



- 1** The circle C has equation $(x - 3)^2 + (y + 5)^2 = 20$
- Write down the coordinates of the centre of C .
 - Find the radius of C , giving your answer as a simplified surd.

(2 marks)

a) Compare with $(x - a)^2 + (y - b)^2 = r^2$

Change the signs inside the brackets

$$(3, -5)$$

b) $r^2 = 20$

$$r = \sqrt{20}$$

$$r = \sqrt{4} \times \sqrt{5}$$

$$r = 2\sqrt{5}$$

$$2\sqrt{5}$$

- 2** The circle C has equation $x^2 + y^2 - 8x + 6y + 9 = 0$. Find the coordinates of the centre of C and the radius of C .

(3 marks)

Group the x terms and the y terms

$$x^2 - 8x + y^2 + 6y + 9 = 0$$

Complete the square for each

$$(x - 4)^2 - 16 + (y + 3)^2 - 9 + 9 = 0$$

$$(x - 4)^2 + (y + 3)^2 - 16 = 0$$

$$(x - 4)^2 + (y + 3)^2 = 16$$

$$\text{centre } (4, -3), \text{ radius } 4$$

- 3 A circle has centre $(-2, 1)$ and passes through the point $(1, 5)$. Find an equation for the circle.

(3 marks)

Radius = distance from the centre to the point

$$r^2 = (1 - (-2))^2 + (5 - 1)^2$$

$$r^2 = 3^2 + 4^2$$

$$r^2 = 25$$

Use $(x - a)^2 + (y - b)^2 = r^2$

$$(x + 2)^2 + (y - 1)^2 = 25$$

- 4 The points $A(-1, 4)$ and $B(5, -2)$ are the ends of a diameter of the circle C .

a. Find an equation for C .

b. Determine, giving a reason, whether the origin lies inside or outside C .

(5 marks)

a) Centre is the midpoint of the diameter

$$\text{centre} = \left(\frac{-1 + 5}{2}, \frac{4 + (-2)}{2} \right)$$

$$\text{centre} = (2, 1)$$

Radius from the centre to B

$$r^2 = (5 - 2)^2 + (-2 - 1)^2$$

$$r^2 = 9 + 9 = 18$$

$$(x - 2)^2 + (y - 1)^2 = 18$$

b) Distance from the centre $(2, 1)$ to $O(0, 0)$

$$d^2 = 2^2 + 1^2 = 5$$

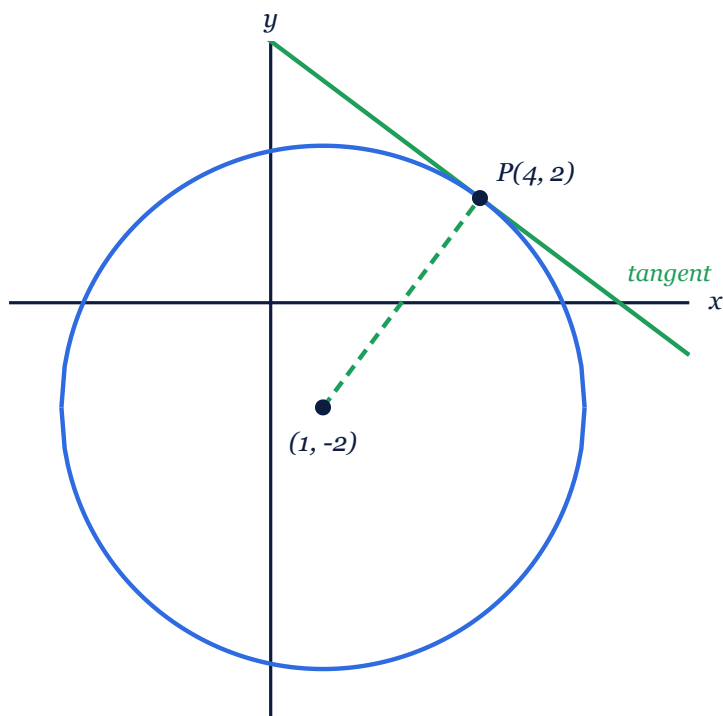
Compare with $r^2 = 18$

$5 < 18$, so the origin is closer to the centre than the radius

the origin lies inside C

5

The circle C has equation $(x - 1)^2 + (y + 2)^2 = 25$. The point $P(4, 2)$ lies on C . Find an equation of the tangent to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.



