



- 1 The diagram shows triangle ABC , with $AB = 7$ cm, $AC = 10$ cm and angle $BAC = 50^\circ$. Find the length of BC . Give your answer to 3 significant figures.

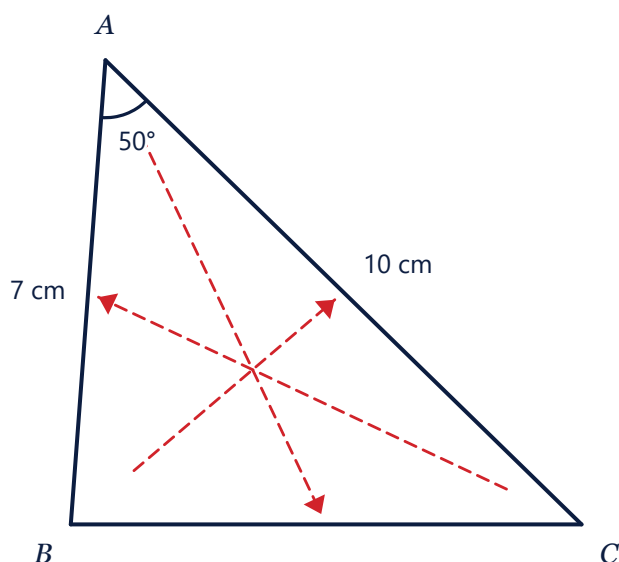


Diagram NOT accurately drawn

(3 marks)

Two sides and the included angle: cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$BC^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 50^\circ$$

$$= 149 - 140 \cos 50^\circ$$

$$= 59.009...$$

$$BC = \sqrt{59.009...}$$

$$= 7.6817...$$

$$BC = 7.68 \text{ cm}$$

2

In triangle ABC , $AB = 12$ cm, angle $ABC = 47^\circ$ and angle $ACB = 58^\circ$

Find the length of AC . Give your answer to 3 significant figures.

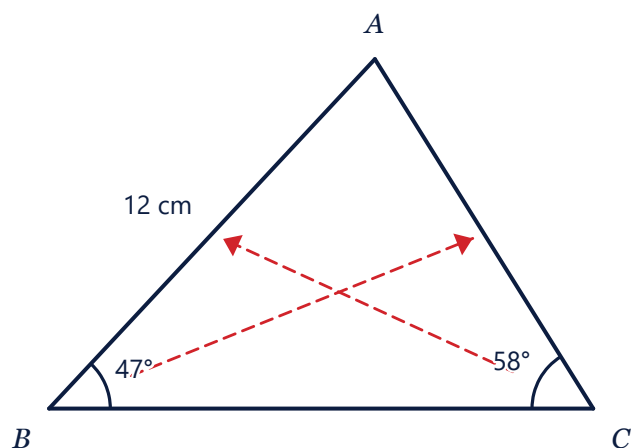


Diagram NOT accurately drawn

(3 marks)

Pairs of opposite sides and angles: sine rule

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} \\ \frac{AC}{\sin 47^\circ} &= \frac{12}{\sin 58^\circ} \\ AC &= \frac{12 \sin 47^\circ}{\sin 58^\circ} \\ &= 10.348\dots\end{aligned}$$

$$AC = 10.3 \text{ cm}$$

3

Triangle ABC has $AB = 5$ cm, $AC = 8$ cm and $BC = 10$ cm.

Find the size of the largest angle in the triangle. Give your answer in degrees to 1 decimal place.

