



1 The population, P , of a town t years after 1 January 2025 is modelled by $P = 2400e^{0.05t}$

a. Write down the population of the town on 1 January 2025 according to the model.

b. Find the population predicted by the model after 10 years.

(3 marks)

a)

$$P = 2400e^0$$

$$P = 2400$$

$$\boxed{2400}$$

b)

$$P = 2400e^{0.05 \times 10}$$

$$P = 2400e^{0.5}$$

$$P = 3956.9...$$

Population is a whole number

$$\boxed{3957}$$

2 The value, $\text{£}V$, of a car t years after it was bought is modelled by $V = 18\,000e^{-0.15t}$

Find how long it takes for the value of the car to halve, giving your answer in years to 3 significant figures.

(3 marks)

$$18000e^{-0.15t} = 9000$$

$$e^{-0.15t} = 0.5$$

Take $\ln$ of both sides

$$-0.15t = \ln 0.5$$

$$t = \frac{\ln 0.5}{-0.15}$$

$$t = 4.6209...$$

$$\boxed{4.62 \text{ years}}$$

3

The curve C has equation $y = 3e^{2x}$

- Find $\frac{dy}{dx}$
- Find the exact gradient of C at the point where $x = \ln 2$
- Find an equation of the tangent to C at the point where $x = 0$

(4 marks)

a)

The derivative of e^{kx} is ke^{kx}

$$\frac{dy}{dx} = 3 \times 2e^{2x}$$

$$\frac{dy}{dx} = 6e^{2x}$$

$$\boxed{\frac{dy}{dx} = 6e^{2x}}$$

b)

$$\frac{dy}{dx} = 6e^{2 \ln 2}$$

$$= 6e^{\ln 4}$$

$$= 6 \times 4$$

$$= 24$$

$$\boxed{24}$$

c)

$$x = 0: y = 3e^0 = 3$$

$$\frac{dy}{dx} = 6e^0 = 6$$

$$y - 3 = 6(x - 0)$$

$$y = 6x + 3$$

$$\boxed{y = 6x + 3}$$

4

The mass, m grams, of a radioactive sample t days after it was made is modelled by $m = 40e^{-0.02t}$

- Show that $\frac{dm}{dt} = -0.02m$
- Find the rate at which the mass is decreasing when $t = 50$
- Find the value of t when the mass is decreasing at 0.5 grams per day.

(7 marks)

a)

The derivative of e^{kt} is ke^{kt}

$$\frac{dm}{dt} = 40 \times (-0.02)e^{-0.02t}$$

$$\frac{dm}{dt} = -0.02 \times 40e^{-0.02t}$$

$$\frac{dm}{dt} = -0.02m$$

b)

$$\frac{dm}{dt} = -0.8e^{-0.02 \times 50}$$

$$\frac{dm}{dt} = -0.8e^{-1}$$

$$\frac{dm}{dt} = -0.2943...$$

decreasing at 0.294 g per day

c)

Decreasing at 0.5 means $\frac{dm}{dt} = -0.5$

$$-0.8e^{-0.02t} = -0.5$$

$$e^{-0.02t} = 0.625$$

$$-0.02t = \ln 0.625$$

$$t = 23.50...$$

$$t = 23.5$$

5

The number of bacteria, N , in a sample t hours after it is prepared is modelled by $N = Ae^{kt}$, where A and k are constants.

When $t = 0$ there are 500 bacteria and when $t = 4$ there are 800 bacteria.

- Find the value of A and show that $k = \frac{1}{4} \ln 1.6$
- Use the model to predict the number of bacteria when $t = 10$
- Give a reason why the model may not be reliable for large values of t

