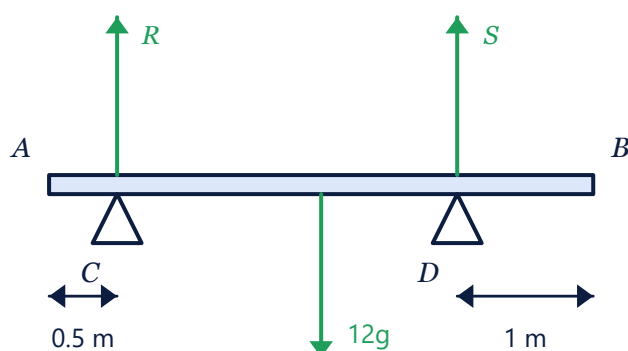




- 1 A uniform beam AB has length 4 m and mass 12 kg. The beam rests horizontally on two supports, one at C and one at D , where $AC = 0.5$ m and $DB = 1$ m, as shown in the diagram.

Find the magnitude of the reaction on the beam at C and the magnitude of the reaction on the beam at D .



(4 marks)

Uniform: the weight $12g$ acts at the midpoint, 2 m from A

Let the reactions be R at C and S at D

Take moments about C (this removes R)

$$S \times 2.5 = 12g \times 1.5$$

$$S = \frac{12g \times 1.5}{2.5}$$

$$S = 7.2g = 70.56$$

$$\text{reaction at } D = 70.6 \text{ N}$$

Resolve vertically

$$R + S = 12g$$

$$R = 12g - 7.2g$$

$$R = 4.8g = 47.04$$

$$\text{reaction at } C = 47.0 \text{ N}$$

2

A uniform plank PQ has length 6 m and mass 20 kg. The plank is held in a horizontal position by two light vertical ropes, one attached at P and one attached at the point R on the plank, where $PR = 5$ m. A bucket of mass 5 kg hangs from the end Q . The bucket is modelled as a particle.

Find the tension in each rope.

(4 marks)

Plank weight $20g$ at the midpoint, 3 m from P

Let the tensions be T at P and U at R

Take moments about P

$$U \times 5 = 20g \times 3 + 5g \times 6$$

$$5U = 60g + 30g$$

$$5U = 90g$$

$$U = 18g = 176.4$$

$$\text{tension at } R = 176 \text{ N}$$

Resolve vertically

$$T + U = 20g + 5g$$

$$T = 25g - 18g$$

$$T = 7g = 68.6$$

$$\text{tension at } P = 68.6 \text{ N}$$

3

A non-uniform plank AB has length 3.6 m and mass 15 kg. The plank hangs in a horizontal position from two light vertical strings, one at A and one at B . The tension in the string at B is one and a half times the tension in the string at A .

- Find the tension in each string.
- Find the distance of the centre of mass of the plank from A .

(5 marks)

a) Let the tension at A be T , so the tension at B is $1.5T$

Resolve vertically

$$T + 1.5T = 15g$$

$$2.5T = 15g$$

$$T = 6g = 58.8$$

$$1.5T = 9g = 88.2$$

$$\boxed{A: 58.8 \text{ N}, B: 88.2 \text{ N}}$$

b) Let the centre of mass G be x m from A

Take moments about A

$$9g \times 3.6 = 15g \times x$$

$$x = \frac{9 \times 3.6}{15}$$

$$x = 2.16$$

$$\boxed{2.16 \text{ m from } A}$$

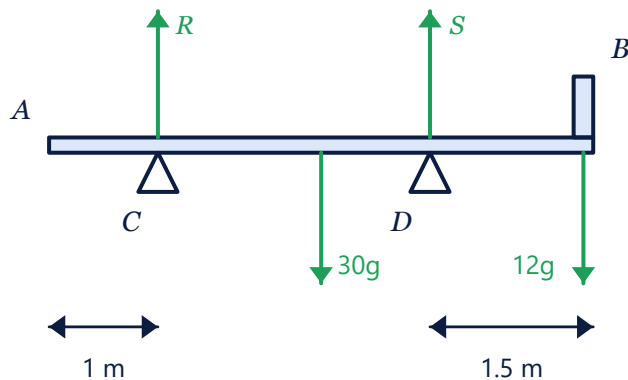
4

A uniform plank AB has length 5 m and mass 30 kg. It rests horizontally on two supports at C and D , where $AC = 1$ m and $DB = 1.5$ m, as shown in the diagram. A child of mass 12 kg stands on the plank at B . The child is modelled as a particle.