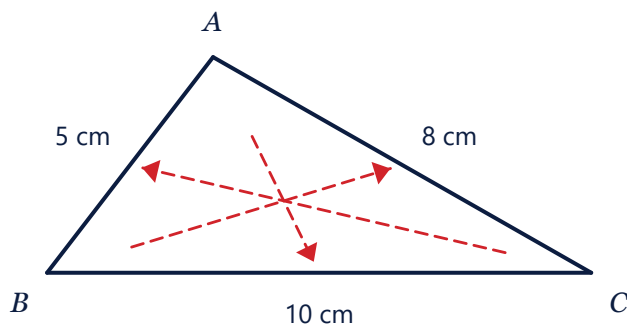


Diagram NOT accurately drawn

(3 marks)

Largest angle is opposite the longest side, BC

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\begin{aligned} \cos A &= \frac{8^2 + 5^2 - 10^2}{2 \times 8 \times 5} \\ &= \frac{-11}{80} \end{aligned}$$

$$\begin{aligned} A &= \cos^{-1}(-0.1375) \\ &= 97.90\dots^\circ \end{aligned}$$

$$\boxed{\text{angle } BAC = 97.9^\circ}$$

4

The diagram shows triangle ABC with $AB = 6.4$ cm, $AC = 8.5$ cm and angle $BAC = 112^\circ$

Find the area of the triangle. Give your answer to 3 significant figures.

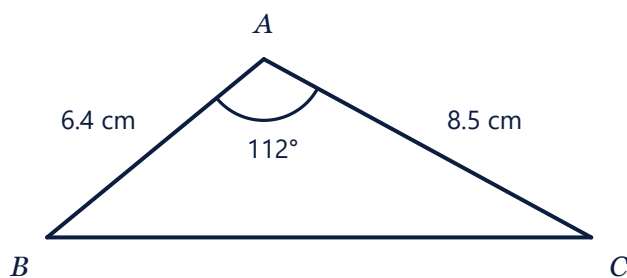


Diagram NOT accurately drawn

(2 marks)

Two sides and the angle between them

$$\begin{aligned}\text{area} &= \frac{1}{2}bc \sin A \\ &= \frac{1}{2} \times 6.4 \times 8.5 \times \sin 112^\circ \\ &= 25.219\dots\end{aligned}$$

$$\boxed{25.2 \text{ cm}^2}$$

5

In triangle ABC , $AB = 14$ cm, $BC = 9$ cm and angle $ABC = 63^\circ$

- Find the length of AC , giving your answer to 3 significant figures.
- Find the size of angle BAC , giving your answer to 1 decimal place.

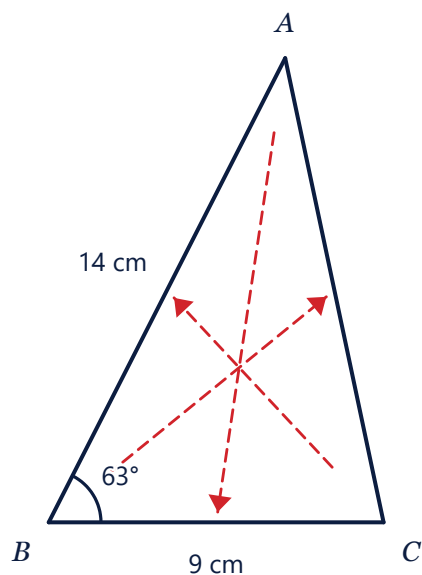


Diagram NOT accurately drawn

(5 marks)

a) Two sides and the included angle: cosine rule

$$\begin{aligned} AC^2 &= 14^2 + 9^2 - 2 \times 14 \times 9 \times \cos 63^\circ \\ &= 277 - 252 \cos 63^\circ \\ &= 162.594... \end{aligned}$$

$$AC = 12.751...$$

$$\boxed{AC = 12.8 \text{ cm}}$$

b) Sine rule, using the unrounded AC

$$\begin{aligned} \frac{\sin A}{9} &= \frac{\sin 63^\circ}{12.751...} \\ \sin A &= \frac{9 \sin 63^\circ}{12.751...} \\ &= 0.62888... \end{aligned}$$

A is opposite the shortest side, so it is acute

$$\begin{aligned} A &= \sin^{-1}(0.62888...) \\ &= 38.97...^\circ \end{aligned}$$

$$\boxed{\text{angle } BAC = 39.0^\circ}$$

6

In triangle ABC , $AB = 11$ cm, $BC = 8$ cm and angle $BAC = 35^\circ$. The diagram shows one possible shape for the triangle.

