



**1** The line  $L$  has equation  $3x - 4y + 12 = 0$

- Find the gradient of  $L$ .
- Find the coordinates of the points where  $L$  crosses the  $x$ -axis and the  $y$ -axis.

(3 marks)

a) Rearrange into the form  $y = mx + c$

$$4y = 3x + 12$$

$$y = \frac{3}{4}x + 3$$

$$\text{gradient} = \frac{3}{4}$$

b) The  $y$ -axis is where  $x = 0$

$$y = \frac{3}{4}(0) + 3$$

$$y = 3$$

The  $x$ -axis is where  $y = 0$

$$\begin{array}{rcl} 3x + 12 & = & 0 \\ -12 & & \\ \hline 3x & = & -12 \\ \div 3 & & \\ x & = & -4 \end{array}$$

$$(-4, 0) \text{ and } (0, 3)$$

2

The points  $A$  and  $B$  have coordinates  $(-2, 7)$  and  $(4, -2)$  respectively.

Find an equation of the line through  $A$  and  $B$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

(3 marks)

Gradient = change in  $y \div$  change in  $x$

$$m = \frac{-2 - 7}{4 - (-2)}$$

$$m = \frac{-9}{6}$$

$$m = -\frac{3}{2}$$

Use  $y - y_1 = m(x - x_1)$  with  $A(-2, 7)$

$$y - 7 = -\frac{3}{2}(x + 2)$$

Multiply both sides by 2 to clear the fraction

$$2y - 14 = -3(x + 2)$$

$$2y - 14 = -3x - 6$$

Collect everything on one side

$$3x + 2y - 8 = 0$$

Check with  $B(4, -2)$ :  $12 - 4 - 8 = 0$

$$\boxed{3x + 2y - 8 = 0}$$

3

The line  $l_1$  has equation  $2x - 5y + 3 = 0$

The line  $l_2$  is perpendicular to  $l_1$  and passes through the point  $P(5, -1)$ .

Find an equation of  $l_2$  in the form  $y = mx + c$

(3 marks)

Gradient of  $l_1$ : rearrange to  $y = mx + c$

$$5y = 2x + 3$$

$$y = \frac{2}{5}x + \frac{3}{5}$$

$$m_1 = \frac{2}{5}$$

Perpendicular:  $m_1 \times m_2 = -1$

$$m_2 = -\frac{5}{2}$$

Use  $y - y_1 = m(x - x_1)$  with  $P(5, -1)$

$$y + 1 = -\frac{5}{2}(x - 5)$$

$$y + 1 = -\frac{5}{2}x + \frac{25}{2}$$

$$y = -\frac{5}{2}x + \frac{25}{2} - 1$$

$$\boxed{y = -\frac{5}{2}x + \frac{23}{2}}$$

4

The points  $A(-3, 4)$  and  $B(5, 8)$  are joined to form the line segment  $AB$ .

Find an equation of the perpendicular bisector of  $AB$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

(4 marks)

The bisector passes through the midpoint

$$M = \left( \frac{-3 + 5}{2}, \frac{4 + 8}{2} \right)$$

$$M = (1, 6)$$

Gradient of  $AB$

$$m = \frac{8 - 4}{5 - (-3)}$$

$$m = \frac{4}{8} = \frac{1}{2}$$

Perpendicular gradient: negative reciprocal

$$m = -2$$

Line through  $M(1, 6)$  with gradient  $-2$

$$y - 6 = -2(x - 1)$$

$$y - 6 = -2x + 2$$

$$y = -2x + 8$$

$$\boxed{2x + y - 8 = 0}$$

5

The points  $A(1, k)$  and  $B(k, 9)$ , where  $k$  is a constant, lie on a line that is perpendicular to the line with equation  $3x + y = 7$

Find the value of  $k$ .

