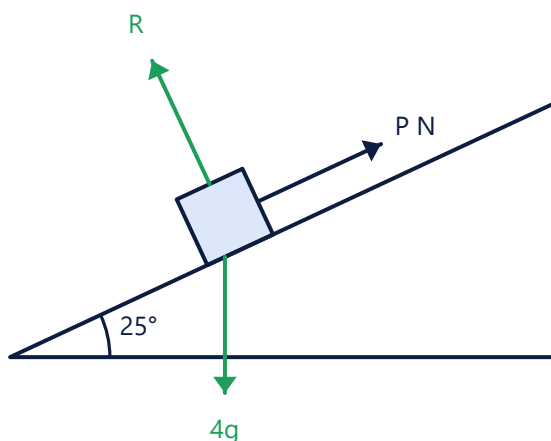




1 A particle of mass 4 kg is held at rest on a smooth plane inclined at 25° to the horizontal. It is held by a force of magnitude P newtons acting up a line of greatest slope of the plane, as shown in the diagram.

- Find the value of P .
- Find the magnitude of the normal reaction of the plane on the particle.



(4 marks)

a) Resolve parallel to the plane (up the plane positive)

$$P - 4g \sin 25^\circ = 0$$

$$P = 4g \sin 25^\circ$$

$$P = 16.566\dots$$

$$P = 16.6$$

b) Resolve perpendicular to the plane

$$R - 4g \cos 25^\circ = 0$$

$$R = 4g \cos 25^\circ$$

$$R = 35.527\dots$$

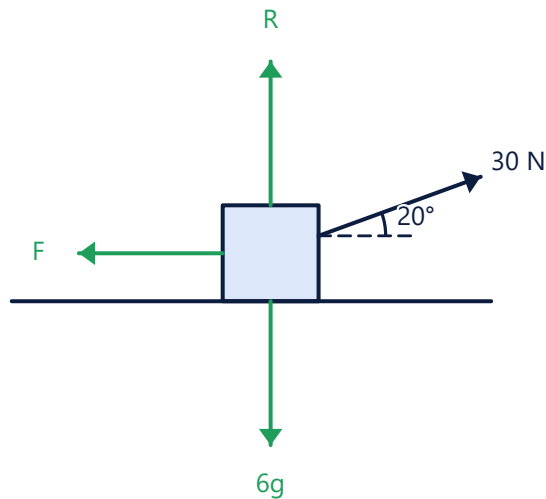
$$R = 35.5 \text{ N}$$

2

A box of mass 6 kg rests on rough horizontal ground. A rope is attached to the box and pulled with a force of 30 N at an angle of 20° above the horizontal, as shown in the diagram. The box is on the point of moving.

The box is modelled as a particle.

- Find the magnitude of the normal reaction of the ground on the box.
- Find the coefficient of friction between the box and the ground.



(5 marks)

a) Resolve vertically

$$R + 30 \sin 20^\circ = 6g$$

$$R = 58.8 - 30 \sin 20^\circ$$

$$R = 48.539\dots$$

$$\boxed{R = 48.5 \text{ N}}$$

b) Resolve horizontally

$$F = 30 \cos 20^\circ$$

$$F = 28.190\dots$$

On the point of moving, so friction is limiting: $F = \mu R$

$$28.190\dots = \mu \times 48.539\dots$$

$$\mu = \frac{28.190\dots}{48.539\dots}$$

$$\mu = 0.5807\dots$$

$$\boxed{\mu = 0.581}$$

3

A sledge of mass 12 kg is at rest on a rough slope inclined at 15° to the horizontal. The coefficient of friction between the sledge and the slope is 0.4

The sledge is modelled as a particle.

- Find the magnitude of the frictional force acting on the sledge.
- Show that the sledge is not in limiting equilibrium.

(5 marks)

a) The sledge is at rest, so friction acts up the slope

Resolve parallel to the slope

$$F = 12g \sin 15^\circ$$

$$F = 30.437...$$

$$F = 30.4 \text{ N}$$

b) Resolve perpendicular to the slope

$$R = 12g \cos 15^\circ$$

$$R = 113.59...$$

Largest friction possible

$$\mu R = 0.4 \times 113.59...$$

$$\mu R = 45.437...$$

$$30.4 < 45.4$$

$$F < \mu R, \text{ so friction is not limiting}$$

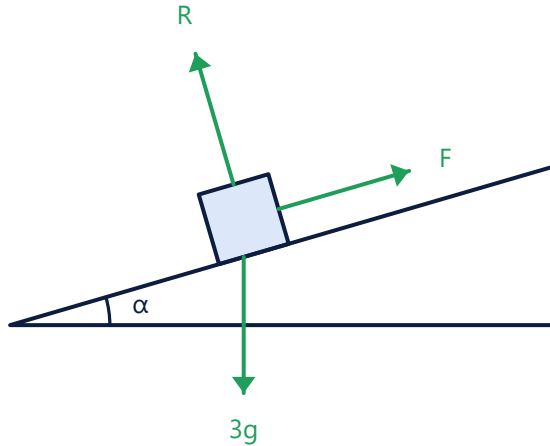
4

A rough plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{7}{24}$

A small parcel of mass 3 kg is released from rest on the plane and slides down a line of greatest slope. The coefficient of friction between the parcel and the plane is 0.1

