



Q	Answer	Marks
1	$(n - 1) + n + (n + 1) = 3n$, which is a multiple of 3 because n is an integer.	2
2	For example $x = 1, y = -2$: $1 > -2$ but $1^2 = 1$ is not greater than $(-2)^2 = 4$, so the statement is false.	2
3	The primes are 31, 37, 41, 43, 47: $31 = 30 + 1, 37 = 36 + 1, 41 = 42 - 1, 43 = 42 + 1, 47 = 48 - 1$, so every one is $6k \pm 1$.	3
4	$(2n + 3)^2 - (2n + 1)^2 = 8n + 8 = 8(n + 1)$, a multiple of 8.	3
5	$x^2 - 6x + 11 = (x - 3)^2 + 2$, and $(x - 3)^2 \geq 0$, so $x^2 - 6x + 11 \geq 2 > 0$.	3
6	n even ($n = 2k$): $2(2k^2 + 5k + 1) + 1$. n odd ($n = 2k + 1$): $2(2k^2 + 7k + 4) + 1$. Both odd, so it is always odd.	4
7	a) True: $n^2 + (n + 1)^2 = 2(n^2 + n) + 1$, odd b) False: $n = 11$ gives $121 = 11 \times 11$, not prime	5
8	a) Only four cases checked; it must hold for all (infinitely many) positive integers b) $n^3 - n = (n - 1)n(n + 1)$, a product of three consecutive integers, so divisible by 2 and by 3, so by 6	4
9	Gradient AB = $1/2$, gradient BC = -2 , product -1 so angle ABC = 90° . AB = BC = $\sqrt{20}$, so the triangle is right-angled and isosceles.	4
10	a) $(x - y)^2 \geq 0$ so $x^2 + y^2 \geq 2xy$ b) $x = \sqrt{a}, y = \sqrt{b}$ in a) gives $a + b \geq 2\sqrt{ab}$, so $(a + b)/2 \geq \sqrt{ab}$	5
11	$n(n + 1)(n + 2)(n + 3) + 1 = (n^2 + 3n)(n^2 + 3n + 2) + 1 = (n^2 + 3n + 1)^2$, a perfect square.	4
12	$n = 3k$: $9(3k^3)$. $n = 3k + 1$: $9(3k^3 + 3k^2 + k) + 1$. $n = 3k - 1$: $9(3k^3 - 3k^2 + k) - 1$. Every integer is one of these forms.	5

Full worked solutions, written out by hand, are on the solutions sheet for this topic.