

Projectiles

Worked solutions

A-level Maths topic practice

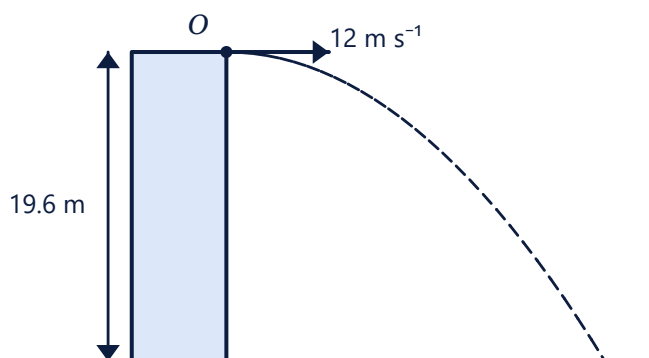


This topic online

1

A ball is thrown horizontally with speed 12 m s^{-1} from a point O at the top of a vertical wall. The point O is 19.6 m above horizontal ground. The ball is modelled as a particle moving freely under gravity.

- Find the time taken for the ball to reach the ground.
- Find the horizontal distance from the foot of the wall to the point where the ball lands.
- Find the speed of the ball just before it hits the ground.



(5 marks)

a) Vertically (down positive): $u = 0$, $a = 9.8$, $s = 19.6$

$$s = ut + \frac{1}{2}at^2$$

$$19.6 = 0 + 4.9t^2$$

$$t^2 = 4$$

$$t = 2 \text{ s}$$

b) Horizontally: constant speed 12

$$x = 12 \times 2$$

$$x = 24 \text{ m}$$

c) Vertical velocity on landing

$$v_y = u + at = 0 + 9.8 \times 2$$

$$v_y = 19.6$$

Horizontal velocity is still 12

$$\text{speed} = \sqrt{12^2 + 19.6^2}$$

$$\text{speed} = \sqrt{528.16}$$

$$\text{speed} = 22.981\dots$$

$$23.0 \text{ m s}^{-1}$$

2

A particle is projected from a point O on horizontal ground with speed 20 m s^{-1} at an angle of 30° above the horizontal. The particle moves freely under gravity.

Find

- the greatest height of the particle above the ground,
- the time of flight,
- the horizontal distance from O to the point where the particle lands.

(6 marks)

Split the velocity into components

$$\text{horizontal: } 20 \cos 30^\circ = 17.320\dots$$

$$\text{vertical: } 20 \sin 30^\circ = 10$$

a) Vertically (up positive): $u = 10$, $v = 0$, $a = -9.8$

$$v^2 = u^2 + 2as$$

$$0 = 10^2 - 2(9.8)s$$

$$s = \frac{100}{19.6}$$

$$s = 5.1020\dots$$

$$\boxed{5.10 \text{ m}}$$

b) Vertically: Lands when $s = 0$

$$0 = 10t - 4.9t^2$$

$$0 = t(10 - 4.9t)$$

$$t = \frac{10}{4.9} \quad (t \neq 0)$$

$$t = 2.0408\dots$$

$$\boxed{2.04 \text{ s}}$$

c) Horizontally: distance = speed $\times$ time

$$x = 17.320\dots \times 2.0408\dots$$

$$x = 35.347\dots$$

$$\boxed{35.3 \text{ m}}$$

3

A small stone is projected from a point on horizontal ground with speed 25 m s^{-1} at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$

The stone is modelled as a particle moving freely under gravity.

- Find the speed of the stone 2 seconds after it is projected.
- Find the direction of motion of the stone at this time.

(6 marks)

