

Differentiation: first principles and basic rules

AS Pure Mathematics | Edexcel 8MA0 | 12 questions, 54 marks

A-level Maths topic practice



Scan for the solutions

Name: _____ **Date:** _____ **Score:** _____ / 54

1 A curve has equation $y = 4x^5 - 3x^2 + 7x - 9$

a. Find $\frac{dy}{dx}$

b. Find the gradient of the curve at the point where $x = -1$

(3 marks)

Answer _____

2 $f(x) = \frac{5}{x^2} + 8\sqrt{x} - 3, \quad x > 0$

Find $f'(x)$, giving each term in its simplest form.

(3 marks)

Answer _____

3 $y = 2x^3 - 8x^{\frac{3}{2}}, \quad x > 0$

Find

a. $\frac{dy}{dx}$

b. $\frac{d^2y}{dx^2}$

(3 marks)

Answer _____

4 $f(x) = 2x^2 + 3x$

Prove, from first principles, that $f'(x) = 4x + 3$

(4 marks)

Answer _____

5 Prove, from first principles, that the derivative of x^3 is $3x^2$

(4 marks)

Answer _____

6 The curve C has equation $y = 3x^2 - x$

The point $P(1, 2)$ lies on C . The point Q also lies on C and has x coordinate $1 + h$, where $h \neq 0$

- Show that the gradient of the chord PQ is $5 + 3h$
- Explain how your answer to part (a) gives the gradient of the tangent to C at P , and state its value.

(4 marks)

Answer _____

7 $y = \frac{x^2 + 6}{2\sqrt{x}}, \quad x > 0$

Show that $\frac{dy}{dx} = \frac{3(x^2 - 2)}{4x\sqrt{x}}$

(4 marks)

Answer _____

8 A curve has equation $y = (3 - \sqrt{x})^2$, $x > 0$

a. Find $\frac{dy}{dx}$

b. Find the value of x at the point on the curve where the gradient is $\frac{1}{2}$

(5 marks)

Answer _____

9 $f(x) = ax^3 + bx^2 - 3x$, where a and b are constants.

Given that $f'(2) = 17$ and $f''(2) = 22$, find the value of a and the value of b .

(5 marks)

Answer _____

10 The curve C has equation $y = \frac{16}{x} + kx^{\frac{3}{2}}$, $x > 0$, where k is a constant.

The gradient of C at the point where $x = 4$ is 5

a. Find the value of k .

b. Find the value of $\frac{d^2y}{dx^2}$ at the point where $x = 4$

(6 marks)

Answer _____

11

Water drains out of a tank. The depth of water, h cm, t minutes after the water starts to drain is modelled by

$$h = 64 - 16\sqrt{t} + t, \quad 0 \leq t \leq 64$$

- Find $\frac{dh}{dt}$
- Find the rate at which the depth is changing when $t = 16$, stating whether the depth is rising or falling.
- Find the time at which the depth is falling at 3 cm per minute.

(6 marks)

Answer _____

12

$f(x) = ax^2 + bx + c$, where a , b and c are constants.

- Prove, from first principles, that $f'(x) = 2ax + b$

The curve with equation $y = f(x)$

- passes through the point $(0, 5)$
- has gradient 1 at the point where $x = 1$

and $f''(x) = 6$

- Find $f(x)$.

(7 marks)

Answer _____

End of questions | Total 54 marks