The parcel is modelled as a particle.

- Find the acceleration of the parcel.
- Find the speed of the parcel when it has slid 5 m.



(7 marks)

$$\tan \alpha = \frac{7}{24} : \text{a } 7, 24, 25 \text{ triangle}$$

$$\sin \alpha = \frac{7}{25}$$

$$\cos \alpha = \frac{24}{25}$$

a) Resolve perpendicular to the plane

$$R = 3g \cos \alpha$$

$$R = 3 \times 9.8 \times \frac{24}{25}$$

$$R = 28.224$$

Sliding, so friction is limiting

$$F = \mu R$$

$$F = 0.1 \times 28.224 = 2.8224$$

$F = ma$ down the plane

$$3g \sin \alpha - F = 3a$$

$$3 \times 9.8 \times \frac{7}{25} - 2.8224 = 3a$$

$$8.232 - 2.8224 = 3a$$

$$5.4096 = 3a$$

$$a = 1.8032$$

$$a = 1.80 \text{ m s}^{-2}$$

b) $u = 0$, $a = 1.8032$, $s = 5$, use $v^2 = u^2 + 2as$

$$v^2 = 0 + 2 \times 1.8032 \times 5$$

$$v^2 = 18.032$$

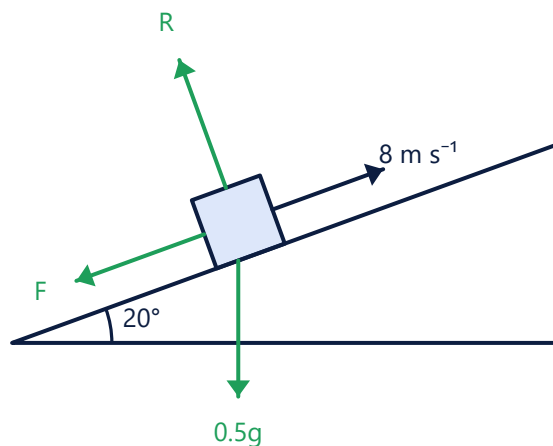
$$v = 4.2464\dots$$

$$v = 4.25 \text{ m s}^{-1}$$

5

A puck of mass 0.5 kg is projected with speed 8 m s^{-1} up a line of greatest slope of a rough plane inclined at 20° to the horizontal. The coefficient of friction between the puck and the plane is 0.3 . The puck is modelled as a particle.

- Find the deceleration of the puck as it moves up the plane.
- Find the distance the puck travels up the plane before it comes to rest.
- Determine whether the puck remains at rest, giving a reason for your answer.



(8 marks)

