

# Integration: further methods

A-level Pure Mathematics (Year 2) | Edexcel 9MA0 | 12 questions, 67 marks

A-level Maths topic practice



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Name: \_\_\_\_\_

Date: \_\_\_\_\_

Score: \_\_\_\_\_ / 67

1 Find

a.  $\int (6e^{3x} + \frac{5}{x}) dx, \quad x > 0$

b.  $\int (3 \sin 2x + 4 \sec^2 x) dx$

c.  $\int \sqrt{4x + 1} dx$

(5 marks)

Answer \_\_\_\_\_

2 Find the exact value of  $\int_1^3 \frac{4x}{x^2 + 3} dx$ , giving your answer in the form  $\ln k$ , where  $k$  is an integer.

(3 marks)

Answer \_\_\_\_\_

3 Using a suitable trigonometric identity, find the exact value of

$\int_0^{\pi/3} 4 \sin^2 x dx$

(4 marks)

Answer \_\_\_\_\_

4 Find  $\int x \cos 3x \, dx$

(4 marks)

Answer \_\_\_\_\_

5 Given that  $k$  is a positive constant, show that

$$\int_k^{2k} \frac{x+k}{x^2} \, dx$$

is independent of  $k$ , and state its exact value.

(4 marks)

Answer \_\_\_\_\_

6 Find the exact value of

$$\int_1^e x^2 \ln x \, dx$$

giving your answer in the form  $ae^3 + b$ , where  $a$  and  $b$  are rational.

(5 marks)

Answer \_\_\_\_\_

7

- a. Use the substitution  $u^2 = 2x - 1$  to show that  $\int_1^5 \frac{x}{\sqrt{2x-1}} dx$  can be written as  $\int_1^3 \frac{u^2 + 1}{2} du$

- b. Hence find the exact value of  $\int_1^5 \frac{x}{\sqrt{2x-1}} dx$

(6 marks)

Answer \_\_\_\_\_

8

$$f(x) = \frac{7x + 5}{(x + 2)(2x + 1)}, \quad x \geq 0$$

- a. Express  $f(x)$  in partial fractions.
- b. Hence show that  $\int_0^4 f(x) dx$  equals  $\ln k$ , stating the value of the integer  $k$ .

(6 marks)

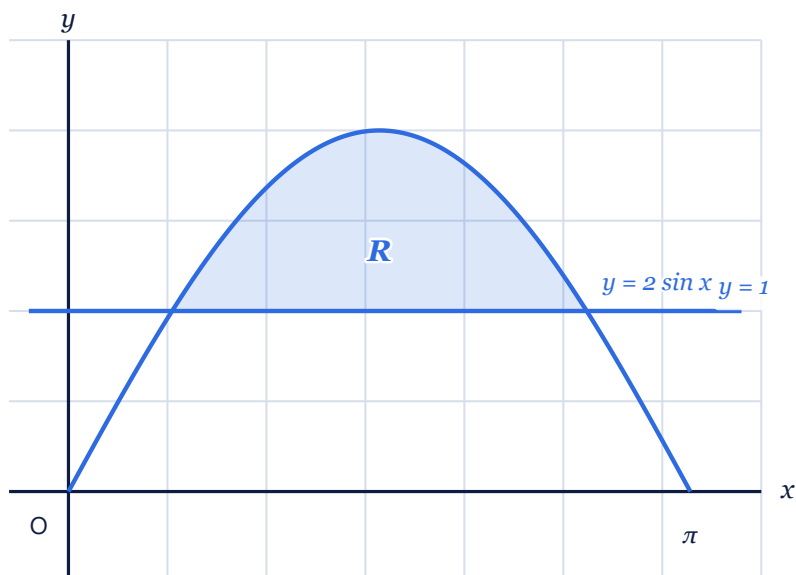
Answer \_\_\_\_\_

9

The diagram shows part of the curve with equation  $y = 2 \sin x$ ,  $0 \leq x \leq \pi$ , and the line with equation  $y = 1$

The region  $R$ , shown shaded, is bounded by the curve and the line.

- Find the exact  $x$  coordinates of the points where the line meets the curve.
- Find the exact area of  $R$ , giving your answer in the form  $a\sqrt{3} - b\pi$ , where  $a$  and  $b$  are rational.