Find the two possible values of angle ACB , giving your answers to 1 decimal place.

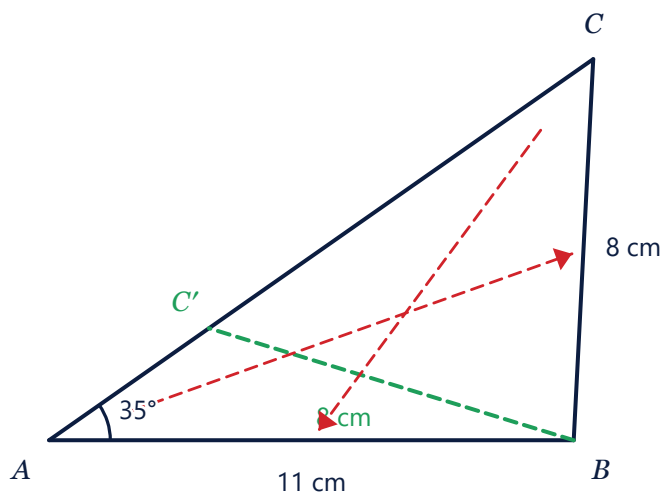


Diagram NOT accurately drawn

(4 marks)

Sine rule for an angle

$$\frac{\sin C}{11} = \frac{\sin 35^\circ}{8}$$

$$\sin C = \frac{11 \sin 35^\circ}{8}$$

$$= 0.78866...$$

$$C = \sin^{-1}(0.78866...) = 52.06...^\circ$$

$\sin(180^\circ - C) = \sin C$ gives a second angle

$$C = 180^\circ - 52.06...^\circ = 127.93...^\circ$$

Check it fits: $35^\circ + 127.9^\circ < 180^\circ$

$$\text{angle } ACB = 52.1^\circ \text{ or } 127.9^\circ$$

7

Triangle ABC has area 30 cm^2 . $AB = 8 \text{ cm}$ and angle $BAC = 40^\circ$

a. Find the length of AC .

b. Find the length of BC .

Give your answers to 3 significant figures.

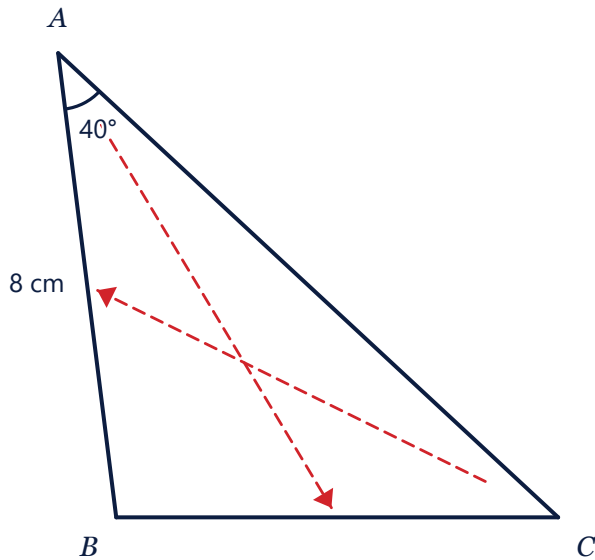


Diagram NOT accurately drawn

(5 marks)

a) Use area = $\frac{1}{2}bc \sin A$

$$\frac{1}{2} \times 8 \times AC \times \sin 40^\circ = 30$$

$$4 \sin 40^\circ \times AC = 30$$

$$AC = \frac{30}{4 \sin 40^\circ}$$

$$= 11.668...$$

$$AC = 11.7 \text{ cm}$$

b) Cosine rule with the unrounded AC

$$BC^2 = 8^2 + 11.668...^2 - 2 \times 8 \times 11.668... \times \cos 40^\circ$$

$$= 57.130...$$

$$BC = 7.558...$$

$$BC = 7.56 \text{ cm}$$

8

The diagram shows triangle ABC with $AB = x$ cm, $AC = (x + 2)$ cm, $BC = 7$ cm and angle $BAC = 120^\circ$