(3 marks)

Gradient of the given line

$$y = -3x + 7$$

$$m = -3$$

So AB has gradient  $\frac{1}{3}$  (negative reciprocal)

Gradient of AB from the coordinates

$$\frac{9 - k}{k - 1} = \frac{1}{3}$$

Cross-multiply

$$3(9 - k) = k - 1$$

$$27 - 3k = k - 1$$

$$+1 \quad 28 - 3k = k \quad +1$$

$$+3k \quad 28 = 4k \quad +3k$$

$$k = 7$$

6

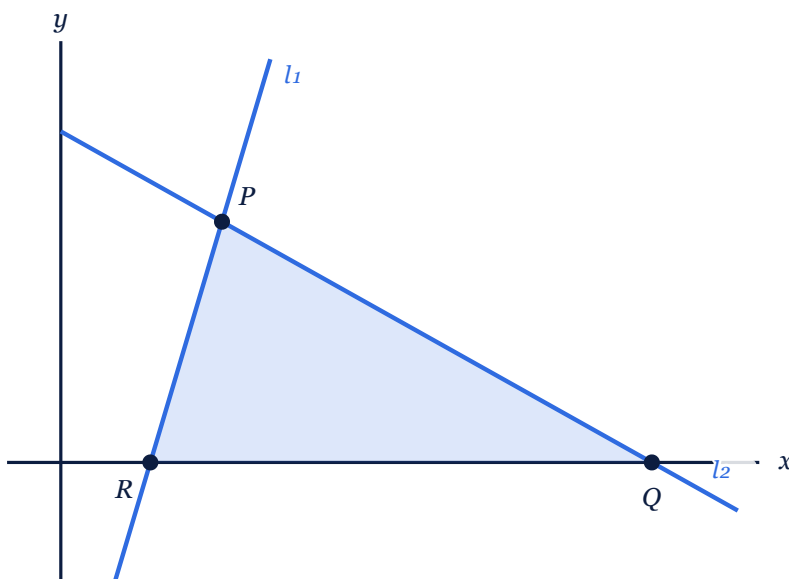
The line  $l_1$  has equation  $y = 3x - 5$  and the line  $l_2$  has equation  $x + 2y - 11 = 0$

The lines meet at the point  $P$ , as shown in the sketch.

a. Find the coordinates of  $P$ .

The line  $l_1$  crosses the  $x$ -axis at  $R$  and the line  $l_2$  crosses the  $x$ -axis at  $Q$ .

b. Find the exact area of triangle  $PQR$ .



(5 marks)

a) Substitute  $y = 3x - 5$  into  $l_2$

$$x + 2(3x - 5) - 11 = 0$$

$$x + 6x - 10 - 11 = 0$$

$$7x - 21 = 0$$

$$x = 3$$

$$y = 3(3) - 5 = 4$$

$$P(3, 4)$$

b) R: put  $y = 0$  in  $l_1$

$$3x - 5 = 0$$

$$x = \frac{5}{3}$$

Q: put  $y = 0$  in  $l_2$

$$x - 11 = 0$$

$$x = 11$$

Base  $QR$  on the  $x$ -axis, height =  $y$ -coordinate of  $P$

$$QR = 11 - \frac{5}{3} = \frac{28}{3}$$

$$\text{Area} = \frac{1}{2} \times \frac{28}{3} \times 4$$

$$\boxed{\text{Area} = \frac{56}{3}}$$

7

The points  $A(-1, 2)$ ,  $B(3, 5)$  and  $C(6, 1)$  are three vertices of the square  $ABCD$ .

- Show that  $AB = BC$ .
- Show that  $AB$  is perpendicular to  $BC$ .
- Find the area of the square  $ABCD$ .
- Find the coordinates of  $D$ .

