



- 1 Without using a calculator, find the exact value of

$$2 \log_3 6 - \log_3 4$$

(3 marks)

Power Law: $n \log a = \log a^n$

$$2 \log_3 6 = \log_3 6^2 = \log_3 36$$

Subtraction Law: $\log a - \log b = \log \frac{a}{b}$

$$\begin{aligned} \log_3 36 - \log_3 4 &= \log_3 \frac{36}{4} \\ &= \log_3 9 \end{aligned}$$

$$3^2 = 9$$

$$= 2$$

$$\boxed{2}$$

- 2 Solve the equation $5^x = 18$, giving your answer to 3 significant figures.

(3 marks)

Take $\ln$ of both sides

$$\ln 5^x = \ln 18$$

Power Law brings the x down

$$x \ln 5 = \ln 18$$

$$x = \frac{\ln 18}{\ln 5}$$

$$x = 1.7958\dots$$

$$\boxed{x = 1.80 \text{ (3 s.f.)}}$$

3 Find the exact solution of each equation.

a. $e^{2x-1} = 7$

b. $\ln(3x+2) = 4$

(4 marks)

a)

Take $\ln$ of both sides

$$2x - 1 = \ln 7$$

$$2x = 1 + \ln 7$$

$$x = \frac{(1 + \ln 7)}{2}$$

$$x = \frac{(1 + \ln 7)}{2}$$

b)

Use e to the power of both sides

$$3x + 2 = e^4$$

$$3x = e^4 - 2$$

$$x = \frac{(e^4 - 2)}{3}$$

$$x = \frac{(e^4 - 2)}{3}$$

4

Given that $\log_a 2 = p$ and $\log_a 3 = q$, write each of the following in terms of p and q

a. $\log_a 18$

b. $\log_a \frac{4}{27}$

c. $\log_a (12a)$

(5 marks)

a)

Write 18 using 2 and 3

$$\begin{aligned}\log_a 18 &= \log_a (2 \times 3^2) \\ &= \log_a 2 + 2 \log_a 3 \\ &= p + 2q\end{aligned}$$

$$\boxed{p + 2q}$$

b)

$$\begin{aligned}\log_a \frac{4}{27} &= \log_a 2^2 - \log_a 3^3 \\ &= 2 \log_a 2 - 3 \log_a 3 \\ &= 2p - 3q\end{aligned}$$

$$\boxed{2p - 3q}$$

c)

$$\log_a a = 1$$

$$\begin{aligned}\log_a (12a) &= \log_a (2^2 \times 3 \times a) \\ &= 2 \log_a 2 + \log_a 3 + \log_a a \\ &= 2p + q + 1\end{aligned}$$

$$\boxed{2p + q + 1}$$

5 Solve $\log_2(x + 5) - \log_2(x - 1) = 3$, $x > 1$

(4 marks)

Subtraction Law

$$\log_2 \frac{(x + 5)}{(x - 1)} = 3$$

Change to index form: $\log_2 N = 3$ means $N = 2^3$

$$\frac{(x + 5)}{(x - 1)} = 8$$

$$x + 5 = 8(x - 1)$$

$$x + 5 = 8x - 8$$

$$13 = 7x$$

$$x = \frac{13}{7}$$

$$\frac{13}{7} > 1, \text{ so it is valid}$$

$$\boxed{x = \frac{13}{7}}$$

6 a. Show that the equation $\log_3 x + \log_3(x - 6) = 3$ can be written as $x^2 - 6x - 27 = 0$

b. Hence solve $\log_3 x + \log_3(x - 6) = 3$, explaining why one root of the quadratic is not a solution.

(5 marks)

a)

Addition Law

$$\log_3 x(x - 6) = 3$$

$$x(x - 6) = 3^3$$

$$x^2 - 6x = 27$$

$$x^2 - 6x - 27 = 0$$

b)

