



- 1 Find the set of values of x for which $5 - 3x < 2x + 20$, giving your answer in set notation.

(2 marks)

$$5 - 3x < 2x + 20$$

Collect the x terms on the side where they stay positive

$$5 < 5x + 20 \quad (+3x)$$

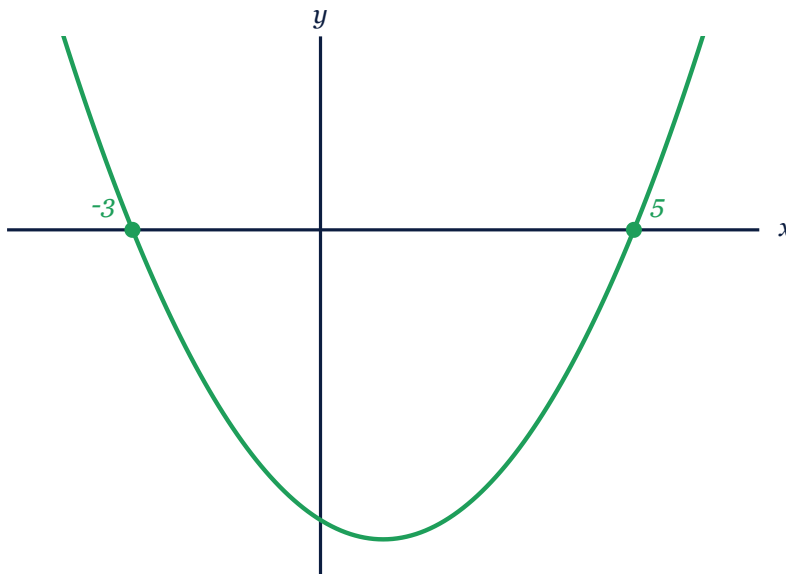
$$-15 < 5x \quad (-20)$$

$$-3 < x \quad (\div 5)$$

Write in set notation

$$\{x : x > -3\}$$

- 2 Solve the inequality $x^2 - 2x - 15 > 0$



(3 marks)

Find the critical values

$$x^2 - 2x - 15 = 0$$

$$(x - 5)(x + 3) = 0$$

$$x = 5 \text{ or } x = -3$$

Sketch: U-shaped curve, above the x -axis outside the roots

> 0 means the parts of the curve above the x -axis

$$x < -3 \text{ or } x > 5$$

3 Find the set of values of x for which $2x^2 + x - 6 \leq 0$, giving your answer in set notation.

(3 marks)

Find the critical values

$$2x^2 + x - 6 = 0$$

$$(2x - 3)(x + 2) = 0$$

$$x = \frac{3}{2} \text{ or } x = -2$$

U-shaped curve: ≤ 0 is between the roots, ends included

$$-2 \leq x \leq \frac{3}{2}$$

$$\boxed{\{x : -2 \leq x \leq \frac{3}{2}\}}$$

4 Find the set of values of x for which both $3x - 7 > 2$ and $x^2 - 9x + 14 < 0$

(4 marks)

Linear inequality

$$3x - 7 > 2$$

$$3x > 9 \text{ (+7)}$$

$$x > 3 \text{ (}\div 3\text{)}$$

Quadratic inequality: critical values

$$(x - 2)(x - 7) = 0$$

$$x = 2 \text{ or } x = 7$$

< 0 is between the roots

$$2 < x < 7$$

Both: overlap $x > 3$ with $2 < x < 7$ on a number line

$$\boxed{3 < x < 7}$$

- 5 Find the set of values of x for which $x^2 + 4x - 1 \geq 0$. Give your answer in set notation, using exact values.

(3 marks)

Critical values: does not factorise, so complete the square

$$(x + 2)^2 - 4 - 1 = 0$$

$$(x + 2)^2 = 5$$

$$x + 2 = \pm\sqrt{5}$$

$$x = -2 \pm \sqrt{5}$$

U-shaped curve: ≥ 0 is outside the roots

$$x \leq -2 - \sqrt{5} \text{ or } x \geq -2 + \sqrt{5}$$

$$\{x : x \leq -2 - \sqrt{5}\} \cup \{x : x \geq -2 + \sqrt{5}\}$$

- 6 The equation $kx^2 + 4x + (k - 3) = 0$, where k is a non-zero constant, has no real roots.

a. Show that $k^2 - 3k - 4 > 0$

b. Hence find the set of possible values of k .

