



Q	Answer	Marks
1	Assuming n^2 is even and n is odd gives $n^2 = 2(2k^2 + 2k) + 1$, which is odd: a contradiction, so n is even.	3
2	Assuming $a + b = p/q$ leads to $b = (pn - mq)/(qn)$, a rational number: a contradiction, so $a + b$ is irrational.	3
3	Assuming $x + 9/x < 6$ gives $(x - 3)^2 < 0$, which is impossible, so $x + 9/x \geq 6$.	3
4	Assuming $\sqrt{2} = a/b$ in lowest terms forces both a and b to be even, which contradicts the fraction being in lowest terms, so $\sqrt{2}$ is irrational.	5
5	Assuming a solution exists, $x - y$ and $x + y$ have the same parity and multiply to 10; they cannot both be odd, and if both are even the product is a multiple of 4, which 10 is not: a contradiction.	4
6	Assuming $\log_2 5 = p/q$ gives $2^p = 5^q$ with p, q positive integers; the left side is even and the right side is odd: a contradiction.	4
7	Expanding gives $1 + 2 \sin x \cos x \leq 1$, so $\sin x \cos x \leq 0$; but $\sin x > 0$ and $\cos x > 0$ for an acute angle: a contradiction, so $\sin x + \cos x > 1$.	3
8	Assuming a finite list $p_1, \dots, p_n$ of all primes, $N = p_1 p_2 \dots p_n + 1$ leaves remainder 1 on division by every prime, so it has a prime factor not in the list (or is prime itself): a contradiction.	5
9	a) shown: if n is not a multiple of 3, n^2 leaves remainder 1 b) shown: $\sqrt{3} = a/b$ in lowest terms forces 3 to divide both a and b	7
10	a) shown: an odd number cubed is odd b) shown: $\sqrt[3]{2} = a/b$ in lowest terms forces a and b both even c) shown: if $5 - \sqrt[3]{2} = r$ were rational, $\sqrt[3]{2} = 5 - r$ would be rational	8

Full worked solutions, written out by hand, are on the solutions sheet for this topic.