$\tan \alpha = \frac{3}{4}$: a 3, 4, 5 triangle

$$\sin \alpha = \frac{3}{5}$$

$$\cos \alpha = \frac{4}{5}$$

$$\text{horizontal: } 25 \times \frac{4}{5} = 20$$

$$\text{vertical: } 25 \times \frac{3}{5} = 15$$

a) Vertically (up positive): $u = 15$, $a = -9.8$, $t = 2$

$$v_y = u + at$$

$$v_y = 15 - 9.8 \times 2$$

$$v_y = -4.6$$

The horizontal velocity stays 20

$$\text{speed} = \sqrt{20^2 + 4.6^2}$$

$$\text{speed} = \sqrt{421.16}$$

$$\text{speed} = 20.522...$$

$$\boxed{20.5 \text{ m s}^{-1}}$$

b) Moving downwards, so the direction is below the horizontal

$$\tan \theta = \frac{4.6}{20}$$

$$\theta = 12.952...$$

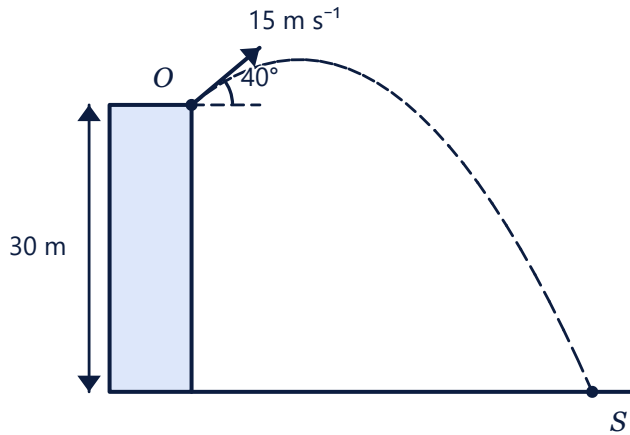
$$\boxed{13.0^\circ \text{ below the horizontal}}$$

4

A stone is thrown from a point O at the top of a vertical cliff. The point O is 30 m above the sea. The stone is thrown with speed 15 m s^{-1} at an angle of 40° above the horizontal, as shown in the diagram. The stone hits the sea at the point S .

The stone is modelled as a particle moving freely under gravity.

- Find the time taken for the stone to travel from O to S .
- Find the distance from the foot of the cliff to S .



(6 marks)

Components

$$\text{vertical: } 15 \sin 40^\circ = 9.6418\dots$$

$$\text{horizontal: } 15 \cos 40^\circ = 11.490\dots$$

a) Vertically (up positive): S is 30 m below O , so $s = -30$

$$s = ut + \frac{1}{2}at^2$$

$$-30 = 9.6418\dots t - 4.9t^2$$

$$4.9t^2 - 9.6418\dots t - 30 = 0$$

Quadratic formula, take the positive root

$$t = \frac{9.6418\dots + \sqrt{(9.6418\dots)^2 + 4 \times 4.9 \times 30}}{9.8}$$

$$t = \frac{9.6418\dots + 26.095\dots}{9.8}$$

$$t = 3.6466\dots$$

$$\boxed{3.65 \text{ s}}$$

b) Horizontally

$$x = 11.490\dots \times 3.6466\dots$$

$$x = 41.902\dots$$

$$41.9 \text{ m}$$

5

A golf ball is hit from a point A on horizontal ground. It leaves the ground with speed $U \text{ m s}^{-1}$ at an angle of 35° above the horizontal and first lands at the point B on the ground, where $AB = 80 \text{ m}$.

The ball is modelled as a particle moving freely under gravity.

a. Show that $U^2 \sin 35^\circ \cos 35^\circ = 392$

b. Find the value of U .

c. Find the time the ball takes to travel from A to B .

(7 marks)

