



- 1 Write $x^2 - 8x + 3$ in the form $(x + p)^2 + q$, where p and q are integers.

(2 marks)

Complete the square: halve the -8

$$x^2 - 8x + 3 = (x - 4)^2 - 16 + 3$$

$(x - 4)^2$ gives an extra +16, so take it away

$$= (x - 4)^2 - 13$$

$$\boxed{(x - 4)^2 - 13}$$

- 2 Write $2x^2 + 12x + 7$ in the form $a(x + b)^2 + c$, where a , b and c are integers.

(3 marks)

Take the 2 out of the x terms only

$$2x^2 + 12x + 7 = 2(x^2 + 6x) + 7$$

Complete the square inside the bracket

$$= 2[(x + 3)^2 - 9] + 7$$

Multiply the 2 back in

$$= 2(x + 3)^2 - 18 + 7$$

$$= 2(x + 3)^2 - 11$$

$$\boxed{2(x + 3)^2 - 11}$$

- 3 $f(x) = 3x^2 - 5x + 4$

Find the value of the discriminant of $f(x)$ and hence state the number of real roots of the equation

$$f(x) = 0$$

(2 marks)

$$a = 3, b = -5, c = 4$$

Discriminant

$$b^2 - 4ac = (-5)^2 - 4 \times 3 \times 4$$

$$= 25 - 48$$

$$= -23$$

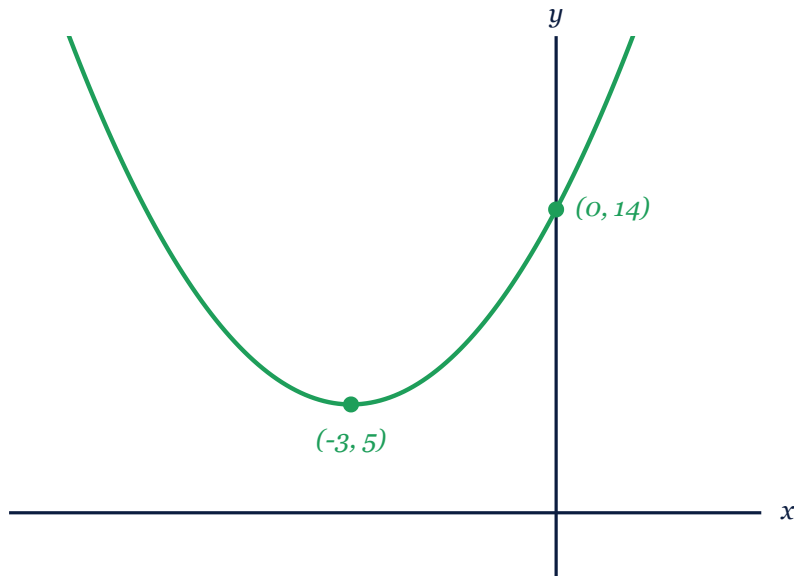
$$b^2 - 4ac < 0$$

$$\boxed{-23, \text{ so no real roots}}$$

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$$f(x) = x^2 + 6x + 14$$

- Write $f(x)$ in the form $(x + a)^2 + b$, where a and b are integers.
- Sketch the curve with equation $y = f(x)$, showing the coordinates of the turning point and of any point where the curve meets the coordinate axes.



(5 marks)

a)

$$\begin{aligned} x^2 + 6x + 14 &= (x + 3)^2 - 9 + 14 \\ &= (x + 3)^2 + 5 \end{aligned}$$

$$(x + 3)^2 + 5$$

b)

Positive x^2 so a U shapeTurning point from a): $x + 3 = 0$ minimum $(-3, 5)$ y-intercept: put $x = 0$

$$y = 0 + 0 + 14 = 14$$

Minimum value $5 > 0$, so it never meets the x-axis

see the sketch

- 5 The equation $3x^2 - 6x + k = 0$, where k is a constant, has two distinct real roots.

Find the range of possible values of k .

(3 marks)

Two distinct real roots: $b^2 - 4ac > 0$

$$a = 3, b = -6, c = k$$

$$(-6)^2 - 4 \times 3 \times k > 0$$

$$36 - 12k > 0$$

$$36 > 12k$$

$$k < 3$$

- 6 Prove that $4x^2 - 12x + 11 > 0$ for all real values of x .