- a. Find the magnitude of the reaction on the plank at C and the magnitude of the reaction on the plank at D .

The child gets off and an adult of mass M kg stands at B . The plank is now on the point of tilting about D .

- b. Find the value of M .
c. State how you have used the modelling assumption that the plank is uniform.



(8 marks)

a) Plank weight $30g$ acts at the midpoint, 2.5 m from A

Let the reactions be R at C and S at D

Take moments about C

$$S \times 2.5 = 30g \times 1.5 + 12g \times 4$$

$$2.5S = 45g + 48g$$

$$2.5S = 93g$$

$$S = 37.2g = 364.56$$

Resolve vertically

$$R + S = 30g + 12g$$

$$R = 42g - 37.2g$$

$$R = 4.8g = 47.04$$

$$\boxed{C: 47.0 \text{ N}, D: 365 \text{ N}}$$

b) On the point of tilting about D : the reaction at C is 0

Take moments about D

the plank weight acts $3.5 - 2.5 = 1$ m from D

$$Mg \times 1.5 = 30g \times 1$$

$$M = \frac{30}{1.5}$$

$$M = 20$$

c) Uniform: the weight acts at the midpoint of the plank

weight of the plank taken to act at its midpoint

5

A uniform rod AB has length 4 m and mass 10 kg. The rod rests horizontally on two supports at C and D , where $AC = 1$ m and $AD = 2.5$ m. A particle of mass 15 kg is placed on the rod at the point P , where $AP = x$ metres.

Given that the rod remains in equilibrium, find the range of possible values of x .

(7 marks)

Rod weight $10g$ acts at the midpoint, 2 m from A

Case 1: P near A , the rod could tilt about C

On the point of tilting about C : reaction at $D = 0$

Take moments about C

$$15g \times (1 - x) = 10g \times (2 - 1)$$

$$15 - 15x = 10$$

$$15x = 5$$

$$x = \frac{1}{3}$$

Case 2: P near B , the rod could tilt about D

On the point of tilting about D : reaction at $C = 0$

Take moments about D

$$15g \times (x - 2.5) = 10g \times (2.5 - 2)$$

$$15x - 37.5 = 5$$

$$15x = 42.5$$

$$x = \frac{17}{6}$$

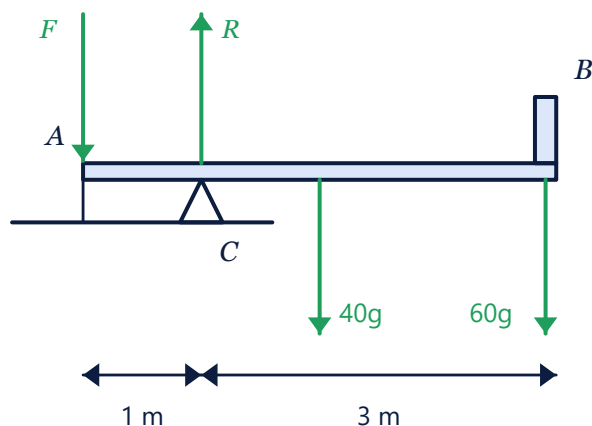
Between these positions the rod stays in equilibrium

$$\frac{1}{3} \leq x \leq \frac{17}{6}$$

6

A diving board AB is modelled as a uniform rod of length 4 m and mass 40 kg. The end A is bolted to the ground and the board rests horizontally on a support at C , where $AC = 1$ m, as shown in the diagram. A diver of mass 60 kg stands at the end B . The diver is modelled as a particle.

