



Scan for the solutions

Name: _____ **Date:** _____ **Score:** _____ / 44

1 Prove that the sum of any three consecutive integers is a multiple of 3 (2 marks)

Answer _____

2 Jamal says
"If x and y are real numbers with $x > y$, then $x^2 > y^2$ "
Use a counterexample to show that Jamal is wrong. (2 marks)

Answer _____

3 Prove by exhaustion that every prime number between 30 and 50 is either one more or one less than a multiple of 6 (3 marks)

Answer _____

4 Prove that the difference between the squares of any two consecutive odd numbers is a multiple of 8 (3 marks)

Answer _____

5 Prove that $x^2 - 6x + 11 > 0$ for all real values of x (3 marks)

Answer _____

6 Prove that $n^2 + 5n + 3$ is odd for every integer n

(4 marks)

Answer _____

7 For each statement, either prove that it is true or use a counterexample to show that it is false.

a. The sum of any two consecutive square numbers is odd.

b. $n^2 - n + 11$ is prime for every positive integer n

(5 marks)

Answer _____

8 Ava is asked to prove that $n^3 - n$ is a multiple of 6 for every positive integer n . She writes " $n = 1$ gives 0, $n = 2$ gives 6, $n = 3$ gives 24 and $n = 4$ gives 60. These are all multiples of 6, so the statement is true."

a. Explain why Ava has not proved the statement.

b. Prove that $n^3 - n$ is a multiple of 6 for every positive integer n

(4 marks)

Answer _____

9 The points A , B and C have coordinates $(1, 2)$, $(5, 4)$ and $(3, 8)$ respectively.

Prove that triangle ABC is a right-angled isosceles triangle.

(4 marks)

Answer _____

10

a. Prove that $x^2 + y^2 \geq 2xy$ for all real numbers x and y

b. Hence prove that, for all positive real numbers a and b , $\frac{a+b}{2} \geq \sqrt{ab}$

(5 marks)

Answer _____

11

Prove that one more than the product of any four consecutive integers is a perfect square.

(4 marks)

Answer _____

12

Prove that the cube of any integer is either a multiple of 9, one more than a multiple of 9 or one less than a multiple of 9

(5 marks)

Answer _____

End of questions | Total 44 marks