a) Moving up the plane, so friction acts down the plane

Resolve perpendicular to the plane

$$R = 0.5g \cos 20^\circ$$

$$R = 4.6045\dots$$

$$F = \mu R = 0.3 \times 4.6045\dots = 1.3813\dots$$

$F = ma$, taking down the plane as positive

$$0.5g \sin 20^\circ + F = 0.5d$$

$$1.6758\dots + 1.3813\dots = 0.5d$$

$$d = 6.1144\dots$$

$$\boxed{\text{deceleration} = 6.11 \text{ m s}^{-2}}$$

b) $u = 8$, $v = 0$, $a = -6.1144\dots$, use $v^2 = u^2 + 2as$

$$0 = 8^2 - 2(6.1144\dots)s$$

$$s = \frac{64}{12.228\dots}$$

$$s = 5.2334\dots$$

$$\boxed{s = 5.23 \text{ m}}$$

c) At rest: compare the pull down the plane with the largest friction

$$0.5g \sin 20^\circ = 1.68 \text{ N}$$

$$\mu R = 1.38 \text{ N}$$

$$1.68 > 1.38$$

friction cannot hold it: the puck slides back down

6

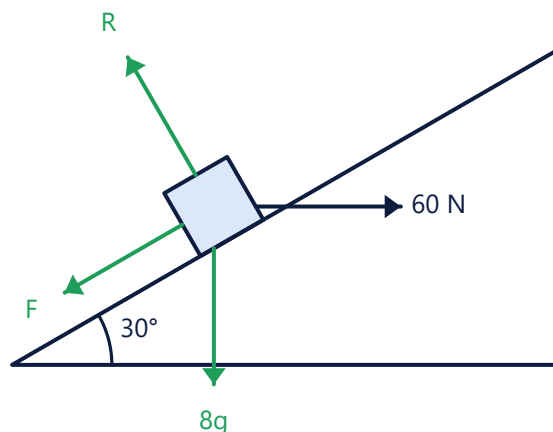
A block of mass 8 kg rests on a rough plane inclined at 30° to the horizontal. A horizontal force of magnitude 60 N acts on the block, in the vertical plane containing a line of greatest slope, pushing towards the plane as shown in the diagram. The block is on the point of moving up the plane.

The block is modelled as a particle.

- Find the magnitude of the normal reaction of the plane on the block.
- Find the coefficient of friction between the block and the plane.

The horizontal force is now removed.

- Find the acceleration of the block down the plane.



(9 marks)

a) Resolve perpendicular to the plane

The horizontal force has a component $60 \sin 30^\circ$ into the plane

$$R = 8g \cos 30^\circ + 60 \sin 30^\circ$$

$$R = 67.896... + 30$$

$$R = 97.896...$$

$$R = 97.9 \text{ N}$$

b) About to move up the plane, so friction acts down the plane

Resolve parallel to the plane

$$60 \cos 30^\circ = 8g \sin 30^\circ + F$$

$$51.961... = 39.2 + F$$

$$F = 12.761...$$

Limiting friction: $F = \mu R$

$$\mu = \frac{12.761...}{97.896...}$$

$$\mu = 0.13035...$$

$$\mu = 0.130$$

$$c) \text{ Now } R = 8g \cos 30^\circ = 67.896\dots$$

$$F = 0.13035\dots \times 67.896\dots = 8.8508\dots$$

$F = ma$ down the plane

$$8g \sin 30^\circ - F = 8a$$

$$39.2 - 8.8508\dots = 8a$$

$$a = 3.7936\dots$$

$$a = 3.79 \text{ m s}^{-2}$$

7

A particle of mass m is on a rough plane inclined at an angle θ to the horizontal. The coefficient of friction between the particle and the plane is μ . A force of magnitude P , acting up a line of greatest slope, pulls the particle up the plane at constant speed.

a. Show that $P = mg(\sin \theta + \mu \cos \theta)$.

The force is replaced by a force of magnitude $\frac{1}{2}P$, also acting up the line of greatest slope. The particle is now at rest and on the point of sliding down the plane.

b. Show that $\mu = \frac{1}{3} \tan \theta$.

(7 marks)

a) Resolve perpendicular to the plane

$$R = mg \cos \theta$$

Moving, so friction is limiting and acts down the plane

$$F = \mu mg \cos \theta$$

Constant speed, so the forces along the plane balance

$$P = mg \sin \theta + F$$

$$P = mg \sin \theta + \mu mg \cos \theta$$

$$P = mg(\sin \theta + \mu \cos \theta)$$

b) Now about to slide down, so friction acts up the plane

$$\frac{1}{2}P + \mu mg \cos \theta = mg \sin \theta$$

Substitute P from part a

$$\frac{1}{2}mg(\sin \theta + \mu \cos \theta) + \mu mg \cos \theta = mg \sin \theta$$

Divide by mg

$$\frac{1}{2} \sin \theta + \frac{1}{2} \mu \cos \theta + \mu \cos \theta = \sin \theta$$

$$\frac{3}{2} \mu \cos \theta = \frac{1}{2} \sin \theta$$

$$\mu = \frac{\sin \theta}{3 \cos \theta}$$

$$\mu = \frac{1}{3} \tan \theta$$

8

A sledge of mass 15 kg is pulled along rough horizontal ground by a rope inclined at 30° above the horizontal. The tension in the rope is 60 N. The coefficient of friction between the sledge and the ground is 0.2

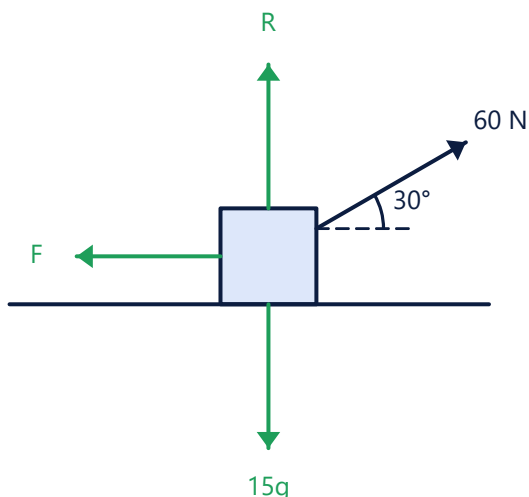
The sledge is modelled as a particle and the rope as light.

a. Find the acceleration of the sledge.