- Find the magnitude of the force exerted on the board by the support at C .
- Find the magnitude of the force exerted on the board by the bolt at A , and state its direction.



(6 marks)

a) Board weight $40g$ acts at the midpoint, 2 m from A
Take moments about A (this removes the bolt force)

$$R \times 1 = 40g \times 2 + 60g \times 4$$

$$R = 80g + 240g$$

$$R = 320g = 3136$$

$$\boxed{3140 \text{ N}}$$

b) Both weights turn the board clockwise about C
so the bolt at A must pull down. Let it be F downwards
Resolve vertically

$$R = F + 40g + 60g$$

$$F = 320g - 100g$$

$$F = 220g = 2156$$

$$\boxed{2160 \text{ N, vertically downwards}}$$

7

A uniform rod AB has length $2a$ and weight W . The rod rests horizontally on two supports, one at A and one at C , where $AC = \frac{3}{2}a$. A particle of weight kW is attached to the rod at B , where k is a constant.

a. Show that the magnitude of the reaction on the rod at A is $\frac{1}{3}W(1 - k)$.

b. Hence write down the greatest possible value of k .

c. Given that the reaction at C is five times the reaction at A , find the value of k .

(7 marks)

a) Weight W acts at the midpoint, a from A

so W is $\frac{1}{2}a$ to the left of C and kW is $\frac{1}{2}a$ to the right of C

Take moments about C , with R the reaction at A

$$R \times \frac{3}{2}a + kW \times \frac{1}{2}a = W \times \frac{1}{2}a$$

Multiply through by $\frac{2}{a}$

$$3R + kW = W$$

$$3R = W - kW$$

$$R = \frac{1}{3}W(1 - k)$$

$$R = \frac{1}{3}W(1 - k) \text{ as required}$$

b) The rod stays in contact at A only if $R \geq 0$

$$1 - k \geq 0$$

$$k = 1$$

c) Resolve vertically, with S the reaction at C

$$R + S = W + kW$$

Substitute $S = 5R$

$$6R = W(1 + k)$$

$$6 \times \frac{1}{3}W(1 - k) = W(1 + k)$$

$$2 - 2k = 1 + k$$

$$3k = 1$$

$$k = \frac{1}{3}$$

8

A non-uniform rod AB has length 3 m. When the rod rests on a single support at the point C , where $AC = 1.2$ m, it balances in a horizontal position. A particle of mass 2 kg is now attached to the rod at B , and the rod now balances horizontally when the support is moved to the point D , where $AD = 1.5$ m.

- Find the mass of the rod.
- The particle is removed from B and attached to the rod at A instead. Find the distance from A of the point at which the rod now balances.

(6 marks)

a) It balances on C alone, so the centre of mass G is at C

$$AG = 1.2 \text{ m}$$

Let the mass of the rod be M kg. Take moments about D

G is $1.5 - 1.2 = 0.3$ m from D , the particle is $3 - 1.5 = 1.5$ m from D

$$Mg \times 0.3 = 2g \times 1.5$$

$$0.3M = 3$$

$$M = 10 \text{ kg}$$

b) Let the new balance point be y m from A

The reaction there is the total weight $12g$. Moments about A

$$12g \times y = 10g \times 1.2 + 2g \times 0$$

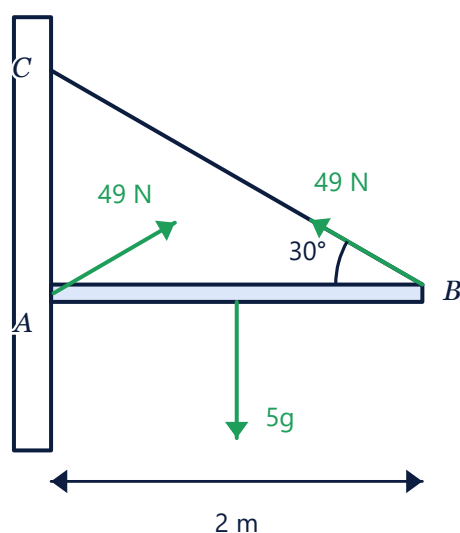
$$12y = 12$$

$$1 \text{ m from } A$$

9

A uniform rod AB has length 2 m and mass 5 kg. The end A is freely hinged to a vertical wall. The rod is held in a horizontal position by a light inextensible string. One end of the string is attached to the rod at B and the other end is attached to the wall at the point C , vertically above A . The string makes an angle of 30° with the rod, as shown in the diagram.

