



Name: _____ Date: _____ Score: _____ / 79

1 $f(x) = x^3 + 2x - 7$

- Show that the equation $f(x) = 0$ has a root α in the interval $[1.5, 1.6]$
- By choosing a suitable interval, show that $\alpha = 1.57$ correct to 2 decimal places. (4 marks)

Answer _____

2 The equation $x^3 - 6x + 2 = 0$ has a root α between 0 and 1

- Show that the equation can be written in the form $x = \frac{x^3 + 2}{6}$
- Use the iteration formula $x_{n+1} = \frac{x_n^3 + 2}{6}$ with $x_1 = 0.5$ to find the values of x_2 , x_3 and x_4 , giving each answer to 4 decimal places.
- Write down the value of α correct to 3 decimal places. (5 marks)

Answer _____

3 The equation $x^4 - 3x - 5 = 0$ has a root β close to $x = 2$

- Taking $x_1 = 2$ as a first approximation, apply the Newton-Raphson method twice to find x_2 and x_3 . Give each answer to 4 decimal places. (4 marks)

Answer _____

4

The table shows values of $y = e^{-x^2}$ for $0 \leq x \leq 1$, with values of y given to 4 decimal places.

x	0	0.25	0.5	0.75	1
y	1	0.9394		0.5698	0.3679

- Complete the table, giving the missing value of y to 4 decimal places.
- Use the trapezium rule with all the values of y in the completed table to find an estimate for $\int_0^1 e^{-x^2} dx$, giving your answer to 3 significant figures.

(4 marks)

Answer _____

5

The curve C has equation $y = e^{0.5x} - x^2$

C has a turning point P whose x coordinate α lies between 0 and 1

- Show that α is a solution of the equation $x = 0.25e^{0.5x}$
- Using the iteration formula $x_{n+1} = 0.25e^{0.5x_n}$ with $x_1 = 0.3$, find the values of x_2 , x_3 and x_4 , giving each answer to 4 decimal places.
- Using a suitable interval and a suitable function that should be stated, show that $\alpha = 0.2888$ to 4 decimal places.

(8 marks)

Answer _____

6

(i) $f(x) = \frac{3}{x-2} + x, \quad x \neq 2$

A student finds $f(1.9)$ and $f(2.1)$ and says that, because the signs are different, the equation $f(x) = 0$ has a root between 1.9 and 2.1

- a. Find $f(1.9)$ and $f(2.1)$
- b. Explain why the student's conclusion is not correct.

(ii) $g(x) = 4x^2 - 16x + 15$

- c. Show that $g(1)$ and $g(3)$ have the same sign.
- d. By considering $g(2)$, explain why the equation $g(x) = 0$ has at least two roots between 1 and 3, even though $g(1)$ and $g(3)$ have the same sign.

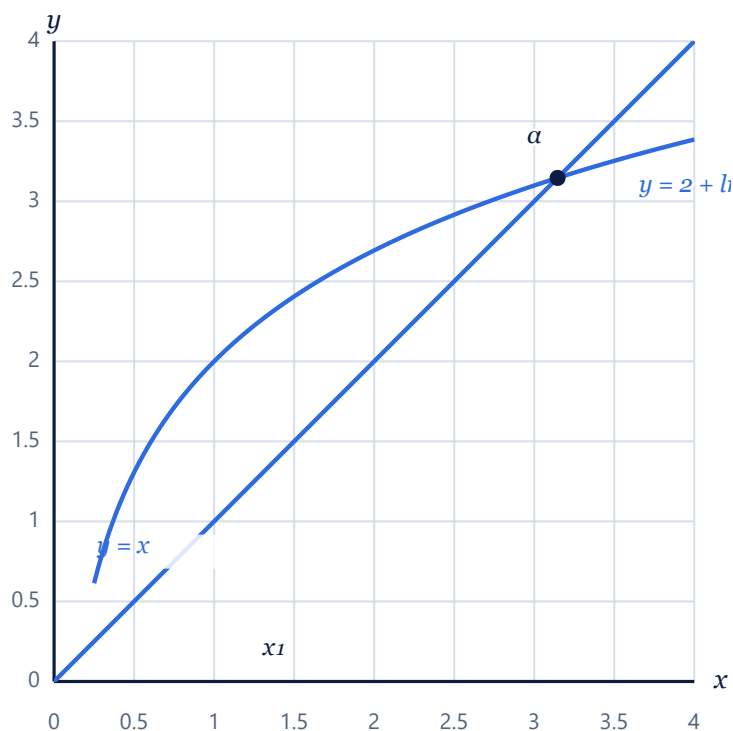
(6 marks)

Answer _____

7

The diagram shows part of the curve with equation $y = 2 + \ln x$ and the line $y = x$. They meet at the point where $x = \alpha$

- Show that α lies between 3 and 3.5
- Use the iteration formula $x_{n+1} = 2 + \ln x_n$ with $x_1 = 1.5$ to find x_2 and x_3 , giving each answer to 4 decimal places.
- On the diagram, draw a staircase diagram to show how the iteration in part (b) moves towards α . Mark x_2 and x_3 on the x -axis.
- A different rearrangement gives the iteration formula $x_{n+1} = e^{x_n - 2}$. Using $x_1 = 3$, find x_2 , x_3 and x_4 to 3 decimal places, and explain why this formula cannot be used to find α .



