

# Binomial expansion for any rational $n$

A-level Pure Mathematics (Year 2) | Edexcel 9MA0 | 12 questions, 76 marks

**A-level Maths topic practice**



Scan for the solutions

**Name:** \_\_\_\_\_ **Date:** \_\_\_\_\_ **Score:** \_\_\_\_\_ / 76

**1**

- a. Find the first four terms, in ascending powers of  $x$ , of the binomial expansion of  $(1 + 3x)^{-2}$ , giving each term in its simplest form.
- b. State the range of values of  $x$  for which the expansion is valid.

(4 marks)

Answer \_\_\_\_\_

**2**

In the binomial expansion of  $(1 + kx)^{-3}$ , where  $k$  is a negative constant, the coefficient of  $x^2$  is 54

- a. Find the value of  $k$ .
- b. Find the coefficient of  $x^3$  in the expansion.

(4 marks)

Answer \_\_\_\_\_

**3**

- a. Find the first four terms, in ascending powers of  $x$ , of the binomial expansion of  $\frac{1}{\sqrt{4+x}}$ , giving each coefficient as a simplified fraction.
- b. State the range of values of  $x$  for which the expansion is valid.

(5 marks)

Answer \_\_\_\_\_

**4**

- a. Find the first four terms, in ascending powers of  $x$ , of the binomial expansion of  $\sqrt{1+8x}$ , giving each term in its simplest form.
- b. Explain why substituting  $x = \frac{1}{8}$  into this expansion would not give a valid approximation for  $\sqrt{2}$
- c. By substituting  $x = \frac{1}{32}$  into your expansion, find an approximation for  $\sqrt{5}$ , giving your answer to 4 decimal places.

(7 marks)

Answer \_\_\_\_\_

5

- a. Show that the binomial expansion of  $(8 - 3x)^{\frac{1}{3}}$ , in ascending powers of  $x$  up to and including the term in  $x^2$ , is

$$2 - \frac{1}{4}x - \frac{1}{32}x^2$$

- b. State the range of values of  $x$  for which the expansion is valid.  
 c. Use the expansion with a suitable value of  $x$  to find an estimate for  $\sqrt[3]{7.97}$ , giving your answer to 6 decimal places.

(6 marks)

Answer \_\_\_\_\_

6

- The binomial expansion of  $(a + bx)^{-2}$ , where  $a$  and  $b$  are positive constants, in ascending powers of  $x$ , begins

$$\frac{1}{9} - \frac{4}{27}x + cx^2 + \dots$$

- a. Find the values of  $a$  and  $b$ .  
 b. Find the value of  $c$ .  
 c. State the range of values of  $x$  for which the expansion is valid.

(6 marks)

Answer \_\_\_\_\_

7

- a. Find the binomial expansion of  $(2 + x)^{-2}$ , in ascending powers of  $x$  up to and including the term in  $x^2$ , giving each coefficient as a simplified fraction.
- b. Hence find the first three terms, in ascending powers of  $x$ , of the expansion of  $\frac{3 - x}{(2 + x)^2}$
- c. State the range of values of  $x$  for which this expansion is valid.

(6 marks)

Answer \_\_\_\_\_

8

- a. Use binomial expansions to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2$$

- b. By substituting  $x = \frac{1}{9}$  into both sides, find an approximation for  $\sqrt{5}$ , giving your answer as a fraction in its simplest form.
- c. Find the percentage error in this approximation, giving your answer to 2 significant figures.

(8 marks)

Answer \_\_\_\_\_

9

The binomial expansion of  $(1 + ax)^n$ , in ascending powers of  $x$ , begins

$$1 - 6x + 24x^2 + \dots$$

where  $a$  and  $n$  are constants and  $n$  is not a positive integer.

- Find the value of  $n$  and the value of  $a$ .
- Find the coefficient of  $x^3$ .
- State the range of values of  $x$  for which the expansion is valid.

(6 marks)

Answer \_\_\_\_\_

10

$$f(x) = \frac{4 + 5x}{(1 + 2x)(1 - x)}$$

- Express  $f(x)$  in partial fractions.
- Hence find the series expansion of  $f(x)$ , in ascending powers of  $x$  up to and including the term in  $x^3$ , giving each coefficient in its simplest form.
- State the range of values of  $x$  for which the expansion is valid, giving a reason.

(9 marks)

Answer \_\_\_\_\_

11

- a. Find the binomial expansion of  $\frac{1}{\sqrt{1+x^2}}$ , in ascending powers of  $x$  up to and including the term in  $x^4$
- b. Hence find an approximation for

$$\int_0^{0.2} \frac{1}{\sqrt{1+x^2}} dx$$

giving your answer to 6 significant figures.

- c. Explain why this approximation is likely to be very accurate.

(7 marks)

Answer \_\_\_\_\_

12

In physics, the total energy  $E$  of a particle of mass  $m$  moving at speed  $v$  is

$$E = mc^2 \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

where  $c$  is the speed of light and  $0 \leq v < c$ . The kinetic energy is  $K = E - mc^2$

a. Use a binomial expansion to show that, when  $v$  is small compared with  $c$ ,

$$K \approx \frac{1}{2}mv^2 + \frac{3}{8} \frac{mv^4}{c^2}$$

b. Explain why the expansion is valid for all speeds allowed in the model.

c. A particle moves at  $v = 0.1c$ . Find the percentage error when its kinetic energy is approximated by  $\frac{1}{2}mv^2$ , giving your answer to 2 significant figures.

(8 marks)

Answer \_\_\_\_\_

End of questions | Total 76 marks