(5 marks)

a) No real roots: $b^2 - 4ac < 0$

$$4^2 - 4(k)(k - 3) < 0$$

$$16 - 4k^2 + 12k < 0$$

$$0 < 4k^2 - 12k - 16$$

$$0 < k^2 - 3k - 4 \quad (\div 4)$$

$$k^2 - 3k - 4 > 0 \text{ shown}$$

b) Critical values

$$(k - 4)(k + 1) = 0$$

$$k = 4 \text{ or } k = -1$$

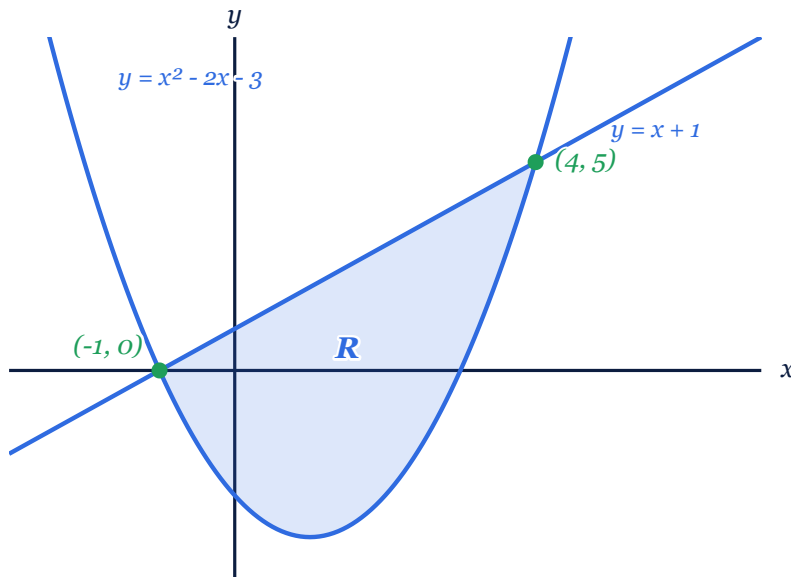
> 0 is outside the roots

$$k < -1 \text{ or } k > 4$$

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The diagram shows the curve with equation $y = x^2 - 2x - 3$ and the line with equation $y = x + 1$. The region R , shown shaded, is bounded by the curve and the line. R does not include its boundary.

- Find the coordinates of the points where the line meets the curve.
- Use inequalities to define the region R .



(5 marks)

a) Set the equations equal

$$x^2 - 2x - 3 = x + 1$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4 \text{ or } x = -1$$

$$y = 4 + 1 = 5$$

$$y = -1 + 1 = 0$$

$(-1, 0)$ and $(4, 5)$

b) R is above the curve and below the line

Boundary not included, so use strict inequalities

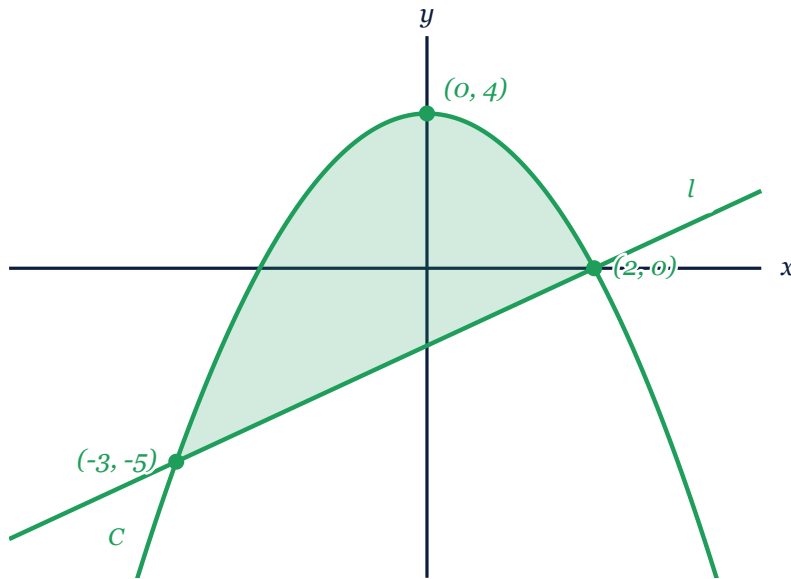
$$y > x^2 - 2x - 3 \text{ and } y < x + 1$$

8

The curve C has equation $y = 4 - x^2$ and the line l has equation $y = x - 2$

a. Find the coordinates of the points where l meets C .

b. On the axes, sketch C and l , and shade the region that satisfies both $y \leq 4 - x^2$ and $y \geq x - 2$



(5 marks)

a) Set the equations equal

$$4 - x^2 = x - 2$$

$$0 = x^2 + x - 6$$

$$0 = (x + 3)(x - 2)$$

$$x = -3 \text{ or } x = 2$$

$$y = -3 - 2 = -5$$

$$y = 2 - 2 = 0$$

$$\boxed{(-3, -5) \text{ and } (2, 0)}$$

b) C is an n shape: roots ± 2 , maximum at $(0, 4)$

l has gradient 1 and y-intercept -2

Shade below C and above l , between $x = -3$ and $x = 2$

Solid lines, because both inequalities include the boundary

$\boxed{\text{see the shaded region on the diagram}}$

9

The line with equation $y = kx - 2$, where k is a constant, meets the curve with equation $y = x^2 + 2x + 2$ at two distinct points. Find the set of possible values of k .