(9 marks)

Answer

8

$$f(x) = x^3 - 3x^2 + 5$$

The equation $f(x) = 0$ has exactly one real root γ

a. Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{2x_n^3 - 3x_n^2 - 5}{3x_n^2 - 6x_n}$$

b. Using this formula with $x_1 = -1$, find x_2 and x_3 , giving each answer to 4 decimal places.

c. Explain why the Newton-Raphson method cannot be used with $x_1 = 2$

(6 marks)

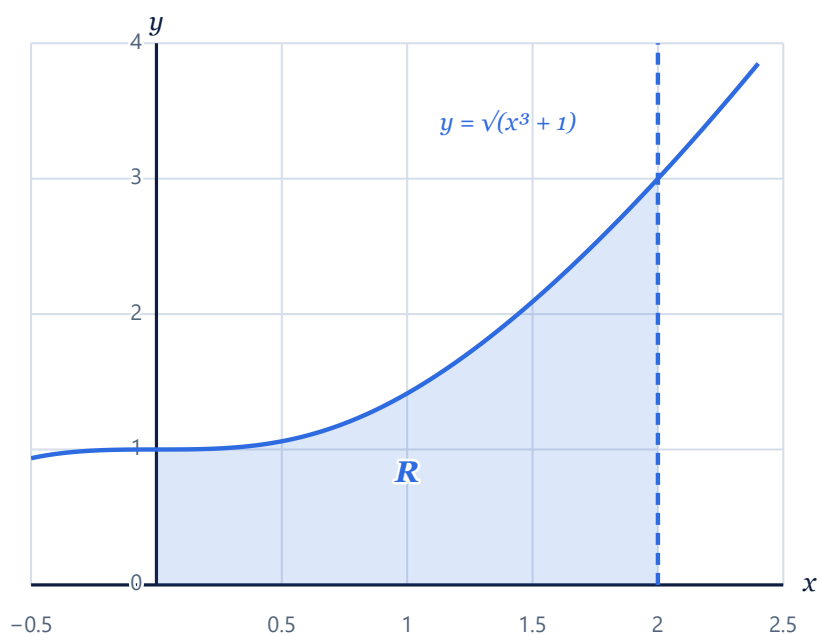
Answer

9

The diagram shows part of the curve with equation $y = \sqrt{x^3 + 1}$. The region R , shown shaded, is bounded by the curve, the x -axis, the y -axis and the line $x = 2$

x	0	0.5	1	1.5	2
y	1		1.4142		3

- Complete the table, giving the values of y to 4 decimal places.
- Use the trapezium rule with all the values in the table to find an estimate for the area of R , giving your answer to 3 significant figures.
- Using the diagram, state with a reason whether your answer to part (b) is an overestimate or an underestimate.
- Explain how the trapezium rule could be used to give a more accurate estimate.



(7 marks)

Answer _____

10

The table shows some values of x and y for a continuous curve with equation $y = f(x)$

x	1	1.5	2	2.5	3
y	2.4	3.1	3.5	3.2	2.6

- a. Use the trapezium rule with all the values in the table to find an estimate for $\int_1^3 f(x) \, dx$
- b. Using your answer to part (a), find an estimate for $\int_1^3 (3f(x) - 2x) \, dx$
- c. Using your answer to part (a), write down an estimate for $\int_0^2 f(x + 1) \, dx$, giving a reason.

(6 marks)

Answer _____

11

The region R is bounded by the curve with equation $y = x \ln x$, the x -axis and the line $x = 3$

- Show that the curve meets the x -axis at $x = 1$ for $x > 0$
- Use the trapezium rule with four strips of equal width to find an estimate for the area of R , giving your answer to 4 decimal places.
- Use integration by parts to find the exact area of R , giving your answer in the form $a \ln 3 + b$, where a and b are rational.
- Find the percentage error in the estimate from part (b), and explain, by considering $\frac{d^2y}{dx^2}$, why the trapezium rule gives an overestimate here.

(10 marks)

Answer

12

The concentration of a medicine in a patient's blood, C mg per litre, t hours after it is taken, is modelled by

$$C = 8te^{-0.5t}, \quad t \geq 0$$

- a. Find the maximum concentration according to the model, giving your answer to 3 significant figures.

The medicine is effective while $C \geq 4$. The concentration first reaches 4 mg per litre when $t = 0.715$ (3 decimal places). It falls back to 4 mg per litre when $t = T$

- b. Show that T is a root of the equation $h(t) = 0$, where $h(t) = 2te^{-0.5t} - 1$, and show that $4 < T < 5$
- c. Taking $t_1 = 4.5$ as a first approximation, apply the Newton-Raphson method twice to find an approximation for T , giving your answer to 3 decimal places.
- d. Hence find, giving your answer in hours to 2 decimal places, how long the medicine is effective for.

(10 marks)

Answer _____

End of questions | Total 79 marks