- Find the tension in the string.
- Find the magnitude of the force exerted on the rod by the hinge, and the angle this force makes with the rod.



(8 marks)

a) Take moments about A (this removes the hinge force)

Only the component of T perpendicular to AB has a moment

$$T \sin 30^\circ \times 2 = 5g \times 1$$

$$T \times \frac{1}{2} \times 2 = 5g$$

$$T = 5g = 49$$

49 N

b) Let the hinge force have components X (horizontal) and Y (vertical)

Resolve horizontally

$$X = T \cos 30^\circ$$

$$X = 49 \times \frac{\sqrt{3}}{2} = 42.43...$$

Resolve vertically

$$Y + T \sin 30^\circ = 5g$$

$$Y = 49 - 24.5 = 24.5$$

Combine the components

$$|F| = \sqrt{(42.43^2 + 24.5^2)}$$

$$|F| = 49$$

Angle with the rod

$$\tan \theta = \frac{24.5}{42.43}$$

$$\theta = 30^\circ$$

49 N at 30° above AB

10

A non-uniform beam AB has length 6 m and mass 50 kg. The beam rests horizontally on two supports at C and D , where $AC = 1$ m and $DB = 2$ m. When a woman of mass 30 kg stands on the beam at B , the beam is on the point of tilting about D . The woman is modelled as a particle.

a. Find the distance of the centre of mass of the beam from A .

The woman now walks along the beam and stands at A .

b. Show that the beam does not tilt.

c. Find the magnitude of the reaction on the beam at each support when the woman stands at A .

d. Explain what is meant by modelling the woman as a particle.

(9 marks)

a) On the point of tilting about D : reaction at $C = 0$

Let G be x m from A . D is $6 - 2 = 4$ m from A

Take moments about D

$$30g \times 2 = 50g \times (4 - x)$$

$$60 = 200 - 50x$$

$$50x = 140$$

$$x = 2.8$$

2.8 m from A

b) Could it tilt about C ? Compare moments about C

$$\text{woman: } 30g \times 1 = 30g$$

$$\text{beam: } 50g \times (2.8 - 1) = 90g, \text{ the other way}$$

$90g > 30g$ so the reaction at D is still positive

the beam does not tilt

c) Take moments about C , with S the reaction at D

$$S \times 3 + 30g \times 1 = 50g \times 1.8$$

$$3S = 90g - 30g$$

$$S = 20g = 196$$

Resolve vertically

$$R + S = 80g$$

$$R = 60g = 588$$

$C: 588 \text{ N}, D: 196 \text{ N}$

d) Particle: ignore her size

her whole weight acts at the single point A

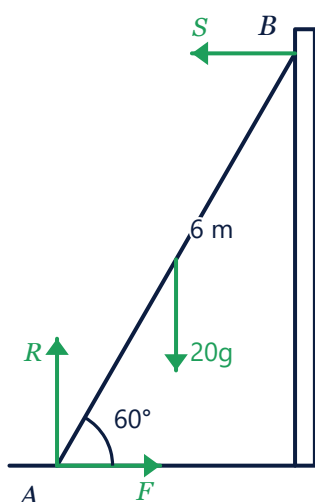
11

A uniform ladder AB has length 6 m and mass 20 kg. The ladder rests with its end A on rough horizontal ground and its end B against a smooth vertical wall. The ladder makes an angle of 60° with the ground and lies in a vertical plane perpendicular to the wall, as shown in the diagram. The ladder is modelled as a uniform rod.

- a. Show that the coefficient of friction between the ladder and the ground must be at least $\frac{1}{2\sqrt{3}}$.