(6 marks)

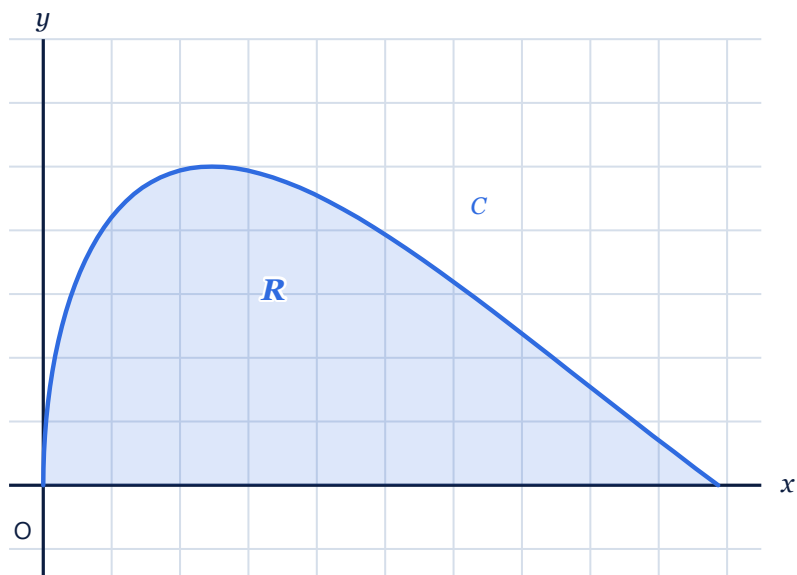
Answer \_\_\_\_\_

10 The curve  $C$  has parametric equations

$$x = t^2, \quad y = \sin t, \quad 0 \leq t \leq \pi$$

The region  $R$ , shown shaded in the diagram, is bounded by  $C$  and the  $x$ -axis.

- Show that the area of  $R$  is given by  $\int_0^\pi 2t \sin t \, dt$
- Hence find the exact area of  $R$ .



(7 marks)

Answer \_\_\_\_\_

11

The gates at a summer music festival open at 10 am. The rate at which people arrive,  $R$  hundred people per hour,  $t$  hours after the gates open is modelled by

$$R = 20t e^{-0.5t}, \quad 0 \leq t \leq 4$$

- Using differentiation, find, according to the model, the greatest rate of arrival, giving your answer in hundreds of people per hour to 3 significant figures.
- Show that, according to the model, the number of people who arrive in the first 4 hours is  $100(80 - 240e^{-2})$
- Hence find the number of people, to the nearest 10, who arrive in the first 4 hours.

(8 marks)

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Answer

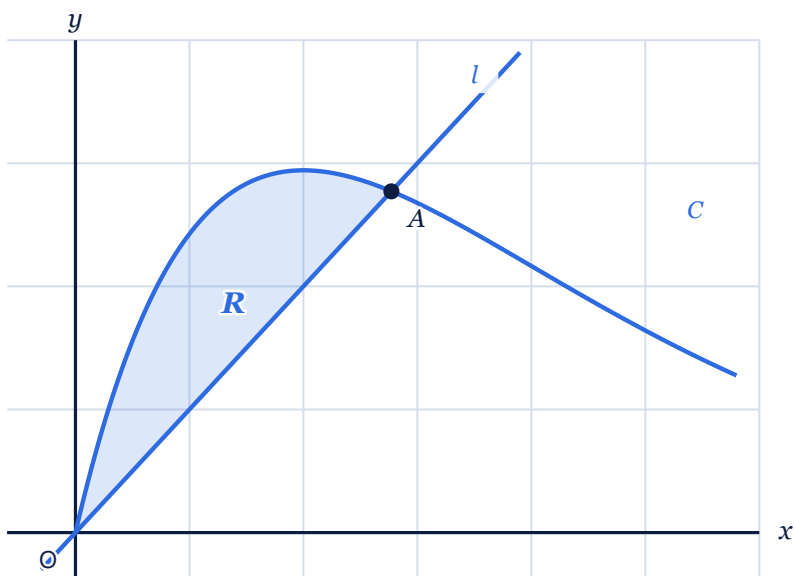
12

The diagram shows part of the curve  $C$  with equation  $y = 4xe^{-x}$ ,  $x \geq 0$ , and the line  $l$  with equation  $y = x$

$C$  and  $l$  meet at the origin  $O$  and at the point  $A$ .

The finite region  $R$ , shown shaded, is bounded by  $C$  and  $l$ .

- Find the exact  $x$  coordinate of  $A$ .
- Show that the exact area of  $R$  is  $3 - 2 \ln 2 - 2(\ln 2)^2$



(9 marks)

Answer \_\_\_\_\_

End of questions | Total 67 marks