- Show that $x^2 + 2x - 15 = 0$
- Hence find the value of x .
- Find the exact area of the triangle.

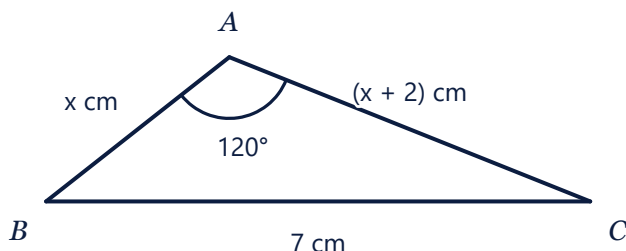


Diagram NOT accurately drawn

(6 marks)

a) Cosine rule with the 120° angle

$$7^2 = x^2 + (x + 2)^2 - 2x(x + 2) \cos 120^\circ$$

$$\cos 120^\circ = -\frac{1}{2}$$

$$49 = x^2 + x^2 + 4x + 4 + x(x + 2)$$

$$49 = 3x^2 + 6x + 4$$

$$0 = 3x^2 + 6x - 45$$

Divide by 3

$$x^2 + 2x - 15 = 0$$

$$x^2 + 2x - 15 = 0 \text{ as required}$$

b) Factorise

$$(x + 5)(x - 3) = 0$$

A length is positive, reject $x = -5$

$$x = 3$$

c) $AB = 3$, $AC = 5$, $\sin 120^\circ = \frac{\sqrt{3}}{2}$

$$\begin{aligned} \text{area} &= \frac{1}{2} \times 3 \times 5 \times \sin 120^\circ \\ &= \frac{15}{2} \times \frac{\sqrt{3}}{2} \\ &= \frac{15\sqrt{3}}{4} \end{aligned}$$

$$\frac{15\sqrt{3}}{4} \text{ cm}^2$$

9

A boat leaves harbour A and sails 12 km on a bearing of 050° to a buoy B . It then sails 8 km on a bearing of 150° to a lighthouse C .

- Show that angle $ABC = 80^\circ$
- Find the distance AC , giving your answer to 3 significant figures.
- Find the bearing of C from A , giving your answer to the nearest degree.

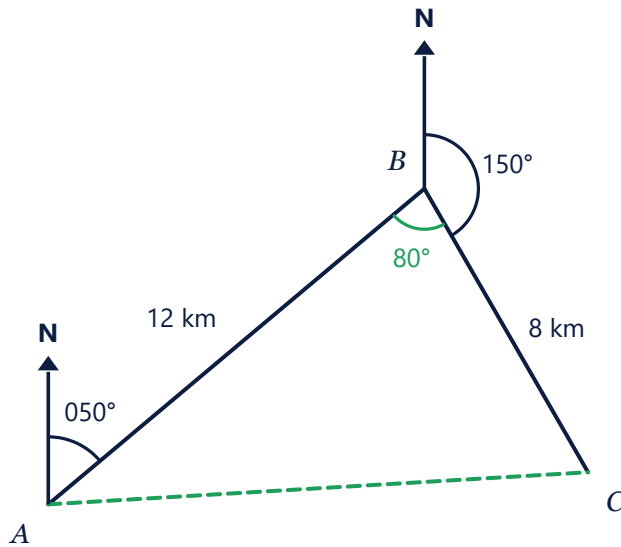


Diagram NOT accurately drawn

(6 marks)