a) Let the time of flight be T

Vertically (up positive): back to ground level, $s = 0$

$$0 = U \sin 35^\circ \times T - 4.9T^2$$

$$T = \frac{U \sin 35^\circ}{4.9} \quad (T \neq 0)$$

Horizontally

$$U \cos 35^\circ \times T = 80$$

Substitute for T

$$U \cos 35^\circ \times \frac{U \sin 35^\circ}{4.9} = 80$$

$$U^2 \sin 35^\circ \cos 35^\circ = 80 \times 4.9$$

$$U^2 \sin 35^\circ \cos 35^\circ = 392$$

b) Rearrange

$$U^2 = \frac{392}{\sin 35^\circ \cos 35^\circ}$$

$$U^2 = 834.31...$$

$$U = 28.884...$$

$$U = 28.9$$

c) Use T from part a

$$T = \frac{28.884... \times \sin 35^\circ}{4.9}$$

$$T = 3.3811...$$

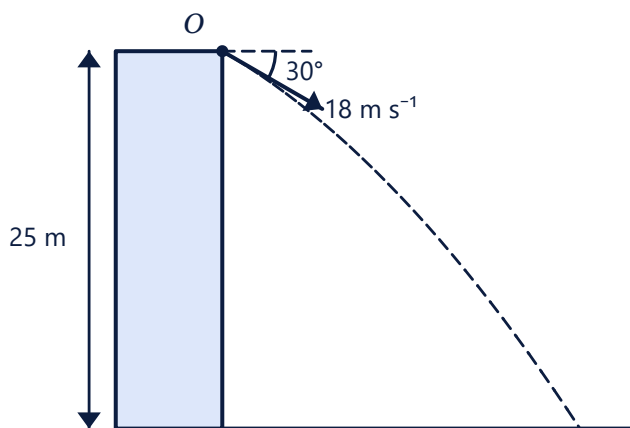
$$3.38 \text{ s}$$

6

A ball is thrown from a point O on a balcony, 25 m above horizontal ground. The ball is thrown with speed 18 m s^{-1} at an angle of 30° **below** the horizontal, as shown in the diagram.

The ball is modelled as a particle moving freely under gravity. Find

- the vertical component of the velocity of the ball just before it hits the ground,
- the time taken for the ball to reach the ground,
- the speed and direction of motion of the ball just before it hits the ground.



(8 marks)

Take **DOWN** as positive vertically

$$\text{vertical: } 18 \sin 30^\circ = 9$$

$$\text{horizontal: } 18 \cos 30^\circ = 15.588\dots$$

$$\text{a) } u = 9, a = 9.8, s = 25$$

$$v^2 = u^2 + 2as$$

$$v^2 = 9^2 + 2(9.8)(25)$$

$$v^2 = 81 + 490 = 571$$

$$v = 23.895\dots$$

$$\boxed{23.9 \text{ m s}^{-1} \text{ downwards}}$$

$$\text{b) } v = u + at$$

$$23.895\dots = 9 + 9.8t$$

$$t = \frac{14.895\dots}{9.8}$$

$$t = 1.5199\dots$$

$$\boxed{1.52 \text{ s}}$$

c) Combine the components

$$\text{speed} = \sqrt{(15.588\dots)^2 + 571}$$

$$\text{speed} = \sqrt{(243 + 571)} = \sqrt{814}$$

$$\text{speed} = 28.530\dots$$

$$\tan \theta = \frac{23.895\dots}{15.588\dots}$$

$$\theta = 56.881\dots^\circ$$

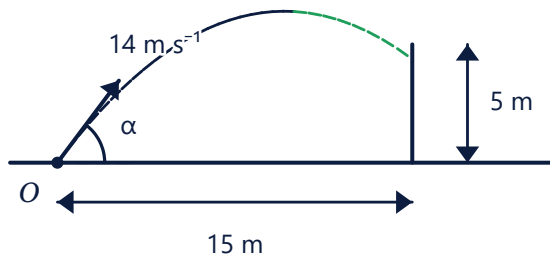
28.5 m s⁻¹ at 56.9° below the horizontal

7

A particle is projected from a point O on horizontal ground with speed 14 m s^{-1} at an angle α above the horizontal, where $\tan \alpha = \frac{4}{3}$

At time t seconds the particle is x metres horizontally and y metres vertically from O . The particle moves freely under gravity.

- Show that $y = \frac{4}{3}x - \frac{5}{72}x^2$
- Use your answer to part (a) to find the horizontal distance from O to the point where the particle lands.
- A vertical wall of height 5 m stands 15 m horizontally from O , in the plane of the motion. Determine whether the particle passes over the wall.



(9 marks)