(6 marks)

a)

$$t = 0: N = Ae^0 = A$$

$$A = 500$$

$$t = 4: 500e^{4k} = 800$$

$$e^{4k} = 1.6$$

$$4k = \ln 1.6$$

$$k = \frac{1}{4} \ln 1.6$$

$$k = 0.1175\dots$$

b)

$$N = 500e^{10k}$$

$$N = 500e^{2.5 \ln 1.6}$$

$$N = 500 \times 1.6^{2.5}$$

$$N = 1619.0\dots$$

1619 bacteria

c)

growth cannot continue forever: food and space run out

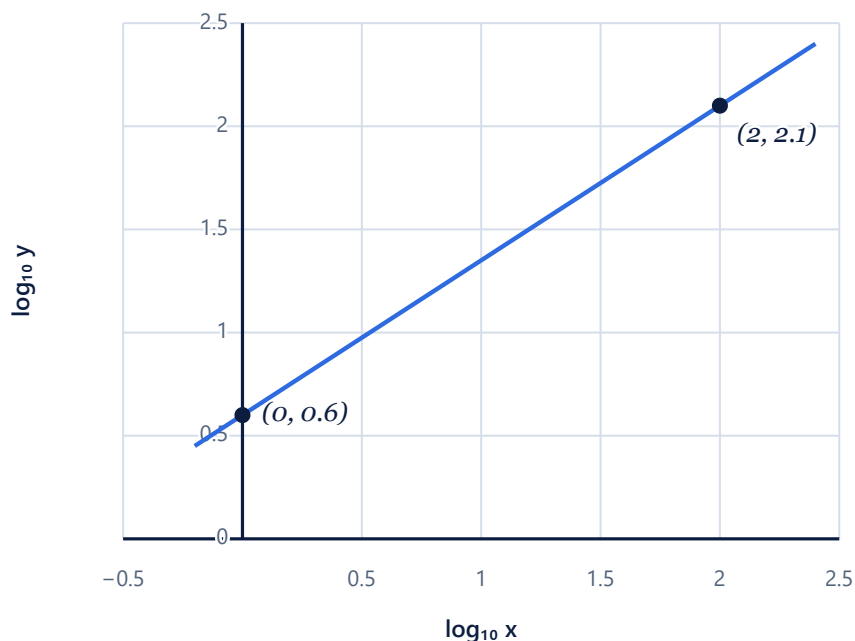
6

Two variables x and y satisfy $y = ax^n$, where a and n are constants.

a. Show that $\log_{10} y = \log_{10} a + n \log_{10} x$

A graph of $\log_{10} y$ against $\log_{10} x$ is a straight line through the points $(0, 0.6)$ and $(2, 2.1)$

b. Find the value of n and the value of a , giving a to 3 significant figures.



(5 marks)

a)

Take $\log_{10}$ of both sides

$$\log y = \log(ax^n)$$

$$\log y = \log a + \log x^n$$

$$\log y = \log a + n \log x$$

b)

Compare with $Y = mX + c$: gradient = n , intercept = $\log a$

$$n = \frac{(2.1 - 0.6)}{(2 - 0)}$$

$$n = 0.75$$

$$\log a = 0.6$$

$$a = 10^{0.6}$$

$$a = 3.981...$$

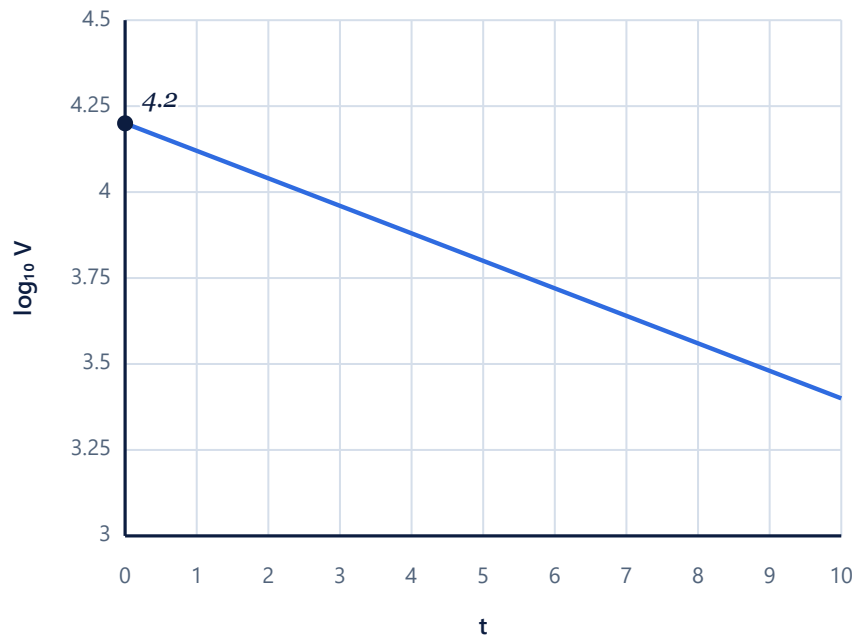
$$\boxed{n = 0.75, a = 3.98}$$

7

The value, £ V , of a machine t years after it was bought is modelled by $V = ab^t$, where a and b are constants.

