



Name: _____ **Date:** _____ **Score:** _____ / 81

- 1** Find the general solution of the differential equation

$$\frac{dy}{dx} = (2x + 1)y, \quad y > 0$$

giving your answer in the form $y = f(x)$.

(3 marks)

Answer _____

- 2** Given that $y = 2$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{e^{2x}}{y}, \quad y > 0$$

giving your answer in the form $y = f(x)$.

(5 marks)

Answer _____

3

A scientist grows bacteria in a dish. The number of bacteria, N , t hours after the start of the experiment increases at a rate proportional to N . Initially there are 500 bacteria and after 3 hours there are 2000 bacteria.

- By forming and solving a differential equation, show that $N = 500e^{kt}$, where k is a positive constant.
- Find the exact value of k .
- Find, according to the model, the time taken for the number of bacteria to reach 10 000. Give your answer in hours to 3 significant figures.

(5 marks)

Answer _____

4

The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = y \tan x, \quad 0 \leq x < \frac{\pi}{2}, \quad y > 0$$

Given that $y = 3$ when $x = 0$,

- show that $y = 3 \sec x$
- find the exact value of x for which $y = 6$

(5 marks)

Answer _____

- 5 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = x e^{-y}$$

and $y = 0$ when $x = 2$

- Solve the differential equation, giving your answer in the form $y = \ln f(x)$
- State the set of values of x , with $x > 0$, for which your solution is defined.

(5 marks)

Answer _____

- 6 A young tree is planted in a park. Its height, h metres, t years after planting is modelled by the differential equation

$$\frac{dh}{dt} = \frac{6-h}{t+1}, \quad t \geq 0, \quad 1 \leq h < 6$$

The tree is 1 metre tall when it is planted.

- Show that $h = 6 - \frac{5}{t+1}$
- Find, according to the model, the height of the tree 4 years after planting.
- State the height that the model predicts the tree approaches in the long term.

(6 marks)

Answer _____

7

A block of ice is left to melt. Its mass, m kg, t hours after it was put out is modelled as decreasing at a rate proportional to the square root of m . The initial mass is 100 kg and after 5 hours the mass is 64 kg.

- Write down a differential equation linking m and t .
- Solve your differential equation to show that $m = (10 - 0.4t)^2$
- Find, according to the model, the time taken for the ice to melt completely.

(7 marks)

Answer _____

8

A mug of tea is made in a kitchen where the room temperature is a constant 20°C . The temperature of the tea, $\theta^{\circ}\text{C}$, t minutes after it is made is modelled by

$$\frac{d\theta}{dt} = -k(\theta - 20)$$

where k is a positive constant. The tea is made at 90°C and after 10 minutes its temperature is 60°C .

- Show that $\theta = 20 + 70e^{-kt}$
- Show that $k = \frac{1}{10} \ln \frac{7}{4}$
- Find, according to the model, the time taken for the tea to cool to 30°C . Give your answer to the nearest minute.
- State what happens to the temperature of the tea in the long term, according to the model.

(8 marks)

Answer

9

Water is pumped into a garden pond at a constant rate of 0.6 m^3 per hour. At the same time water leaves the pond through an overflow pipe at a rate of $0.2V \text{ m}^3$ per hour, where $V \text{ m}^3$ is the volume of water in the pond t hours after the pump is switched on.

- Show that $5 \frac{dV}{dt} = 3 - V$
- Given that $V = 1$ when $t = 0$, show that $V = 3 - 2e^{-0.2t}$
- Find, according to the model, the time taken for the volume to reach 2.5 m^3 . Give your answer in hours to 3 significant figures.
- Explain why, according to the model, the volume of water in the pond can never reach 3 m^3 .

(8 marks)

Answer

10

After a storm a puddle on a flat playground is modelled as a circle. At time t hours the radius of the puddle is r cm and its area is A cm². In the model the area decreases at a rate inversely proportional to r .

- Show that $\frac{dr}{dt} = -\frac{c}{r^2}$, where c is a positive constant.
- Given that the radius is 30 cm at $t = 0$ and 20 cm at $t = 1$, show that $r^3 = 27\,000 - 19\,000t$
- Find, according to the model, the time taken for the puddle to dry up completely. Give your answer to the nearest minute.

(9 marks)

Answer

11

The number of fish, P hundred, in a lake t years after a group of fish was released into it is modelled by the differential equation

$$\frac{dP}{dt} = \frac{P(10 - P)}{20}, \quad 0 < P < 10$$

The group released contained 200 fish.

a. Express $\frac{1}{P(10 - P)}$ in partial fractions.

b. Show that $P = \frac{10}{1 + 4e^{-0.5t}}$

c. Find, according to the model, the time taken for the number of fish to reach 800. Give your answer in the form $a \ln 2$ years, where a is an integer.

d. State the maximum number of fish the lake can support, according to the model.

(10 marks)

Answer

12

A gardener measures a seedling. Its height, h cm, t weeks after the first measurement is modelled by the differential equation

$$\frac{dh}{dt} = t e^{-0.5t} \sqrt{h}, \quad t \geq 0$$

When $t = 0$ the seedling is 4 cm tall.

- Show that $\sqrt{h} = 4 - (t + 2)e^{-0.5t}$
- Find, according to the model, the height of the seedling 4 weeks after the first measurement. Give your answer to 3 significant figures.
- Find the height that the model predicts the seedling approaches in the long term, and give one reason why the model may not be realistic for a real plant.

(10 marks)

Answer

End of questions | Total 81 marks