$$\tan \alpha = \frac{4}{3}: \text{ a 3, 4, 5 triangle}$$

$$\sin \alpha = \frac{4}{5}$$

$$\cos \alpha = \frac{3}{5}$$

$$\text{horizontal: } 14 \times \frac{3}{5} = 8.4$$

$$\text{vertical: } 14 \times \frac{4}{5} = 11.2$$

a) Write x and y in terms of t

$$x = 8.4t$$

$$y = 11.2t - 4.9t^2$$

Eliminate t

$$t = \frac{x}{8.4}$$

$$y = 11.2\left(\frac{x}{8.4}\right) - 4.9\left(\frac{x}{8.4}\right)^2$$

$$y = \frac{4}{3}x - \frac{4.9x^2}{70.56}$$

$$y = \frac{4}{3}x - \frac{5}{72}x^2$$

$$y = \frac{4}{3}x - \frac{5}{72}x^2$$

b) Lands when $y = 0$

$$0 = x\left(\frac{4}{3} - \frac{5}{72}x\right)$$

$$\frac{5}{72}x = \frac{4}{3} \quad (x \neq 0)$$

$$x = \frac{4}{3} \times \frac{72}{5}$$

$$x = 19.2 \text{ m}$$

c) Height when $x = 15$

$$y = \frac{4}{3}(15) - \frac{5}{72}(15^2)$$

$$y = 20 - 15.625$$

$$y = 4.375$$

$$4.375 < 5$$

no, it does not clear the wall

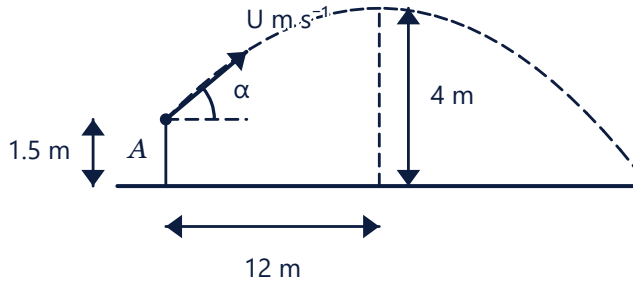
8

A girl throws a ball from a point A , 1.5 m above horizontal ground. The ball leaves A with speed U m s⁻¹ at an angle α above the horizontal. The highest point reached by the ball is 4 m above the ground, and at this moment the ball has travelled 12 m horizontally from A .

The ball is modelled as a particle moving freely under gravity.

a. Show that $U \sin \alpha = 7$

b. Find the value of U and the size of the angle α .



(8 marks)

a) Vertically (up positive) from A to the top: rise = 4 - 1.5 = 2.5

$u = U \sin \alpha$, $v = 0$, $a = -9.8$, $s = 2.5$

$$v^2 = u^2 + 2as$$

$$0 = (U \sin \alpha)^2 - 2(9.8)(2.5)$$

$$(U \sin \alpha)^2 = 49$$

$$U \sin \alpha = 7$$

b) Time to the top: $v = u + at$

$$0 = 7 - 9.8t$$

$$t = \frac{7}{9.8} = \frac{5}{7}$$

Horizontally: 12 m in this time

$$U \cos \alpha \times \frac{5}{7} = 12$$

$$U \cos \alpha = 16.8$$

Combine the two components

$$U^2 = 7^2 + 16.8^2$$

$$U^2 = 331.24$$

$$U = 18.2$$

$$\tan \alpha = \frac{7}{16.8} = \frac{5}{12}$$

$$\alpha = 22.619...^{\circ}$$

$$U = 18.2, \alpha = 22.6^{\circ}$$

9

A firework is launched from a point on horizontal ground with speed 24.5 m s^{-1} at an angle of 60° above the horizontal. The firework is modelled as a particle moving freely under gravity.

- Find the length of time for which the firework is at least 15 m above the ground.
- Write down the least speed of the firework during its flight.
- Find the speed of the firework when it is 15 m above the ground.

(8 marks)