(3 marks)

Complete the square

$$\begin{aligned} 4x^2 - 12x + 11 &= 4(x^2 - 3x) + 11 \\ &= 4\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 11 \\ &= 4\left(x - \frac{3}{2}\right)^2 - 9 + 11 \\ &= 4\left(x - \frac{3}{2}\right)^2 + 2 \end{aligned}$$

A square is never negative

$$4\left(x - \frac{3}{2}\right)^2 \geq 0 \text{ for all } x$$

$$\text{so } 4\left(x - \frac{3}{2}\right)^2 + 2 \geq 2 > 0$$

$$4x^2 - 12x + 11 > 0 \text{ for all real } x$$

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$$g(x) = 5 + 8x - 2x^2$$

- Write $g(x)$ in the form $a - b(x - c)^2$, where a , b and c are integers.
- Hence state the maximum value of $g(x)$ and the value of x at which it occurs.

(5 marks)

a)

Take -2 out of the x terms

$$\begin{aligned} 5 + 8x - 2x^2 &= -2(x^2 - 4x) + 5 \\ &= -2[(x - 2)^2 - 4] + 5 \\ &= -2(x - 2)^2 + 8 + 5 \end{aligned}$$

$$\boxed{13 - 2(x - 2)^2}$$

b)

 $2(x - 2)^2 \geq 0$, so g is biggest when it is 0

$$x - 2 = 0$$

$$x = 2$$

$$\boxed{\text{maximum } 13 \text{ when } x = 2}$$

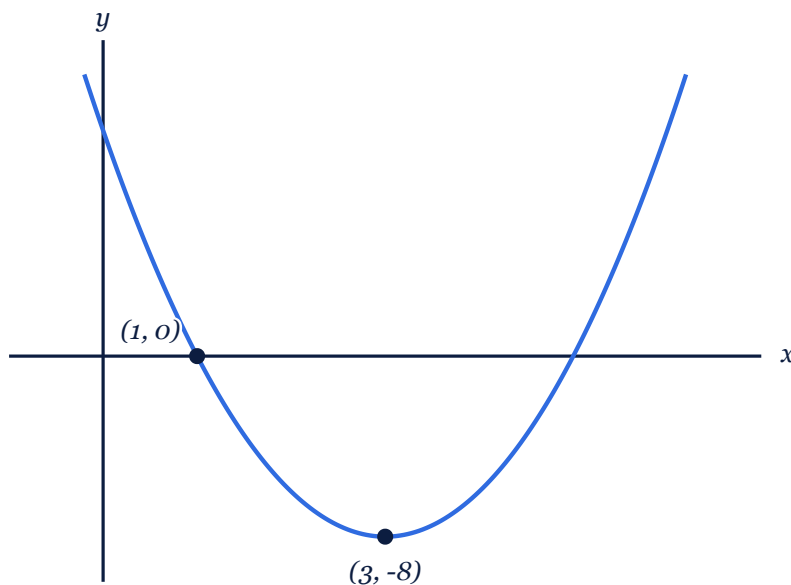
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The figure shows a sketch of the curve with equation $y = ax^2 + bx + c$, where a , b and c are constants.

The curve has a minimum turning point at $(3, -8)$ and crosses the x -axis at $(1, 0)$.

a. Find the values of a , b and c .

b. Find the coordinates of the other point where the curve crosses the x -axis.



(4 marks)

a)

Use completed square form with turning point $(3, -8)$

$$y = a(x - 3)^2 - 8$$

Substitute $(1, 0)$

$$0 = a(1 - 3)^2 - 8$$

$$0 = 4a - 8$$

$$a = 2$$

Expand

$$y = 2(x^2 - 6x + 9) - 8$$

$$y = 2x^2 - 12x + 10$$

$$\boxed{a = 2, b = -12, c = 10}$$

b)

Symmetrical about $x = 3$: 1 is 2 to the left, so go 2 to the right

$$x = 3 + 2 = 5$$

$$\boxed{(5, 0)}$$

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The equation $x^2 + 2px + (3p + 4) = 0$, where p is a constant, has equal roots.

a. Find the possible values of p .

b. For each value of p , find the root of the equation.

