



Q	Answer	Marks
1	$f(1.5) = -0.625$, f continuous, so a root lies in $[1.5, 1.6]$; $f(1.565) = -0.0370$, so $1.565 < \alpha < 1.575$ and $\alpha = 1.57$ to 2 dp	4
2	a) $6x = x^3 + 2$ so $x = \sqrt[3]{\frac{x^3 + 2}{6}}$ b) $x_2 = 0.3542, x_3 = 0.3407, x_4 = 0.3399$ c) $\alpha = 0.340$	5
3	$x_2 = 1.8276, x_3 = 1.7961$	4
4	a) $y = 0.7788$ b) 0.743	4
5	a) $\frac{dy}{dx} = 0.5e^{0.5x} - 2x = 0$ so $x = 0.25e^{0.5x}$ b) $x_2 = 0.2905, x_3 = 0.2891, x_4 = 0.2889$ c) $g(x) = 0.5e^{0.5x} - 2x$: $g(0.28875) > 0, g(0.28885) < 0$, so $\alpha = 0.2888$	8
6	a) $f(1.9) = -28.1, f(2.1) = 32.1$ b) f is not continuous: there is an asymptote at $x = 2$, so the sign change is not caused by a root c) $g(1) = 3$ and $g(3) = 3$, both positive d) $g(2) = -1 < 0$: sign changes on $[1, 2]$ and on $[2, 3]$, g continuous, so at least two roots	6
7	a) $f(3) = -0.0986$, so $3 < \alpha < 3.5$ b) $x_2 = 2.4055, x_3 = 2.8777$ c) staircase (see diagram) climbing up towards α d) $x_2 = 2.718, x_3 = 2.051, x_4 = 1.052$: the values move away from α (diverge from it)	9
8	a) $x_{n+1} = x_n - \frac{(x_n^3 - 3x_n^2 + 5)}{(3x_n^2 - 6x_n)} = \frac{(2x_n^3 - 3x_n^2 - 5)}{(3x_n^2 - 6x_n)}$ b) $x_2 = -1.1111, x_3 = -1.1038$ c) $f'(2) = 0$: the tangent is horizontal and never meets the x -axis (division by zero)	6
9	a) 1.0607 and 2.0917 b) 3.28 c) overestimate: the curve bends upwards (convex), so each trapezium top lies above the curve d) use more strips of smaller width	7
10	a) 6.15 b) 10.45 c) 6.15 : the curve is translated 1 unit left and the limits move with it	6
11	a) $x \ln x = 0, x > 0$, so $\ln x = 0$ and $x = 1$ b) 2.9666 c) $\frac{9}{2} \ln 3 - 2$ d) 0.776% ; $y'' = 1/x > 0$, convex, so trapezia lie above the curve	10
12	a) 5.89 mg per litre (at $t = 2$) b) $8te^{-0.5t} = 4$ gives $2te^{-0.5t} - 1 = 0$; $h(4) = 0.0827, h(5) = -0.179$ c) $T = 4.307$	10

d) 3.59 hours	
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Full worked solutions, written out by hand, are on the solutions sheet for this topic.