(4 marks)

Centre of C

$$\text{centre} = (1, -2)$$

Gradient of the radius from the centre to P

$$m = \frac{2 - (-2)}{4 - 1} = \frac{4}{3}$$

Tangent $\perp$ radius: negative reciprocal

$$\text{tangent gradient} = -\frac{3}{4}$$

Use $y - b = m(x - a)$ with $P(4, 2)$

$$y - 2 = -\frac{3}{4}(x - 4)$$

$\times 4$

$$4y - 8 = -3(x - 4) \quad \times 4$$

$$4y - 8 = -3x + 12$$

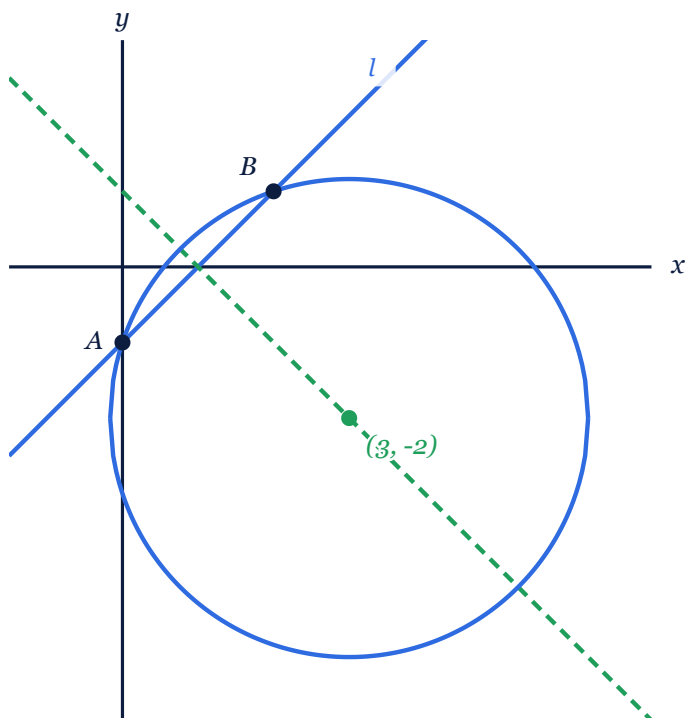
$$3x + 4y - 20 = 0$$

$$\boxed{3x + 4y - 20 = 0}$$

6

The circle C has equation $x^2 + y^2 - 6x + 4y + 3 = 0$. The line l with equation $y = x - 1$ meets C at the points A and B .

- Find the coordinates of A and B .
- Find the exact length of the chord AB .
- Show that the perpendicular bisector of AB passes through the centre of C .



(7 marks)

a) Substitute $y = x - 1$ into the circle

$$x^2 + (x - 1)^2 - 6x + 4(x - 1) + 3 = 0$$

$$x^2 + x^2 - 2x + 1 - 6x + 4x - 4 + 3 = 0$$

$$2x^2 - 4x = 0$$

$$2x(x - 2) = 0$$

$$x = 0 \text{ or } x = 2$$

$$y = 0 - 1 = -1$$

$$y = 2 - 1 = 1$$

$$\boxed{A(0, -1), B(2, 1)}$$

b) Distance formula

$$AB^2 = (2 - 0)^2 + (1 - (-1))^2$$

$$AB^2 = 4 + 4 = 8$$

$$AB = \sqrt{8} = 2\sqrt{2}$$

$$2\sqrt{2}$$

c) Midpoint of AB

$$M = \left(\frac{0+2}{2}, \frac{-1+1}{2} \right) = (1, 0)$$

Gradient of AB is 1, so the perpendicular gradient is -1

$$y - 0 = -1(x - 1)$$

$$y = 1 - x$$

Centre of C: halve the x and y coefficients and change sign

$$\text{centre} = (3, -2)$$

$$1 - 3 = -2$$

$$(3, -2) \text{ lies on } y = 1 - x, \text{ so shown}$$

7

The circle C has equation $(x - 1)^2 + (y - 2)^2 = 5$. The line l has equation $y = 2x + k$, where k is a constant.

a. Show that the x coordinates of any points where l meets C satisfy $5x^2 + (4k - 10)x + k^2 - 4k = 0$

b. Given that l is a tangent to C, find the possible values of k.