The coefficient of friction between the ladder and the ground is in fact 0.35. A man of mass 75 kg climbs slowly up the ladder. The man is modelled as a particle.

- b. Find the greatest distance up the ladder, measured from A , that the man can climb before the ladder slips.



(10 marks)

a) Resolve vertically

$$R = 20g$$

Resolve horizontally

$$F = S$$

Take moments about A

$$S \times 6 \sin 60^\circ = 20g \times 3 \cos 60^\circ$$

$$S \times 3\sqrt{3} = 30g$$

$$S = \frac{10g}{\sqrt{3}}$$

No slipping: $F \leq \mu R$

$$\frac{10g}{\sqrt{3}} \leq \mu \times 20g$$

$$\mu \geq \frac{10}{20\sqrt{3}}$$

$$\mu \geq \frac{1}{2\sqrt{3}} \text{ as required}$$

b) Let the man be d m up the ladder when it is about to slip

Resolve vertically

$$R = 20g + 75g = 95g$$

Limiting friction

$$F = 0.35 \times 95g = 33.25g$$

Resolve horizontally

$$S = F = 33.25g$$

Take moments about A

$$S \times 6 \sin 60^\circ = 20g \times 3 \cos 60^\circ + 75g \times d \cos 60^\circ$$

$$33.25g \times 5.196 = 30g + 37.5g \times d$$

$$172.77 = 30 + 37.5d$$

$$d = \frac{142.77}{37.5}$$

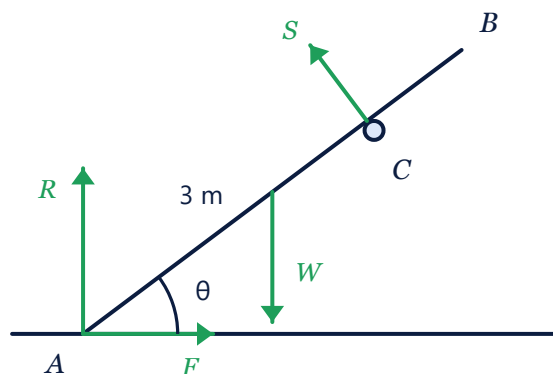
$$d = 3.807...$$

$$3.81 \text{ m}$$

12

A uniform rod AB has length 4 m and weight W . The rod rests with its end A on rough horizontal ground and rests against a small smooth peg at the point C on the rod, where $AC = 3$ m. The rod is in a vertical plane perpendicular to the peg and makes an angle θ with the ground, where $\tan \theta = \frac{3}{4}$, as shown in the diagram. The rod is in limiting equilibrium.

- Show that the magnitude of the reaction of the peg on the rod is $\frac{8}{15}W$.
- Find the exact value of the coefficient of friction between the rod and the ground.
- State the direction of the friction on the rod at A , giving a reason.



(10 marks)

$\tan \theta = \frac{3}{4}$: use a 3, 4, 5 triangle

$$\sin \theta = \frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

a) Smooth peg: the reaction S is perpendicular to the rod

Take moments about A

$$S \times 3 = W \times 2 \cos \theta$$

$$3S = W \times 2 \times \frac{4}{5}$$

$$3S = \frac{8}{5}W$$

$$S = \frac{8}{15}W$$

$$S = \frac{8}{15}W \text{ as required}$$

b) S makes angle θ with the vertical. Resolve horizontally

$$F = S \sin \theta$$

$$F = \frac{8}{15}W \times \frac{3}{5} = \frac{8}{25}W$$

Resolve vertically

$$R + S \cos \theta = W$$

$$R = W - \frac{8}{15}W \times \frac{4}{5}$$

$$R = W - \frac{32}{75}W = \frac{43}{75}W$$

Limiting equilibrium: $F = \mu R$

$$\mu = \frac{F}{R}$$

$$\mu = \frac{8}{25} \times \frac{75}{43}$$

$$\mu = \frac{24}{43}$$

c) The peg reaction pushes the foot of the rod away from the peg

friction acts horizontally towards the peg