$$(x - 9)(x + 3) = 0$$

$$x = 9 \text{ or } x = -3$$

You cannot take the log of a negative number

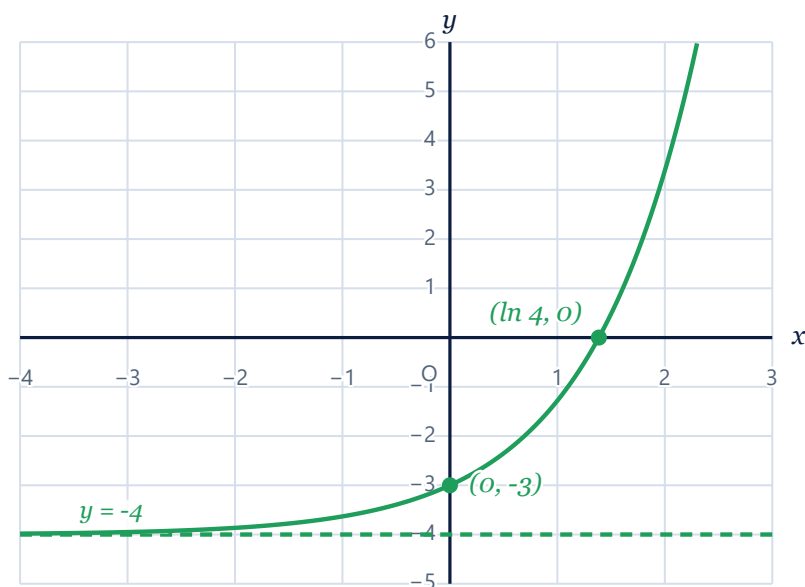
$x = -3$ gives $\log_3(-3)$, which is undefined

$$\boxed{x = 9}$$

7

Sketch the graph of $y = e^x - 4$

On your sketch show the coordinates of any points where the graph meets the coordinate axes, giving exact values, and state the equation of the asymptote.



(4 marks)

$y = e^x$ moved down 4

y-intercept: put $x = 0$

$$y = e^0 - 4$$

$$y = -3$$

x-intercept: put $y = 0$

$$e^x = 4$$

$$x = \ln 4$$

Asymptote: as $x \rightarrow -\infty$, $e^x \rightarrow 0$

$$= y \rightarrow -4$$

$$(0, -3), (\ln 4, 0), \text{ asymptote } y = -4$$

8

Solve the equation $3^{x+1} = 5^{2x-1}$

giving your answer in the form $\frac{\ln a}{\ln b}$, where a and b are rational numbers, and then to 3 significant figures.

(5 marks)

Take $\ln$ of both sides

$$(x+1) \ln 3 = (2x-1) \ln 5$$

$$x \ln 3 + \ln 3 = 2x \ln 5 - \ln 5$$

Collect the x terms on one side

$$\ln 3 + \ln 5 = 2x \ln 5 - x \ln 3$$

$$\ln 3 + \ln 5 = x(2 \ln 5 - \ln 3)$$

Use the Laws of Logs

$$\ln 15 = x(\ln 25 - \ln 3)$$

$$\ln 15 = x \ln \frac{25}{3}$$

$$x = \frac{\ln 15}{\ln (25/3)}$$

$$x = 1.2772...$$

$$x = \frac{\ln 15}{\ln (25/3)} = 1.28$$

9

a. Show that the substitution $u = 2^x$ turns the equation $2^{2x} - 7(2^x) + 12 = 0$ into $u^2 - 7u + 12 = 0$

b. Hence solve $2^{2x} - 7(2^x) + 12 = 0$, giving any non-exact answer to 3 significant figures.

(6 marks)

a)

Index law: $2^{2x} = (2^x)^2$

$$2^{2x} = u^2$$

$$u^2 - 7u + 12 = 0$$

b)

$$(u - 3)(u - 4) = 0$$

$$u = 3 \text{ or } u = 4$$

$$2^x = 4$$

$$x = 2$$

$$2^x = 3$$

$$x \ln 2 = \ln 3$$

$$x = \frac{\ln 3}{\ln 2}$$

$$x = 1.5849...$$

$$x = 2 \text{ or } x = 1.58$$

10

Solve the simultaneous equations

$$\log_2 x + \log_2 y = 6$$

$$\log_2 x - 2 \log_2 y = 3$$

(6 marks)

