

Algebraic methods and partial fractions

A-level Pure Mathematics (Year 2) | Edexcel 9MA0 | 12 questions, 72 marks

A-level Maths topic practice



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1 Simplify fully

a. $\frac{3x^2 - 12}{x^2 + x - 6}$

b. $\frac{x^2 + 5x}{x^2 - 1} \div x^2 \frac{+ 3x - 10}{x^2 - x}$

(5 marks)

Answer _____

2 Express $\frac{2x^2 + 6x - 2}{x(x - 1)(x + 2)}$ in partial fractions.

(4 marks)

Answer _____

3 Given that

$$2x^3 + \frac{3x^2 - 5x + 4}{x + 2} \equiv Ax^2 + Bx + C + \frac{D}{x + 2}, \quad x \neq -2$$

find the values of the constants A , B , C and D .

(4 marks)

Answer _____

4

$$f(x) = 2 - \frac{4}{x+1} - \frac{16}{x^2 - 2x - 3}, \quad x > 3$$

a. Show that $f(x) = \frac{2x-10}{x-3}$

b. Hence solve the equation $f(x) = x - 6$

(7 marks)

Answer _____

5

Express $\frac{3x^2 - x + 8}{(x-3)(x+1)^2}$ in partial fractions.

(4 marks)

Answer _____

6

Express $\frac{2x^2 + 1}{(x-2)(x+1)}$ in the form $A + \frac{B}{x-2} + \frac{C}{x+1}$, where A , B and C are constants to be found.

(4 marks)

Answer _____

7 Given that

$$\frac{x^4 + 2x^3 - 3x^2 + x - 5}{x^2 + x - 1} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 + x - 1}$$

find the values of the constants a , b , c , d and e .

(4 marks)

Answer _____

8

a. Express $\frac{4x + 3}{(2x - 1)(x + 2)}$ in partial fractions.

b. Hence find the exact value of $\int_1^3 \frac{4x + 3}{(2x - 1)(x + 2)} dx$, giving your answer in the form $\ln k$, where k is a rational number.

(6 marks)

Answer _____

9

$f(x) = \frac{kx + 6}{(x - 1)(x + 2)}$, $x > 1$, where k is a positive constant.

a. Express $f(x)$ in partial fractions in terms of k .

b. Given that $\int_2^{10} f(x) dx = 10 \ln 3$, find the value of k .

(7 marks)

Answer _____

10 The function f is defined by

$$f(x) = \frac{3x^2 + 13x - 8}{(x - 1)(x + 3)}, \quad x > 1$$

- Express $f(x)$ in the form $A + \frac{B}{x - 1} + \frac{C}{x + 3}$, where A , B and C are constants.
- Hence find $f'(x)$ and explain why f is a decreasing function.
- State the range of f .

(9 marks)

Answer

11

$$f(x) = \frac{18x^2 - 9x + 8}{(3x - 1)^2(x + 2)}, \quad x > \frac{1}{3}$$

Given that $f(x)$ can be written in the form

$$\frac{A}{3x - 1} + \frac{B}{(3x - 1)^2} + \frac{C}{x + 2}$$

where A , B and C are constants,

- a. (i) find the value of B and the value of C ,
(ii) show that $A = 0$

- b. Hence find the exact value of $\int_1^2 f(x) \, dx$, giving your answer in the form $p + q \ln r$,
where p , q and r are rational.

(8 marks)

Answer _____

12

Rainwater flows into a garden water butt. The rate of flow, R litres per hour, t hours after the rain starts is modelled by

$$R = \frac{30t}{(t+1)(t+6)}, \quad 0 \leq t \leq 10$$

- Express $\frac{30t}{(t+1)(t+6)}$ in partial fractions.
- Hence find, according to the model, the exact volume of water that flows into the water butt in the first 4 hours. Give your answer in the form $a \ln 5 - b \ln 3$, where a and b are integers.
- Using your answer to part (a), find the greatest rate of flow according to the model, giving your answer to 3 significant figures.

(10 marks)

Answer _____

End of questions | Total 72 marks