(7 marks)

a) Distance =  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$AB = \sqrt{(3 - (-1))^2 + (5 - 2)^2}$$

$$AB = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$BC = \sqrt{(6 - 3)^2 + (1 - 5)^2}$$

$$BC = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\boxed{AB = BC = 5}$$

b) Find both gradients

$$m(AB) = \frac{5 - 2}{3 - (-1)} = \frac{3}{4}$$

$$m(BC) = \frac{1 - 5}{6 - 3} = -\frac{4}{3}$$

$$\frac{3}{4} \times -\frac{4}{3} = -1$$

so  $AB$  is perpendicular to  $BC$

c) Side length 5

$$\text{Area} = 5^2$$

$$\boxed{\text{Area} = 25}$$

d)  $B$  to  $C$  is +3 across, -4 up, so  $A$  to  $D$  is the same

$$D = (-1 + 3, 2 - 4)$$

$$\boxed{D(2, -2)}$$

8

The point  $A$  has coordinates  $(2, -1)$ .

The point  $B$  lies on the line with equation  $y = 2x + 1$  and the distance  $AB$  is  $2\sqrt{5}$

Find the possible coordinates of  $B$ .

(5 marks)

Any point on the line is  $(t, 2t + 1)$

Square the distance:  $AB^2 = (2\sqrt{5})^2 = 20$

$$(t - 2)^2 + (2t + 1 - (-1))^2 = 20$$

$$(t - 2)^2 + (2t + 2)^2 = 20$$

$$t^2 - 4t + 4 + 4t^2 + 8t + 4 = 20$$

$$5t^2 + 4t + 8 = 20$$

$$5t^2 + 4t - 12 = 0$$

Factorise

$$(5t - 6)(t + 2) = 0$$

$$t = \frac{6}{5} \text{ or } t = -2$$

Find  $y = 2t + 1$  for each

$$t = -2: y = -4 + 1 = -3$$

$$t = \frac{6}{5}: y = \frac{12}{5} + 1 = \frac{17}{5}$$

$$\boxed{(-2, -3) \text{ or } \left(\frac{6}{5}, \frac{17}{5}\right)}$$

9

The line  $l_1$  has equation  $4x + 3y = 24$  and crosses the  $x$ -axis at  $A$  and the  $y$ -axis at  $B$ .

The line  $l_2$  is parallel to  $l_1$  and passes through the point  $C(9, 8)$ .

a. Find an equation of  $l_2$  in the form  $ax + by = c$

The line  $l_2$  crosses the  $x$ -axis at  $D$ .

b. Find the coordinates of  $D$ .

c. Explain why  $ABCD$  is a parallelogram and find its area.

(6 marks)

a) Parallel lines: same  $4x + 3y$ , different constant

$$4x + 3y = c$$

Substitute  $C(9, 8)$

$$c = 4(9) + 3(8) = 60$$

$$4x + 3y = 60$$

b) Put  $y = 0$

$$4x = 60$$

$$x = 15$$

$$D(15, 0)$$

c) Find  $A$  and  $B$  from  $l_1$

$$y = 0: 4x = 24, x = 6, \text{ so } A(6, 0)$$

$$x = 0: 3y = 24, y = 8, \text{ so } B(0, 8)$$

$AB$  is on  $l_1$  and  $DC$  is on  $l_2$ , so  $AB \parallel DC$

$B$  and  $C$  have  $y = 8$ ,  $A$  and  $D$  have  $y = 0$

so  $BC \parallel AD$  (both horizontal)

Two pairs of parallel sides: a parallelogram

Area = base  $\times$  perpendicular height

$$AD = 15 - 6 = 9$$

$$\text{Area} = 9 \times 8$$

$$\text{Area} = 72$$

10

A household's monthly gas bill, £ $C$ , is modelled by  $C = a + bE$ , where  $E$  is the number of kWh of gas used that month and  $a$  and  $b$  are constants.

In March the household used 820 kWh and the bill was £162.40. In June the household used 300 kWh and the bill was £68.80

- Find the values of  $a$  and  $b$ .
- Interpret the value of  $b$  in the context of the model.
- Use the model to estimate the bill for a month in which 1500 kWh are used, and comment on the reliability of your estimate.