Combine each equation into a single log

$$\log_2 xy = 6$$

$$xy = 2^6 = 64$$

$$\log_2 \frac{x}{y^2} = 3$$

$$\frac{x}{y^2} = 2^3 = 8$$

$$x = 8y^2$$

Substitute into $xy = 64$

$$8y^2 \times y = 64$$

$$y^3 = 8$$

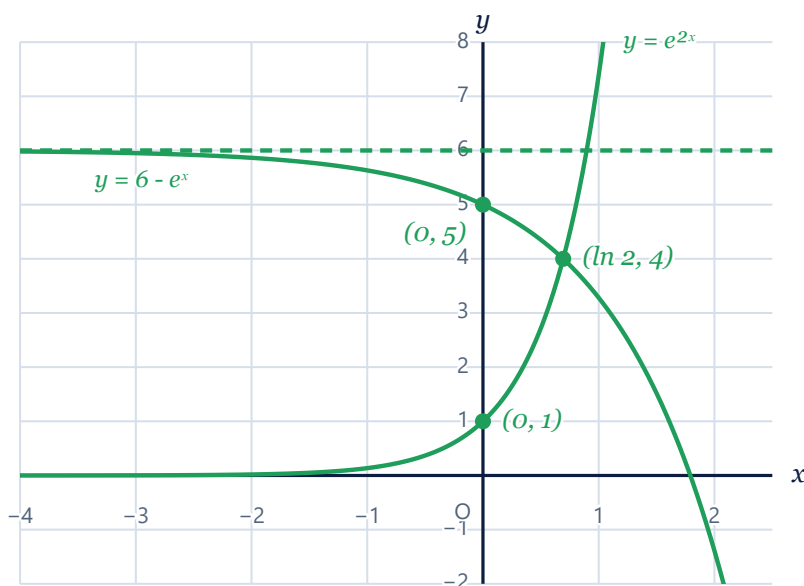
$$y = 2$$

$$x = 8(2)^2 = 32$$

Check: $\log_2 32 - 2\log_2 2 = 5 - 2 = 3$

$$x = 32, y = 2$$

- a. On the same axes, sketch the curves with equations $y = e^{2x}$ and $y = 6 - e^x$, showing the coordinates of the points where each curve meets the y -axis and the equations of any asymptotes.
- b. Find the exact coordinates of the point where the curves meet, explaining why there is only one such point.



(7 marks)

a)

$$y = e^{2x}$$

$$x = 0: y = e^0 = 1$$

as $x \rightarrow -\infty$, $y \rightarrow 0$: asymptote $y = 0$

$$y = 6 - e^x$$

$$x = 0: y = 6 - 1 = 5$$

as $x \rightarrow -\infty$, $e^x \rightarrow 0$ so $y \rightarrow 6$: asymptote $y = 6$

$$(0, 1), y = 0; (0, 5), y = 6$$

b)

$$e^{2x} = 6 - e^x$$

$$\text{Let } u = e^x, \text{ so } e^{2x} = u^2$$

$$u^2 + u - 6 = 0$$

$$(u + 3)(u - 2) = 0$$

$$e^x = -3 \text{ or } e^x = 2$$

$e^x > 0$ for all x , so $e^x = -3$ has no solution: one point only

$$x = \ln 2$$

$$y = e^{2\ln 2} = e^{\ln 4} = 4$$

$$\boxed{(\ln 2, 4)}$$

12

- a. Prove that $\log_4 x = \frac{1}{2} \log_2 x$ for $x > 0$
- b. Hence solve $\log_2 x + \log_4 x = 6$
- c. Solve $\log_2 x = \log_4(x + 6)$, explaining why only one solution is valid.

(9 marks)

a)

Let $\log_4 x = y$ and write in index form

$$x = 4^y$$

$$x = (2^2)^y = 2^{2y}$$

Take $\log_2$ of both sides

$$\log_2 x = 2y$$

$$\log_2 x = 2 \log_4 x$$

$$\log_4 x = \frac{1}{2} \log_2 x$$

$$\log_4 x = \frac{1}{2} \log_2 x$$

b)

$$\log_2 x + \frac{1}{2} \log_2 x = 6$$

$$\frac{3}{2} \log_2 x = 6$$

$$\log_2 x = 4$$

$$x = 2^4$$

$$x = 16$$

c)

$$\log_2 x = \frac{1}{2} \log_2 (x + 6)$$

$$2 \log_2 x = \log_2 (x + 6)$$

$$\log_2 x^2 = \log_2 (x + 6)$$

$$x^2 = x + 6$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$x = -2$ gives $\log_2 (-2)$, which is undefined

$$x = 3$$