(6 marks)

a) Substitute $y = 2x + k$

$$(x - 1)^2 + (2x + k - 2)^2 = 5$$

$$x^2 - 2x + 1 + 4x^2 + 4(k - 2)x + (k - 2)^2 = 5$$

$$5x^2 + (4k - 8 - 2)x + k^2 - 4k + 4 + 1 - 5 = 0$$

$$5x^2 + (4k - 10)x + k^2 - 4k = 0$$

shown

b) Tangent: one point of contact, so $b^2 - 4ac = 0$

$$(4k - 10)^2 - 4(5)(k^2 - 4k) = 0$$

$$16k^2 - 80k + 100 - 20k^2 + 80k = 0$$

$$100 - 4k^2 = 0$$

$$k^2 = 25$$

$$k = \pm 5$$

$$k = 5 \text{ or } k = -5$$

8

The points $P(-2, 4)$, $Q(4, 6)$ and $R(6, 0)$ lie on the circle C .

- Show that angle PQR is a right angle.
- Hence find an equation for C .

(6 marks)

a) Gradients of PQ and QR

$$\text{gradient } PQ = \frac{6 - 4}{4 - (-2)} = \frac{2}{6} = \frac{1}{3}$$

$$\text{gradient } QR = \frac{0 - 6}{6 - 4} = -3$$

$$\frac{1}{3} \times -3 = -1$$

$$PQ \perp QR, \text{ so angle } PQR = 90^\circ$$

b) Angle in a semicircle: PR is a diameter

$$\text{centre} = \left(\frac{-2 + 6}{2}, \frac{4 + 0}{2} \right) = (2, 2)$$

$$r^2 = (6 - 2)^2 + (0 - 2)^2$$

$$r^2 = 16 + 4 = 20$$

$$(x - 2)^2 + (y - 2)^2 = 20$$

9

The points $A(7, 5)$, $B(-1, 5)$ and $D(5, -1)$ lie on the circle C .

- Find the equation of the perpendicular bisector of AD .
- Hence, or otherwise, find the coordinates of the centre of C .
- Find an equation for C .

(7 marks)

a) Midpoint of AD

$$M = \left(\frac{7+5}{2}, \frac{5+(-1)}{2} \right) = (6, 2)$$

Gradient of AD

$$m = \frac{-1-5}{5-7} = \frac{-6}{-2} = 3$$

Perpendicular gradient is $-\frac{1}{3}$

$$y - 2 = -\frac{1}{3}(x - 6)$$

$$\begin{array}{ccc} \times 3 & & \times 3 \\ 3y - 6 & = & -x + 6 \end{array}$$

$$x + 3y - 12 = 0$$

$$\boxed{x + 3y - 12 = 0}$$

b) The centre is on the perpendicular bisector of every chord

AB is horizontal, midpoint $(3, 5)$, so its perpendicular bisector is vertical

$$x = 3$$

Substitute into $x + 3y - 12 = 0$

$$3 + 3y - 12 = 0$$

$$3y = 9$$

$$y = 3$$

$$\boxed{\text{centre } (3, 3)}$$

c) Radius from the centre to A

$$r^2 = (7-3)^2 + (5-3)^2$$

$$r^2 = 16 + 4 = 20$$

$$\boxed{(x-3)^2 + (y-3)^2 = 20}$$

10

The circle C has equation $x^2 + y^2 - 4x + 2y - 4 = 0$ and centre M . The point P has coordinates $(8, 7)$.

A line from P touches C at the point T .

- Find the centre and radius of C .
- Find the exact length of PT .
- Find the size of angle MPT , giving your answer in degrees to 1 decimal place.

(5 marks)

a) Complete the square

$$(x - 2)^2 - 4 + (y + 1)^2 - 1 - 4 = 0$$

$$(x - 2)^2 + (y + 1)^2 = 9$$

centre $M(2, -1)$, radius 3

b) Distance MP

$$MP^2 = (8 - 2)^2 + (7 - (-1))^2$$

$$MP^2 = 36 + 64 = 100$$

Tangent $\perp$ radius, so triangle MTP has a right angle at T

$$PT^2 = MP^2 - MT^2$$

$$PT^2 = 100 - 9 = 91$$

$$PT = \sqrt{91}$$

$$\sqrt{91}$$

c) In the right-angled triangle, MT is opposite angle MPT

$$\sin MPT = \frac{3}{10}$$

$$MPT = \sin^{-1}(0.3) = 17.457...^\circ$$

$$17.5^\circ$$

11

The equation $x^2 + y^2 - 2kx + 4y + 2k + 4 = 0$, where k is a constant, represents a circle C .

- Find, in terms of k , the coordinates of the centre of C and an expression for the square of its radius.
- Hence find the set of possible values of k .
- Given that C touches the x -axis, find the exact possible values of k .