(5 marks)

Set the equations equal

$$x^2 + 2x + 2 = kx - 2$$

$$x^2 + (2 - k)x + 4 = 0$$

Two distinct points: $b^2 - 4ac > 0$

$$(2 - k)^2 - 4(1)(4) > 0$$

$$4 - 4k + k^2 - 16 > 0$$

$$k^2 - 4k - 12 > 0$$

Critical values

$$(k - 6)(k + 2) = 0$$

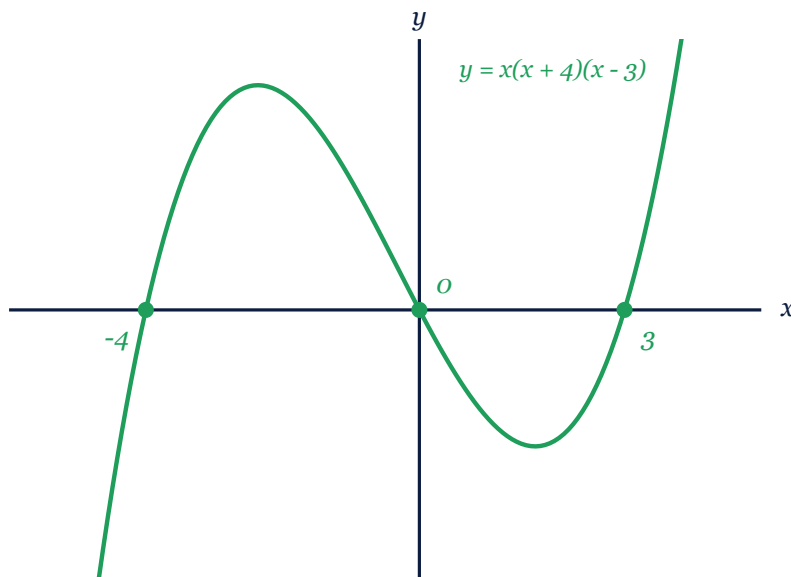
$$k = 6 \text{ or } k = -2$$

> 0 is outside the roots

$$k < -2 \text{ or } k > 6$$

10

Find the set of values of x for which $\frac{12}{x} < x + 1$, $x \neq 0$. Give your answer in set notation.



(6 marks)

x might be negative, so multiply by x^2 (always positive) instead of x

$$12x < x^3 + x^2$$

$$0 < x^3 + x^2 - 12x$$

$$0 < x(x^2 + x - 12)$$

$$0 < x(x+4)(x-3)$$

Critical values

$$x = -4, x = 0, x = 3$$

Sketch the positive cubic through -4, 0 and 3

above the x -axis for $-4 < x < 0$ and for $x > 3$

$$\boxed{\{x : -4 < x < 0\} \cup \{x : x > 3\}}$$

11

A rectangular patio has length $(2x + 3)$ m and width $(x - 1)$ m. The perimeter of the patio must be more than 30 m and its area must be less than 52 m^2 .

a. Show that $x > \frac{13}{3}$

b. Find the set of possible values of x .

(6 marks)

a) Perimeter

$$2(2x + 3) + 2(x - 1) > 30$$

$$4x + 6 + 2x - 2 > 30$$

$$6x + 4 > 30$$

$$6x > 26 \quad (-4)$$

$$x > \frac{13}{3} \quad (\div 6)$$

shown

b) Area

$$(2x + 3)(x - 1) < 52$$

$$2x^2 + x - 3 < 52$$

$$2x^2 + x - 55 < 0$$

Critical values

$$(2x + 11)(x - 5) = 0$$

$$x = -\frac{11}{2} \text{ or } x = 5$$

$$-\frac{11}{2} < x < 5$$

Combine with $x > \frac{13}{3}$ (this also makes the width positive)

$$\frac{13}{3} < x < 5$$

12

Tomasz volleys a ball in a park in Bristol. The height h metres of the ball above the ground t seconds after it is kicked is modelled by

$$h = 1 + 12t - 5t^2$$

- Find, in set notation, the values of t for which the ball is more than 5 m above the ground.
- Find the time at which the model predicts the ball hits the ground. Give your answer to 3 significant figures.
- Hence state a suitable range of values of t for which the model is valid.

(7 marks)

a) Height more than 5 m

$$1 + 12t - 5t^2 > 5$$

$$0 > 5t^2 - 12t + 4$$

Critical values

$$(5t - 2)(t - 2) = 0$$

$$t = 0.4 \text{ or } t = 2$$

< 0 is between the roots

$$\{t : 0.4 < t < 2\}$$

b) Hits the ground when $h = 0$

$$5t^2 - 12t - 1 = 0$$

$$t = \frac{12 \pm \sqrt{(144 + 20)}}{10}$$

$$t = \frac{12 \pm \sqrt{164}}{10}$$

$$t = 2.4806... \text{ (reject the negative root)}$$

$$2.48 \text{ s}$$

c) Before the kick and after landing the model does not apply

$$0 \leq t \leq 2.48$$