Components

$$\text{vertical: } 24.5 \sin 60^{\circ} = 21.217...$$

$$\text{horizontal: } 24.5 \cos 60^{\circ} = 12.25$$

a) Vertically (up positive): find when $s = 15$

$$15 = 21.217...t - 4.9t^2$$

$$4.9t^2 - 21.217...t + 15 = 0$$

$$t = \frac{21.217... \pm \sqrt{(21.217...)^2 - 4(4.9)(15)}}{9.8}$$

$$t = \frac{21.217... \pm 12.497...}{9.8}$$

$$t = 0.88980... \text{ or } 3.4404...$$

Above 15 m between the two times

$$3.4404... - 0.88980... = 2.5505...$$

$$2.55 \text{ s}$$

b) Least speed at the top: vertical velocity is 0

$$12.25 \text{ m s}^{-1}$$

c) Vertical velocity at 15 m: $v^2 = u^2 + 2as$

$$v_y^2 = 21.217...^2 - 2(9.8)(15)$$

$$v_y^2 = 450.1875 - 294 = 156.1875$$

$$\text{speed}^2 = 12.25^2 + 156.1875$$

$$\text{speed}^2 = 306.25$$

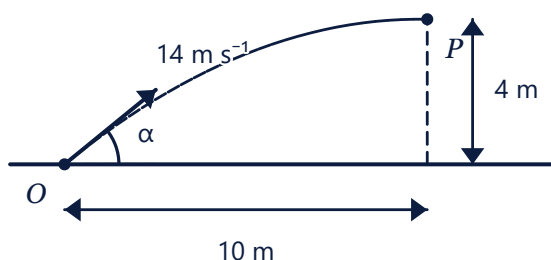
$$\text{speed} = 17.5 \text{ m s}^{-1}$$

10

A ball is projected from a point O on horizontal ground with speed 14 m s^{-1} at an angle α above the horizontal. The ball passes through the point P , which is 10 m horizontally and 4 m vertically from O , as shown in the diagram.

The ball is modelled as a particle moving freely under gravity.

- Show that the time taken to reach P is $\frac{5}{7 \cos \alpha}$ seconds.
- Show that $5 \tan^2 \alpha - 20 \tan \alpha + 13 = 0$
- Find the two possible values of α .
- For the smaller value of α , find the time taken for the ball to reach P .



(10 marks)

a) Horizontally: constant speed $14 \cos \alpha$

$$14 \cos \alpha \times t = 10$$

$$t = \frac{10}{14 \cos \alpha} = \frac{5}{7 \cos \alpha}$$

b) Vertically (up positive)

$$4 = 14 \sin \alpha \times t - 4.9t^2$$

Substitute t

$$4 = 14 \sin \alpha \times \frac{5}{7 \cos \alpha} - 4.9 \times \frac{25}{49 \cos^2 \alpha}$$

$$4 = 10 \tan \alpha - 2.5 \sec^2 \alpha$$

Use $\sec^2 \alpha = 1 + \tan^2 \alpha$

$$4 = 10 \tan \alpha - 2.5 - 2.5 \tan^2 \alpha$$

$$2.5 \tan^2 \alpha - 10 \tan \alpha + 6.5 = 0$$

$$5 \tan^2 \alpha - 20 \tan \alpha + 13 = 0$$

c) Quadratic formula in $\tan \alpha$

$$\tan \alpha = \frac{20 \pm \sqrt{(400 - 260)}}{10}$$

$$\tan \alpha = \frac{20 \pm \sqrt{140}}{10}$$

$$\tan \alpha = 3.1832... \text{ or } 0.81678...$$

$$\alpha = 72.560...^\circ \text{ or } 39.241...^\circ$$

$$\alpha = 72.6^\circ \text{ or } \alpha = 39.2^\circ$$

d) Use part a with $\alpha = 39.241...^\circ$

$$t = \frac{5}{7 \cos 39.241...^\circ}$$

$$t = 0.92226...$$

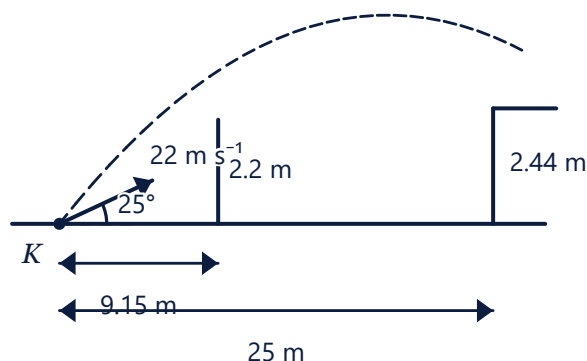
$$0.922 \text{ s}$$

11

A footballer takes a free kick from a point K on horizontal ground, 25 m from the goal line. The ball leaves the ground with speed 22 m s^{-1} at an angle of 25° above the horizontal, in a vertical plane perpendicular to the goal line. A line of defenders stands 9.15 m from K ; they jump to a height of 2.2 m. The crossbar of the goal is 2.44 m above the ground.

The ball is modelled as a particle moving freely under gravity.

- Show that the ball passes over the defenders.
- Determine whether the ball passes under or over the crossbar, and by how much.
- State one limitation of the model that could affect your answer to part (b).



