

Proof by contradiction

A-level Pure Mathematics (Year 2) | Edexcel 9MA0 | 10 questions, 45 marks

A-level Maths topic practice



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Name: _____ Date: _____ Score: _____ / 45

1 Prove by contradiction that, for any integer n , if n^2 is even then n is even. (3 marks)

Answer _____

2 Prove by contradiction that if a is a rational number and b is an irrational number, then $a + b$ is irrational. (3 marks)

Answer _____

3 Prove by contradiction that for all positive real numbers x ,

$$x + \frac{9}{x} \geq 6$$
 (3 marks)

Answer _____

4 You may assume that, for any integer n , if n^2 is even then n is even.
 Prove by contradiction that $\sqrt{2}$ is irrational. (5 marks)

Answer _____

5 Prove by contradiction that there are no integers x and y such that

$$x^2 - y^2 = 10$$

(4 marks)

Answer _____

6 Prove by contradiction that $\log_2 5$ is irrational.

(4 marks)

Answer _____

7 A student is asked to prove by contradiction that

$\sin x + \cos x > 1$ for all values of x with $0^\circ < x < 90^\circ$

The student begins:

Assume that there is an angle x with $0^\circ < x < 90^\circ$ such that $\sin x + \cos x \leq 1$

Both sides are positive, so $(\sin x + \cos x)^2 \leq 1$

Complete the proof.

(3 marks)

Answer _____

8 Prove by contradiction that there are infinitely many prime numbers.

(5 marks)

Answer _____

9

- a. Prove that, for any integer n , if n^2 is a multiple of 3 then n is a multiple of 3
b. Hence prove by contradiction that $\sqrt{3}$ is irrational.

(7 marks)

Answer _____

10

- a. Prove that, for any integer p , if p^3 is even then p is even.
b. Hence prove by contradiction that $\sqrt[3]{2}$ is irrational.
c. Deduce, using a further proof by contradiction, that $5 - \sqrt[3]{2}$ is irrational.

(8 marks)

Answer _____

End of questions | Total 45 marks