The sledge is now pushed along the same ground by a light rod inclined at 30° **below** the horizontal. The thrust in the rod is 60 N.

b. Explain why the acceleration is now less than in part (a).

c. Find the new acceleration.



(8 marks)

a) Resolve vertically

$$R + 60 \sin 30^\circ = 15g$$

$$R = 147 - 30 = 117$$

Moving, so friction is limiting

$$F = 0.2 \times 117 = 23.4$$

$F = ma$ horizontally

$$60 \cos 30^\circ - 23.4 = 15a$$

$$51.961... - 23.4 = 15a$$

$$a = 1.9041...$$

$$a = 1.90 \text{ m s}^{-2}$$

b) The thrust now has a vertical component downwards

so R is larger, so the friction μR is larger

the forward component $60 \cos 30^\circ$ is the same

larger friction, same forward force, so smaller acceleration

c) Resolve vertically

$$R = 15g + 60 \sin 30^\circ$$

$$R = 147 + 30 = 177$$

$$F = 0.2 \times 177 = 35.4$$

$$60 \cos 30^\circ - 35.4 = 15a$$

$$a = 1.1041\dots$$

$$a = 1.10 \text{ m s}^{-2}$$

9

A small crate of mass 3 kg is on a rough ramp inclined at 35° to the horizontal. The coefficient of friction between the crate and the ramp is 0.5

A force of magnitude P newtons, acting up a line of greatest slope of the ramp, keeps the crate at rest. The crate is modelled as a particle.

- Explain why the crate would not stay at rest if $P = 0$
- Find the least possible value of P .
- Find the greatest possible value of P .

(8 marks)

Resolve perpendicular to the ramp

$$R = 3g \cos 35^\circ = 24.083...$$

$$\mu R = 0.5 \times 24.083... = 12.041...$$

$$3g \sin 35^\circ = 16.863...$$

a) With $P = 0$, compare the pull down the ramp with the most friction

$$16.86 > 12.04$$

friction cannot hold it, so it slides down

b) Least P : about to slide DOWN, friction acts UP the ramp

$$P + \mu R = 3g \sin 35^\circ$$

$$P = 16.863... - 12.041...$$

$$P = 4.8216...$$

$$P = 4.82$$

c) Greatest P : about to slide UP, friction acts DOWN the ramp

$$P = 3g \sin 35^\circ + \mu R$$

$$P = 16.863... + 12.041...$$

$$P = 28.904...$$

$$P = 28.9$$

10

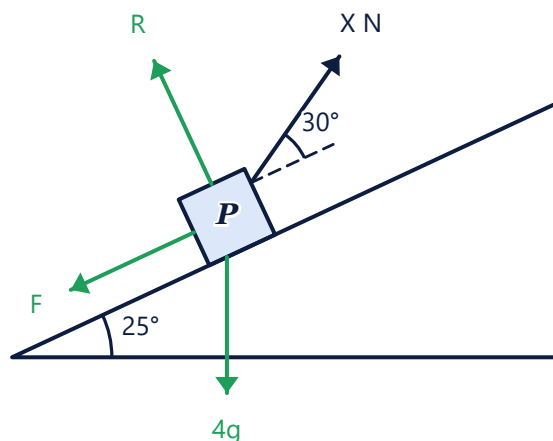
A particle P of mass 4 kg is on a rough plane inclined at 25° to the horizontal. The coefficient of friction between P and the plane is 0.35

A force of magnitude X newtons acts on P at an angle of 30° above a line of greatest slope, in the vertical plane containing that line, as shown in the diagram. The particle is on the point of moving up the plane.

a. Find the value of X .

The force is now removed.

b. Show that P slides down the plane and find its acceleration.