(5 marks)

a)

Equal roots: $b^2 - 4ac = 0$

$$a = 1, b = 2p, c = 3p + 4$$

$$(2p)^2 - 4 \times 1 \times (3p + 4) = 0$$

$$4p^2 - 12p - 16 = 0$$

Divide by 4

$$p^2 - 3p - 4 = 0$$

$$(p - 4)(p + 1) = 0$$

$$p = 4 \text{ or } p = -1$$

b)

$$p = 4$$

$$x^2 + 8x + 16 = 0$$

$$(x + 4)^2 = 0$$

$$x = -4$$

$$p = -1$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

$$x = 1$$

$$p = 4: x = -4 \quad p = -1: x = 1$$

Amara kicks a ball from the top of a wall. The height of the ball above the ground, h metres, t seconds after it is kicked is modelled by

$$h = 2 + 12t - 5t^2$$

- Write h in the form $A - B(t - C)^2$, where A , B and C are constants.
- Hence find, according to the model, the maximum height of the ball and the time at which it reaches it.
- Find the time when the ball hits the ground, giving your answer to 3 significant figures.
- Give one limitation of the model.

(8 marks)

a)

Take -5 out of the t terms

$$\begin{aligned} h &= -5(t^2 - 2.4t) + 2 \\ &= -5[(t - 1.2)^2 - 1.44] + 2 \\ &= -5(t - 1.2)^2 + 7.2 + 2 \end{aligned}$$

$$h = 9.2 - 5(t - 1.2)^2$$

b)

Largest when $(t - 1.2)^2 = 0$

$$9.2 \text{ m at } t = 1.2 \text{ s}$$

c)

Hits the ground when $h = 0$

$$\begin{aligned} 9.2 - 5(t - 1.2)^2 &= 0 \\ (t - 1.2)^2 &= 1.84 \\ t - 1.2 &= \pm\sqrt{1.84} \end{aligned}$$

t must be positive, so take +

$$t = 1.2 + 1.3565...$$

$$t = 2.5565...$$

$$2.56 \text{ s}$$

d)

It ignores air resistance (or wind)

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a. Write $x^2 - 6x + 13$ in the form $(x + p)^2 + q$, where p and q are integers.

b. Hence find the maximum value of $\frac{10}{x^2 - 6x + 13}$, the value of x at which it occurs, and the range of possible values of $\frac{10}{x^2 - 6x + 13}$ for real x .

(5 marks)

a)

$$x^2 - 6x + 13 = (x - 3)^2 - 9 + 13$$

$$(x - 3)^2 + 4$$

b)

Smallest denominator gives the biggest fraction

$$(x - 3)^2 + 4 \geq 4, \text{ smallest when } x = 3$$

$$\max = \frac{10}{4} = \frac{5}{2}$$

Denominator can be huge, so the fraction tends to 0 but is always positive

$$\max \frac{5}{2} \text{ at } x = 3; 0 < \frac{10}{x^2 - 6x + 13} \leq \frac{5}{2}$$

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$f(x) = x^2 - 2kx + 3k + 4$, where k is a constant.

- Write $f(x)$ in the form $(x + a)^2 + b$, where a and b are given in terms of k .
- Given that the curve $y = f(x)$ lies entirely above the x -axis, find the range of possible values of k .
- When $k = 4$, write down the coordinates of the turning point of $y = f(x)$ and describe how the curve meets the x -axis.

(6 marks)

a)

Halve the $-2k$

$$f(x) = (x - k)^2 - k^2 + 3k + 4$$

$$a = -k, b = -k^2 + 3k + 4$$

b)

Above the x -axis: minimum value > 0

$$-k^2 + 3k + 4 > 0$$

$$k^2 - 3k - 4 < 0$$

Critical values

$$(k - 4)(k + 1) = 0$$

$$k = 4 \text{ or } k = -1$$

U-shaped in k , below zero between the roots

$$-1 < k < 4$$

c)

$$b = -16 + 12 + 4 = 0$$

turning point $(k, b) = (4, 0)$
 $(4, 0)$: the curve touches the x -axis there