A graph of $\log_{10} V$ against t is a straight line that crosses the vertical axis at 4.2 and has gradient -0.08

- Write down an equation for the line.
- Find the value of a and the value of b , giving each to 3 significant figures.
- Interpret the value of b in the context of the model.



(6 marks)

a)

$$\log V = 4.2 - 0.08t$$

b)

Take $\log_{10}$ of $V = ab^t$

$$\log V = \log a + t \log b$$

Compare coefficients

$$\log a = 4.2$$

$$a = 10^{4.2} = 15848.9\dots$$

$$\log b = -0.08$$

$$b = 10^{-0.08} = 0.8317\dots$$

$$a = 15800, b = 0.832$$

c)

$$1 - 0.832 = 0.168$$

value is multiplied by 0.832 each year: a 16.8% fall per year

8

A cup of tea is left to cool. Its temperature, θ °C, t minutes after it was made is modelled by

$$\theta = 20 + 75e^{-0.04t}$$

- Find the temperature of the tea when it was made.
- Find the temperature of the tea after 10 minutes.
- Find the time it takes for the tea to cool to 50 °C
- State the temperature that the model suggests the tea will approach in the long term, and what it represents.

(7 marks)

a)

$$\theta = 20 + 75e^0$$

$$\theta = 95$$

$$\boxed{95^\circ\text{C}}$$

b)

$$\theta = 20 + 75e^{-0.4}$$

$$\theta = 20 + 50.27...$$

$$\theta = 70.27...$$

$$\boxed{70.3^\circ\text{C}}$$

c)

$$20 + 75e^{-0.04t} = 50$$

$$75e^{-0.04t} = 30$$

$$e^{-0.04t} = 0.4$$

$$-0.04t = \ln 0.4$$

$$t = 22.907...$$

$$\boxed{22.9 \text{ minutes}}$$

d)

$$\text{As } t \rightarrow \infty, e^{-0.04t} \rightarrow 0$$

$$= \theta \rightarrow 20$$

$$\boxed{20^\circ\text{C, the room temperature}}$$

9

A student measures the time, T seconds, for one swing of a pendulum of length L metres. She plots $\log_{10} T$ against $\log_{10} L$ and finds that the points lie close to a straight line through $(-1, -0.198)$ and $(0.6, 0.602)$

- Explain why this suggests a model of the form $T = aL^n$, where a and n are constants.
- Find the value of n and the value of a , giving a to 3 significant figures.
- Use the model to find the time for one swing of a pendulum of length 2.5 m

(6 marks)

a)

$$T = aL^n$$

$$\log T = \log a + n \log L$$

this is $Y = c + mX$ with $Y = \log T$, $X = \log L$

a straight line of $\log T$ against $\log L$ fits $T = aL^n$

b)

Gradient = n

$$n = \frac{(0.602 - (-0.198))}{(0.6 - (-1))}$$

$$n = \frac{0.8}{1.6}$$

$$n = 0.5$$

Substitute a point to find $\log a$

$$-0.198 = \log a + 0.5(-1)$$

$$\log a = 0.302$$

$$a = 10^{0.302}$$

$$a = 2.004...$$

$$n = 0.5, a = 2.00$$

c)

$$T = 2.004... \times 2.5^{0.5}$$

$$T = 3.169...$$

$$3.17 \text{ s}$$

10

The number of members, P , of an online chess club t months after it started is shown in the table.

t	1	2	3	5
P	162	219	295	538
$\log_{10} P$	2.2095		2.4698	2.7308

The club secretary suggests the model $P = ab^t$, where a and b are constants.

- Show that this model gives $\log_{10} P = \log_{10} a + t \log_{10} b$
- Complete the table, giving the value to 4 decimal places.
- Using the values for $t = 1$ and $t = 5$, find the value of a and the value of b , giving each to 3 significant figures.
- Use the model to predict the number of members after 8 months, and comment on the reliability of this prediction.