(9 marks)

a) Resolve perpendicular to the plane

$$R + X \sin 30^\circ = 4g \cos 25^\circ$$

$$R = 35.527... - 0.5X$$

About to move up, so friction acts down the plane and $F = 0.35R$

Resolve parallel to the plane

$$X \cos 30^\circ = 4g \sin 25^\circ + 0.35R$$

$$0.86602...X = 16.566... + 0.35(35.527... - 0.5X)$$

$$0.86602...X = 16.566... + 12.434... - 0.175X$$

$$1.04102...X = 29.001...$$

$$X = 27.858...$$

$$\boxed{X = 27.9}$$

b) Now $R = 4g \cos 25^\circ = 35.527...$

$$\mu R = 0.35 \times 35.527... = 12.434...$$

$$4g \sin 25^\circ = 16.566...$$

$16.57 > 12.43$, so friction cannot hold P : it slides down

$F = ma$ down the plane

$$16.566... - 12.434... = 4a$$

$$4.1320... = 4a$$

$$a = 1.0330...$$

slides down with $a = 1.03 \text{ m s}^{-2}$

11

A skier of mass 70 kg passes the top of a straight slope with speed 3 m s^{-1} , moving down a line of greatest slope. The slope is 50 m long and is inclined at 12° to the horizontal. At the bottom the skier moves onto horizontal snow, with no change of speed, and continues in a straight line until coming to rest. The coefficient of friction between the skis and the snow is 0.08 everywhere.

The skier is modelled as a particle and air resistance is ignored.

- Find the speed of the skier at the bottom of the slope.
- Find the distance the skier travels on the horizontal snow.
- Air resistance is now included in the model. State, with a reason, how this would change your answer to part (b).

(9 marks)

a) On the slope, resolve perpendicular

$$R = 70g \cos 12^\circ$$

Friction acts up the slope, $F = 0.08R$

$F = ma$ down the slope

$$70g \sin 12^\circ - 0.08 \times 70g \cos 12^\circ = 70a$$

$$a = g(\sin 12^\circ - 0.08 \cos 12^\circ)$$

$$a = 1.2706\dots$$

$u = 3$, $s = 50$, use $v^2 = u^2 + 2as$

$$v^2 = 3^2 + 2(1.2706\dots)(50)$$

$$v^2 = 136.06\dots$$

$$v = 11.664\dots$$

$$v = 11.7 \text{ m s}^{-1}$$

b) On level snow $R = 70g$, so friction $= 0.08 \times 70g$

$$-0.08 \times 70g = 70a$$

$$a = -0.784$$

$u^2 = 136.06\dots$, $v = 0$

$$0 = 136.06\dots - 2(0.784)s$$

$$s = \frac{136.06\dots}{1.568}$$

$$s = 86.777\dots$$

$$s = 86.8 \text{ m}$$

c) Air resistance is an extra force opposing the motion

the distance would be shorter

12

A rough plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{5}{12}$

A particle Q of mass m is projected with speed U up a line of greatest slope of the plane from the point A . As Q moves up the plane it decelerates at $\frac{9}{13}g$. The coefficient of friction between Q and the plane is μ .

- Show that $\mu = \frac{1}{3}$
- Explain why Q does not remain at rest when it stops.
- Find, in terms of g , the acceleration of Q as it slides back down the plane.
- Show that Q passes back through A with speed $\frac{1}{3}U$.

(10 marks)

$\tan \alpha = \frac{5}{12}$: a 5, 12, 13 triangle

$$\sin \alpha = \frac{5}{13}$$

$$\cos \alpha = \frac{12}{13}$$

a) Resolve perpendicular to the plane

$$R = mg \cos \alpha = \frac{12}{13}mg$$

Moving up, so friction μR acts down the plane

$F = ma$ down the plane

$$mg \sin \alpha + \mu R = m \times \frac{9}{13}g$$

$$\frac{5}{13}mg + \frac{12}{13}\mu mg = \frac{9}{13}mg$$

$$5 + 12\mu = 9$$

$$\mu = \frac{4}{12}$$

$$\boxed{\mu = \frac{1}{3}}$$

b) At rest: compare the pull down the plane with the most friction

$$mg \sin \alpha = \frac{5}{13}mg$$

$$\mu R = \frac{1}{3} \times \frac{12}{13}mg = \frac{4}{13}mg$$

$$\boxed{\frac{5}{13}mg > \frac{4}{13}mg, \text{ so } Q \text{ slides back down}}$$

c) Now friction acts up the plane

$$\frac{5}{13}mg - \frac{4}{13}mg = ma$$

$$\boxed{a = \frac{1}{13}g}$$

d) Up the plane: $u = U$, $v = 0$, $a = -\frac{9}{13}g$

$$0 = U^2 - 2\left(\frac{9}{13}g\right)s$$

$$s = \frac{13U^2}{18g}$$

Down the plane from rest: $u = 0$, $a = \frac{1}{13}g$, same s

$$v^2 = 0 + 2\left(\frac{1}{13}g\right)\left(\frac{13U^2}{18g}\right)$$

$$v^2 = \frac{U^2}{9}$$

$$\boxed{v = \frac{1}{3}U}$$