a) Back bearing from B to A

$$050^\circ + 180^\circ = 230^\circ$$

$$\text{angle } ABC = 230^\circ - 150^\circ = 80^\circ$$

$$\text{angle } ABC = 80^\circ \text{ as required}$$

b) Cosine rule

$$AC^2 = 12^2 + 8^2 - 2 \times 12 \times 8 \times \cos 80^\circ$$

$$= 208 - 192 \cos 80^\circ$$

$$= 174.659\dots$$

$$AC = 13.215\dots$$

$$AC = 13.2 \text{ km}$$

c) Sine rule for angle BAC

$$\frac{\sin BAC}{8} = \frac{\sin 80^\circ}{13.215\dots}$$

$$\sin BAC = \frac{8 \sin 80^\circ}{13.215\dots} = 0.59615\dots$$

$$\text{angle } BAC = 36.59\dots^\circ$$

Add to the first bearing

$$\text{bearing} = 50^\circ + 36.59\dots^\circ = 86.59\dots^\circ$$

087°

10

A farmer's field is in the shape of triangle ABC , with $AB = 120$ m, $BC = 95$ m and $AC = 150$ m.

- Find the size of angle ABC , giving your answer to 1 decimal place.
- Find the area of the field in hectares, where 1 hectare = 10 000 m². Give your answer to 2 significant figures.

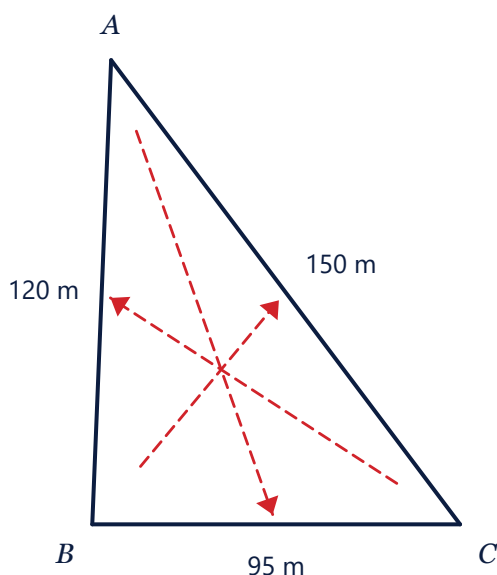


Diagram NOT accurately drawn

(5 marks)

a) Three sides: cosine rule for the angle

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\begin{aligned}\cos B &= \frac{95^2 + 120^2 - 150^2}{2 \times 95 \times 120} \\ &= \frac{925}{22800}\end{aligned}$$

$$\begin{aligned}B &= \cos^{-1}(0.04057...) \\ &= 87.67...\end{aligned}$$

$$\boxed{\text{angle } ABC = 87.7^\circ}$$

b) Area with the unrounded angle

$$\begin{aligned}\text{area} &= \frac{1}{2} \times 120 \times 95 \times \sin 87.67...^\circ \\ &= 5695.3... \text{ m}^2\end{aligned}$$

Divide by 10 000 for hectares

$$5695.3 \div 10000 = 0.5695...$$

$$\boxed{0.57 \text{ hectares}}$$

11

Triangle ABC has $AB = 6$ cm, $BC = 4$ cm and $AC = 5$ cm.

a. Show that $\cos ACB = \frac{1}{8}$

b. Hence find the exact area of triangle ABC , giving your answer in the form $k\sqrt{7}$, where k is a rational number.