(9 marks)

a)

$$\log P = \log(ab^t)$$

$$\log P = \log a + \log b^t$$

$$\log P = \log a + t \log b$$

b)

$$\log 219 = 2.34044\dots$$

$$\boxed{2.3404}$$

c)

Gradient of the line = $\log b$

$$\log b = \frac{(2.7308 - 2.2095)}{(5 - 1)}$$

$$\log b = \frac{0.5213}{4}$$

$$\log b = 0.130325$$

$$b = 10^{0.130325} = 1.3500\dots$$

Intercept = $\log a$, using $t = 1$

$$2.2095 = \log a + 0.130325$$

$$\log a = 2.079175$$

$$a = 10^{2.079175} = 120.0\dots$$

$$\boxed{a = 120, b = 1.35}$$

d)

$$P = 120 \times 1.35^8$$

$$P = 1323.8...$$

$t = 8$ is outside the range of the data: extrapolation

about 1320 members, may be unreliable

- 11 The area, $A \text{ m}^2$, of a pond covered by algae t days after it is first measured is modelled by

$$A = 50e^{kt}$$

where k is a constant. When $t = 0$, the area is increasing at a rate of 4 m^2 per day.

- Show that $k = 0.08$
- Find the area covered when it is increasing at a rate of 20 m^2 per day.
- Find the value of t when this happens.
- The pond has a surface area of 600 m^2 . Find the time when the model predicts the pond will be completely covered, and explain why the model cannot be used after that time.

(8 marks)

a)

$$\frac{dA}{dt} = 50ke^{kt}$$

$$\text{At } t = 0, \frac{dA}{dt} = 4$$

$$50ke^0 = 4$$

$$50k = 4$$

$$k = 0.08$$

b)

$$\frac{dA}{dt} = 0.08 \times 50e^{0.08t}$$

$$\frac{dA}{dt} = 0.08A$$

$$0.08A = 20$$

$$A = 250$$

$$\boxed{250 \text{ m}^2}$$

c)

$$50e^{0.08t} = 250$$

$$e^{0.08t} = 5$$

$$0.08t = \ln 5$$

$$t = 20.11...$$

$$\boxed{t = 20.1}$$

d)

$$50e^{0.08t} = 600$$

$$e^{0.08t} = 12$$

$$t = \frac{\ln 12}{0.08}$$

$$t = 31.06\dots$$

after this the model gives more than 600 m², which is impossible

31.1 days

12

In a forest, the number of grey squirrels, G , and the number of red squirrels, R , t years after the start of a study are modelled by

$$G = 300e^{0.04t} \quad R = 900e^{-0.02t}$$

- Show that the two populations are equal when $t = \frac{50}{3} \ln 3$
- Find the size of each population at this time.
- Show that, at this time, the grey squirrel population is increasing at exactly twice the rate at which the red squirrel population is decreasing.
- State one criticism of the model for the red squirrels in the long term.

(9 marks)

a)

$$300e^{0.04t} = 900e^{-0.02t}$$

Multiply both sides by $e^{0.02t}$

$$e^{0.06t} = 3$$

$$0.06t = \ln 3$$

$$t = \frac{\ln 3}{0.06}$$

$$t = \frac{50}{3} \ln 3$$

$$t = 18.31...$$

b)

$$G = 300e^{0.04 \times 18.31...}$$

$$G = 300 \times 3^{2/3}$$

$$G = 624.0...$$

$R = G$ at this time

624 of each

c)

$$\frac{dG}{dt} = 300 \times 0.04e^{0.04t} = 0.04G$$

$$\frac{dR}{dt} = 900 \times (-0.02)e^{-0.02t} = -0.02R$$

$$\text{At } t = \frac{50}{3} \ln 3, G = R$$

$$\frac{dG}{dt} = 0.04G$$

$$\left| \frac{dR}{dt} \right| = 0.02G$$

$$\frac{dG}{dt} = 2 \times \left| \frac{dR}{dt} \right|$$

grey increasing at twice the rate red is decreasing

d)

R never reaches 0, so the model cannot show the reds dying out