$

Moving up, friction acts down the plane

$$F = \frac{1}{2} \times \frac{8}{5}mg = \frac{4}{5}mg$$

$F = ma$ for A (up the plane)

$$T - 2mg \times \frac{3}{5} - \frac{4}{5}mg = 2ma$$

$$T - 2mg = 2ma$$

$F = ma$ for B (down)

$$3mg - T = 3ma$$

Add to remove T

$$mg = 5ma$$

$$a = \frac{1}{5}g$$

b) Substitute into the equation for B

$$T = 3mg - 3m \times \frac{1}{5}g$$

$$T = \frac{12}{5}mg$$

c) Both parts of the string pull on P with tension T

Angle between them = $90^\circ - \alpha$, and $\cos(90^\circ - \alpha) = \sin \alpha = \frac{3}{5}$

Add the two forces (cosine rule for vectors)

$$F^2 = T^2 + T^2 + 2T \times T \times \frac{3}{5}$$

$$F^2 = \frac{16}{5}T^2$$

$$F = \frac{4}{\sqrt{5}}T$$

$$F = \frac{4}{\sqrt{5}} \times \frac{12}{5}mg$$

$$F = \frac{48}{5\sqrt{5}}mg$$

$$F = \frac{48\sqrt{5}}{25}mg \text{ (about } 4.29mg\text{)}$$

11

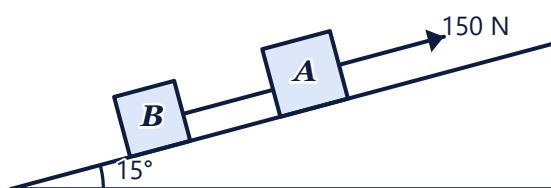
Two boxes, A of mass 10 kg and B of mass 6 kg , are joined by a light inextensible rope and are pulled up a line of greatest slope of a rough ramp inclined at 15° to the horizontal. A is higher up the ramp than B , and the rope is parallel to the ramp. A force of magnitude 150 N , parallel to the ramp, acts on A , as shown in the diagram. The coefficient of friction between each box and the ramp is 0.3 .

The boxes are modelled as particles.

- Find the acceleration of the boxes.
- Find the tension in the rope.
- Show that, for this pulling force, the tension in the rope does not depend on the angle of the ramp, provided both boxes have the same coefficient of friction.

When the boxes are moving at 3 m s^{-1} the rope breaks.

- Find the further distance that B moves up the ramp, and determine whether B then stays at rest.



(10 marks)

a) Whole system (16 kg), up the ramp

$$R = 16g \cos 15^\circ$$

$$\text{friction} = 0.3 \times 16g \cos 15^\circ = 45.437\dots$$

$$16g \sin 15^\circ = 40.582\dots$$

$$150 - 40.582\dots - 45.437\dots = 16a$$

$$63.980\dots = 16a$$

$$a = 3.9987\dots$$

$$\boxed{a = 4.00\text{ m s}^{-2}}$$

b) B alone, up the ramp

$$T - 6g \sin 15^\circ - 0.3 \times 6g \cos 15^\circ = 6a$$

$$T - 15.218\dots - 17.038\dots = 6(3.9987\dots)$$

$$T = 56.25$$

$$\boxed{T = 56.3\text{ N}}$$

c) Let θ be the angle and $k = g(\sin \theta + \mu \cos \theta)$, the same for both boxes

$$\text{whole: } 150 - 16k = 16a$$

$$B: T - 6k = 6a$$

$$T = 6(a + k)$$

$$a + k = \frac{150}{16}$$

$$T = 6 \times \frac{150}{16} = 56.25$$

T does not depend on θ (or μ)

d) B alone, rope broken: down the ramp positive

$$6g \sin 15^\circ + 0.3 \times 6g \cos 15^\circ = 6d$$

$$d = g(\sin 15^\circ + 0.3 \cos 15^\circ) = 5.3762...$$

$$0 = 3^2 - 2(5.3762...)s$$

$$s = \frac{9}{10.752...}$$

$$s = 0.83701...$$

At rest: compare $\tan 15^\circ$ with μ

$$\tan 15^\circ = 0.268 < 0.3$$

0.837 m, then B stays at rest

12

A fixed wedge has two faces, one inclined at 30° to the horizontal and one inclined at 60° to the horizontal, meeting at a ridge. A small smooth pulley is fixed at the ridge. A particle A of mass 4 kg is on the 30° face and a particle B of mass 6 kg is on the 60° face. They are joined by a light inextensible string which passes over the pulley, with each part of the string parallel to a line of greatest slope of its face, as shown in the diagram.

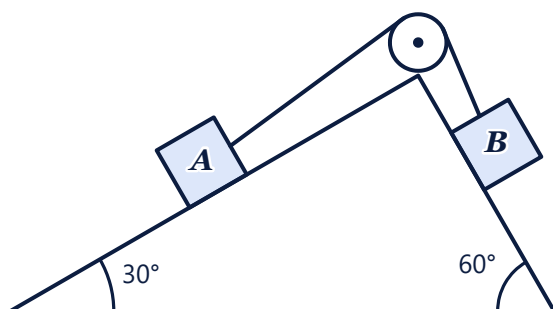
The 30° face is rough, with coefficient of friction 0.2 between A and the face. The 60° face is smooth.

The system is released from rest with the string taut, and B moves down its face.

a. Find the acceleration of the particles and the tension in the string.

After B has moved 1.2 m the string breaks. A does not reach the pulley.

b. Find the further distance that A moves up its face before coming to rest.



(10 marks)

a) $F = ma$ for B , down its face (smooth)

$$6g \sin 60^\circ - T = 6a$$

A : $R = 4g \cos 30^\circ$, moving up so friction $0.2R$ acts down the face

$F = ma$ for A , up its face

$$T - 4g \sin 30^\circ - 0.2 \times 4g \cos 30^\circ = 4a$$

Add to remove T

$$6g \sin 60^\circ - 4g \sin 30^\circ - 0.8g \cos 30^\circ = 10a$$

$$50.922... - 19.6 - 6.7896... = 10a$$

$$24.532... = 10a$$

$$a = 2.4532...$$

$$T = 6(9.8 \sin 60^\circ - 2.4532...)$$

$$T = 36.202...$$

$$\boxed{a = 2.45 \text{ m s}^{-2}, T = 36.2 \text{ N}}$$

b) Speed when the string breaks: $u = 0$, $s = 1.2$

$$v^2 = 2(2.4532...)(1.2) = 5.8878...$$

A alone: down its face positive

$$4g \sin 30^\circ + 0.2 \times 4g \cos 30^\circ = 4d$$

$$d = g(0.5 + 0.2 \cos 30^\circ) = 6.5974...$$

$$0 = 5.8878... - 2(6.5974...)s$$

$$s = \frac{5.8878...}{13.194...}$$

$$s = 0.44622...$$

0.446 m