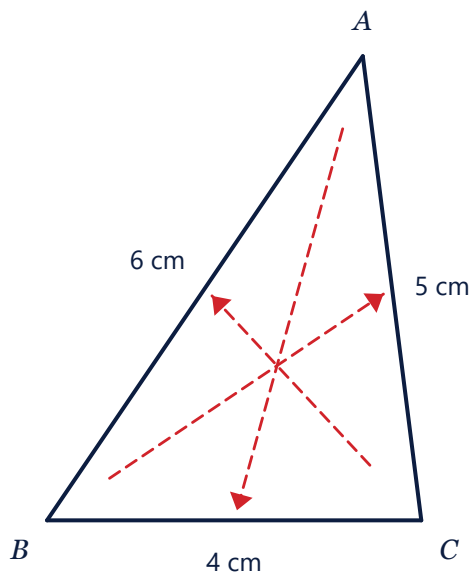


Diagram NOT accurately drawn

(5 marks)

a) AB is opposite angle C

$$\begin{aligned}\cos C &= \frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5} \\ &= \frac{16 + 25 - 36}{40} \\ &= \frac{5}{40} = \frac{1}{8}\end{aligned}$$

$$\cos ACB = \frac{1}{8} \text{ as required}$$

b) Use $\sin^2 C + \cos^2 C = 1$

$$\begin{aligned}\sin^2 C &= 1 - \left(\frac{1}{8}\right)^2 \\ &= \frac{63}{64}\end{aligned}$$

C is an angle in a triangle, so $\sin C > 0$

$$\sin C = \frac{\sqrt{63}}{8} = \frac{3\sqrt{7}}{8}$$

$$\text{area} = \frac{1}{2} \times 4 \times 5 \times \frac{3\sqrt{7}}{8}$$

$$= \frac{60\sqrt{7}}{16}$$

$$= \frac{15}{4}\sqrt{7}$$

$$\boxed{\frac{15}{4}\sqrt{7} \text{ cm}^2}$$

12

In triangle ABC , $AB = 10$ cm, $AC = 6$ cm and angle $ABC = 30^\circ$. The length of BC is a cm. The diagram shows one possible shape for the triangle.

- Show that $a^2 - 10\sqrt{3}a + 64 = 0$
- Hence find the two possible exact lengths of BC .
- Explain why there are two possible triangles.

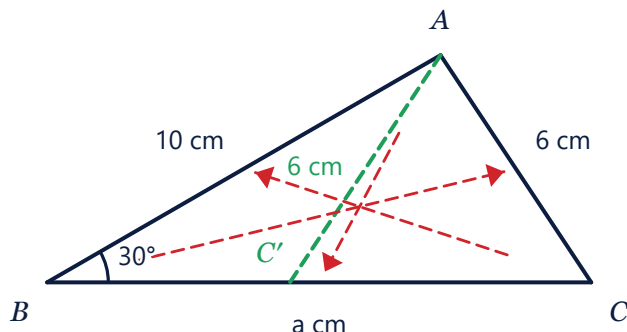


Diagram NOT accurately drawn

(6 marks)

a) Cosine rule with the angle at B

$$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos B$$

$$6^2 = 10^2 + a^2 - 2 \times 10 \times a \times \cos 30^\circ$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$36 = 100 + a^2 - 10\sqrt{3}a$$

$$a^2 - 10\sqrt{3}a + 64 = 0$$

$$a^2 - 10\sqrt{3}a + 64 = 0 \text{ as required}$$

b) Quadratic formula

$$a = \frac{10\sqrt{3} \pm \sqrt{(10\sqrt{3})^2 - 4 \times 64}}{2}$$

$$= \frac{10\sqrt{3} \pm \sqrt{300 - 256}}{2}$$

$$= \frac{10\sqrt{3} \pm \sqrt{44}}{2}$$

$$\sqrt{44} = 2\sqrt{11}$$

$$= 5\sqrt{3} \pm \sqrt{11}$$

$$BC = 5\sqrt{3} + \sqrt{11} \text{ cm or } 5\sqrt{3} - \sqrt{11} \text{ cm}$$

c)

Both roots are positive, so both are lengths.

Height from A to $BC = 10 \sin 30^\circ = 5$ cm, and

$5 < AC = 6 < AB = 10$, so AC can swing to meet the base at two points