(8 marks)

Components

$$\text{horizontal: } 22 \cos 25^\circ = 19.938\dots$$

$$\text{vertical: } 22 \sin 25^\circ = 9.2976\dots$$

a) Time to travel 9.15 m horizontally

$$t = \frac{9.15}{19.938\dots} = 0.45890\dots$$

Height at this time

$$y = 9.2976\dots(0.45890\dots) - 4.9(0.45890\dots)^2$$

$$y = 4.2667\dots - 1.0319\dots$$

$$y = 3.2348\dots$$

$$3.23 > 2.2$$

the ball passes over the defenders

b) Time to travel 25 m horizontally

$$t = \frac{25}{19.938\dots} = 1.2538\dots$$

$$y = 9.2976\dots(1.2538\dots) - 4.9(1.2538\dots)^2$$

$$y = 11.657\dots - 7.7032\dots$$

$$y = 3.9543\dots$$

$$3.9543\dots - 2.44 = 1.5143\dots$$

over the crossbar by 1.51 m

c) A limitation

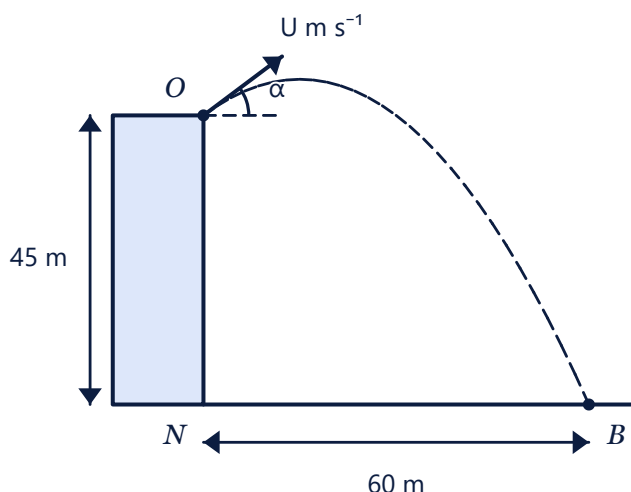
air resistance (or spin) is ignored

12

A small stone is projected from a point O at the top of a vertical cliff. The point O is 45 m vertically above the point N on horizontal ground. The stone is projected with speed $U \text{ m s}^{-1}$ at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$. The stone hits the ground at the point B , where $NB = 60 \text{ m}$, as shown in the diagram.

The stone is modelled as a particle moving freely under gravity.

- Show that $U = 17.5$
- Find the length of time for which the stone is higher than O .
- Find the speed of the stone just before it hits the ground.



(10 marks)

$$\tan \alpha = \frac{3}{4} : \sin \alpha = \frac{3}{5}, \cos \alpha = \frac{4}{5}$$

$$\text{horizontal: } \frac{4}{5}U = 0.8U$$

$$\text{vertical: } \frac{3}{5}U = 0.6U$$

a) Let the time from O to B be T . Horizontally:

$$0.8U \times T = 60$$

$$T = \frac{75}{U}$$

Vertically (up positive): B is 45 m below O

$$-45 = 0.6U \times T - 4.9T^2$$

$$-45 = 0.6U \times \frac{75}{U} - 4.9T^2$$

$$-45 = 45 - 4.9T^2$$

$$T^2 = \frac{90}{4.9} = \frac{900}{49}$$

$$T = \frac{30}{7}$$

$$U = 75 \div \frac{30}{7}$$

$$U = 17.5$$

b) Vertical component $0.6 \times 17.5 = 10.5$. Back level with 0 when $s = 0$

$$0 = 10.5t - 4.9t^2$$

$$t = \frac{10.5}{4.9} = \frac{15}{7}$$

$$\frac{15}{7} \text{ s} = 2.14 \text{ s}$$

c) At B, $t = \frac{30}{7}$

$$v_y = 10.5 - 9.8 \times \frac{30}{7}$$

$$v_y = 10.5 - 42 = -31.5$$

$$v_x = 0.8 \times 17.5 = 14$$

$$\text{speed} = \sqrt{14^2 + 31.5^2}$$

$$\text{speed} = \sqrt{1188.25}$$

$$\text{speed} = 34.471\dots$$

$$34.5 \text{ m s}^{-1}$$