(7 marks)

a) Complete the square in x and y

$$(x - k)^2 - k^2 + (y + 2)^2 - 4 + 2k + 4 = 0$$

$$(x - k)^2 + (y + 2)^2 = k^2 - 2k$$

$$\text{centre } (k, -2), r^2 = k^2 - 2k$$

b) For a real circle $r^2 > 0$

$$k^2 - 2k > 0$$

$$k(k - 2) > 0$$

Critical values 0 and 2; > 0 is outside the roots

$$k < 0 \text{ or } k > 2$$

c) The centre $(k, -2)$ is 2 below the x -axis

Touches the x -axis, so the radius is 2

$$k^2 - 2k = 4$$

$$k^2 - 2k - 4 = 0$$

$$(k - 1)^2 - 1 - 4 = 0$$

$$(k - 1)^2 = 5$$

$$k = 1 \pm \sqrt{5}$$

Check: $1 + \sqrt{5} > 2$ and $1 - \sqrt{5} < 0$, so both are valid

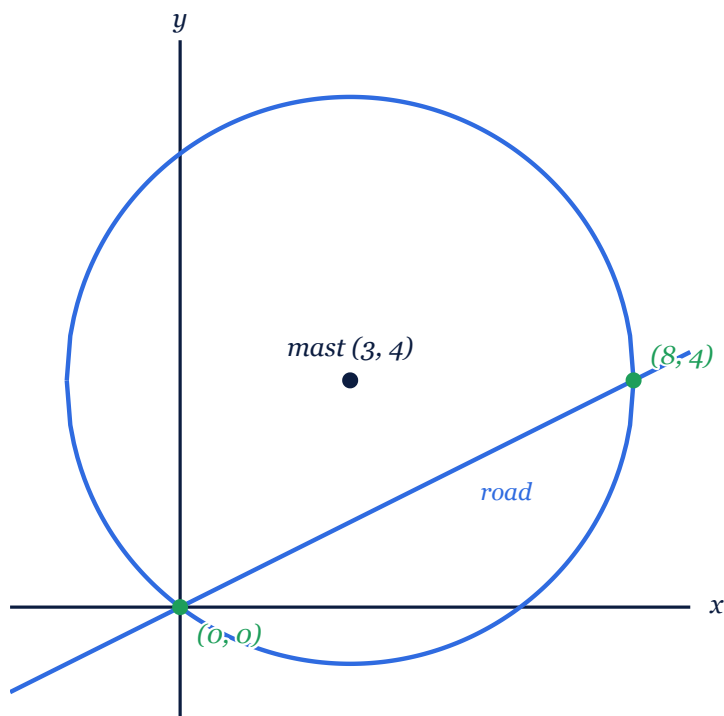
$$k = 1 + \sqrt{5} \text{ or } k = 1 - \sqrt{5}$$

12

A mobile phone mast in rural Cumbria has coordinates (3, 4) relative to a fixed origin O , where the units are kilometres. The mast gives a signal everywhere within 5 km of it, and the boundary of this region is modelled as a circle.

A straight road through a village at O is modelled by the line with equation $y = \frac{1}{2}x$

- Write down an equation for the boundary circle.
- Find the coordinates of the points where the road meets the boundary.
- Find the exact length of road that has a signal.
- A van travels along the road at a constant 60 km/h. Find, to the nearest minute, how long the van has a signal.
- Give one reason why the circle may not be a good model for the region with a signal.



(8 marks)

a) Centre (3, 4), radius 5

$$(x - 3)^2 + (y - 4)^2 = 25$$

b) Substitute $y = \frac{1}{2}x$

$$(x - 3)^2 + \left(\frac{1}{2}x - 4\right)^2 = 25$$

$$x^2 - 6x + 9 + \frac{1}{4}x^2 - 4x + 16 = 25$$

$$\frac{5}{4}x^2 - 10x = 0$$

$$x\left(\frac{5}{4}x - 10\right) = 0$$

$$x = 0 \text{ or } x = 8$$

$$y = 0 \text{ or } y = 4$$

$(0, 0)$ and $(8, 4)$

c) Distance between the two points

$$d^2 = 8^2 + 4^2 = 80$$

$$d = \sqrt{80} = 4\sqrt{5}$$

$4\sqrt{5} \text{ km}$

d) time = distance $\div$ speed

$$t = \frac{4\sqrt{5}}{60} \text{ hours}$$

$$t = 0.14907 \text{ hours}$$

$$t = 0.14907 \times 60 = 8.94 \text{ minutes}$$

9 minutes

e) Any one sensible reason

hills or buildings block the signal, so the edge is not a circle