(6 marks)

a)  $b$  is the gradient of the line through (300, 68.80) and (820, 162.40)

$$b = \frac{162.40 - 68.80}{820 - 300}$$

$$b = \frac{93.60}{520}$$

$$b = 0.18$$

Substitute the June values

$$68.80 = a + 0.18 \times 300$$

$$68.80 = a + 54$$

$$a = 14.80$$

$$a = 14.80, b = 0.18$$

b) Gradient = cost per kWh

Each extra kWh used adds £0.18 to the bill

c) Substitute  $E = 1500$

$$C = 14.80 + 0.18 \times 1500$$

$$C = 14.80 + 270$$

$$£284.80$$

1500 kWh is outside 300 to 820 kWh,

so this is extrapolation: may be unreliable

11

The triangle  $ABC$  has vertices  $A(-4, 1)$ ,  $B(6, -4)$  and  $C(2, 8)$ .

a. Find an equation of the line through  $C$  that is perpendicular to  $AB$ .

This line meets  $AB$  at the point  $N$ .

b. Find the coordinates of  $N$ .

c. Hence find the area of triangle  $ABC$ . You must show all your working.

(8 marks)

a) Gradient of  $AB$

$$m(AB) = \frac{-4 - 1}{6 - (-4)} = \frac{-5}{10} = -\frac{1}{2}$$

Perpendicular gradient = 2

$$y - 8 = 2(x - 2)$$

$$y - 8 = 2x - 4$$

$$\boxed{y = 2x + 4}$$

b) Equation of  $AB$ , through  $A(-4, 1)$

$$y - 1 = -\frac{1}{2}(x + 4)$$

$$y = -\frac{1}{2}x - 1$$

Solve simultaneously with  $y = 2x + 4$

$$2x + 4 = -\frac{1}{2}x - 1$$

$$\frac{5}{2}x = -5$$

$$x = -2$$

$$y = 2(-2) + 4 = 0$$

$$\boxed{N(-2, 0)}$$

c)  $AB$  is the base,  $CN$  is the perpendicular height

$$AB = \sqrt{10^2 + 5^2} = \sqrt{125} = 5\sqrt{5}$$

$$CN = \sqrt{4^2 + 8^2} = \sqrt{80} = 4\sqrt{5}$$

$$\text{Area} = \frac{1}{2} \times 5\sqrt{5} \times 4\sqrt{5}$$

$$\text{Area} = \frac{1}{2} \times 20 \times 5$$

$$\boxed{\text{Area} = 50}$$

Two candles,  $A$  and  $B$ , are lit at the same time. Their heights are modelled as linear functions of the time  $t$  hours after lighting.

- Candle  $A$  is 30 cm tall when lit and 22 cm tall 4 hours later.
- Candle  $B$  is 24 cm tall when lit and burns down completely in 16 hours.

- Find, for each candle, an equation for its height  $h$  cm in terms of  $t$ .
- Find the time at which the two candles are the same height, and that height.
- Find the interval of time for which candle  $A$  is still burning but is shorter than candle  $B$ .
- Give one limitation of the model.

(8 marks)

a) Rate of burning = gradient (negative)

$$A: \text{gradient} = \frac{22 - 30}{4} = -2$$

$$A: h = 30 - 2t$$

$$B: \text{gradient} = \frac{0 - 24}{16} = -1.5$$

$$B: h = 24 - 1.5t$$

b) Set the heights equal

$$\begin{array}{rclcl} 30 - 2t & = & 24 - 1.5t & & \\ +2t & & & +2t & \\ 30 & = & 24 + 0.5t & & \\ -24 & & & -24 & \\ 6 & = & 0.5t & & \\ \div 0.5 & & & \div 0.5 & \\ t & = & 12 & & \\ h & = & 30 - 2(12) = 6 & & \end{array}$$

$$t = 12 \text{ hours, height } 6 \text{ cm}$$

c)  $A$  falls faster, so after  $t = 12$  it is shorter

$A$  burns out when  $h = 0$

$$30 - 2t = 0$$

$$t = 15$$

$$12 < t < 15$$

d) Limitation

Real candles may not burn at a constant rate  
(draughts, the wick, a candle that narrows)