



1

- Write 135° in radians, giving your answer as an exact multiple of π .
- Convert 2.4 radians to degrees, giving your answer to 1 decimal place.
- Find the exact value of $\sin \frac{5\pi}{6}$

(3 marks)

a)

Degrees to radians: $\times \frac{\pi}{180}$

$$135^\circ = 135 \times \frac{\pi}{180}$$

$$\boxed{\frac{3\pi}{4}}$$

b)

Radians to degrees: $\times \frac{180}{\pi}$

$$2.4 \times \frac{180}{\pi} = 137.50\dots$$

$$\boxed{137.5^\circ}$$

c)

$\frac{5\pi}{6} = \pi - \frac{\pi}{6}$, second quadrant: sin is positive

$$\sin \frac{5\pi}{6} = \sin \frac{\pi}{6}$$

$$\boxed{\frac{1}{2}}$$

2

A sector of a circle has radius 8 cm and angle 0.9 radians.

Find

- the length of the arc,
- the perimeter of the sector,
- the area of the sector.

(4 marks)

a)

$$s = r\theta$$

$$= 8 \times 0.9$$

$$\boxed{7.2 \text{ cm}}$$

b)

Two radii and the arc

$$P = 8 + 8 + 7.2$$

$$\boxed{23.2 \text{ cm}}$$

c)

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \times 64 \times 0.9$$

$$\boxed{28.8 \text{ cm}^2}$$

3

Given that θ is small and measured in radians, use the small angle approximations to show that

$$3 \sin 2\theta + \cos 4\theta \approx a + b\theta + c\theta^2$$

where a , b and c are integers to be found.

(3 marks)

$$\sin \theta \approx \theta \text{ and } \cos \theta \approx 1 - \frac{\theta^2}{2}$$

$$3 \sin 2\theta \approx 3(2\theta)$$

$$= 6\theta$$

$$\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$$

$$= 1 - 8\theta^2$$

$$3 \sin 2\theta + \cos 4\theta \approx 1 + 6\theta - 8\theta^2$$

$$\boxed{a = 1, b = 6, c = -8}$$

4

A sector of a circle has radius r cm and angle θ radians. The area of the sector is 20 cm^2 and its perimeter is 18 cm.

Find the two possible pairs of values of r and θ .

(5 marks)

$$\frac{1}{2}r^2\theta = 20$$

$$2r + r\theta = 18$$

$$r\theta = 18 - 2r$$

Write the area as $\frac{1}{2}r(r\theta)$ and substitute

$$\frac{1}{2}r(18 - 2r) = 20$$

$$9r - r^2 = 20$$

$$r^2 - 9r + 20 = 0$$

$$(r - 4)(r - 5) = 0$$

$$\theta = \frac{40}{r^2}$$

$$r = 4: \theta = \frac{40}{16} = 2.5$$

$$r = 5: \theta = \frac{40}{25} = 1.6$$

Both angles are less than 2π

$$r = 4, \theta = 2.5 \text{ or } r = 5, \theta = 1.6$$

5

The diagram shows a circle with centre O and radius 10 cm . The points A and B lie on the circle and angle $AOB = 1.2$ radians. The shaded region is the segment cut off by the chord AB .

- Find the area of the shaded segment.
- Find the perimeter of the shaded segment.

Give each answer to 3 significant figures.

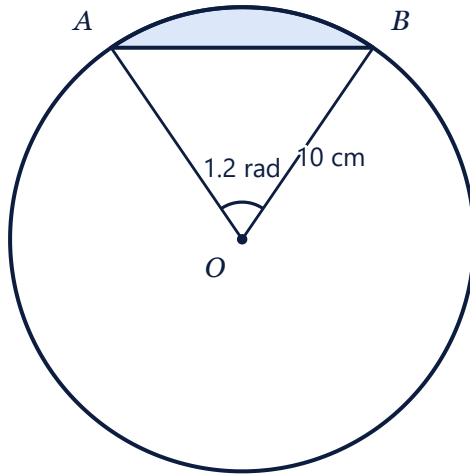


Diagram NOT accurately drawn

(6 marks)

a)

Segment = sector - triangle

$$\begin{aligned}
 A &= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta \\
 &= \frac{1}{2} \times 100 \times (1.2 - \sin 1.2) \\
 &= 50 \times 0.26796... \\
 &= 13.398...
 \end{aligned}$$

$$13.4 \text{ cm}^2$$

b)

$$\begin{aligned}
 \text{arc } AB &= 10 \times 1.2 \\
 &= 12
 \end{aligned}$$

Chord: split the isosceles triangle in half

$$\begin{aligned}
 AB &= 2 \times 10 \sin 0.6 \\
 &= 11.292... \\
 p &= 12 + 11.292...
 \end{aligned}$$

23.3 cm

6

Solve, for $0 \leq x < 2\pi$, the equation

$$2 \cos^2 x + 3 \sin x = 3$$

giving your answers as exact multiples of π .

(5 marks)

Use $\cos^2 x = 1 - \sin^2 x$

$$2(1 - \sin^2 x) + 3 \sin x = 3$$

$$2 - 2 \sin^2 x + 3 \sin x = 3$$

$$2 \sin^2 x - 3 \sin x + 1 = 0$$

$$(2 \sin x - 1)(\sin x - 1) = 0$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6} \text{ or } x = \pi - \frac{\pi}{6}$$

$$\sin x = 1$$

$$x = \frac{\pi}{2}$$

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

7

Solve, for $0 \leq \theta < \pi$, the equation

$$3 \cos(2\theta + 0.5) = 2$$

giving your answers in radians to 3 significant figures.

(5 marks)

$$\cos(2\theta + 0.5) = \frac{2}{3}$$

Let $u = 2\theta + 0.5$ and change the interval

$$0.5 \leq u < 2\pi + 0.5$$

$$0.5 \leq u < 6.783\dots$$

$$u = \cos^{-1}\left(\frac{2}{3}\right)$$

$$= 0.8410\dots$$

Cosine is also positive in the fourth quadrant

$$u = 2\pi - 0.8410\dots$$

$$= 5.4421\dots$$

 $0.8410\dots + 2\pi = 7.124\dots$ is outside the interval

$$\theta = \frac{u - 0.5}{2}$$

$$\theta = \frac{0.8410\dots - 0.5}{2} = 0.1705\dots$$

$$\theta = \frac{5.4421\dots - 0.5}{2} = 2.4710\dots$$

$$\theta = 0.171, 2.47$$

8

The equation $\sin \theta + \cos \theta = 1.1$ has a small positive root, $\theta = \alpha$, where θ is measured in radians.

- Use the small angle approximations to show that α is approximately a root of $\theta^2 - 2\theta + 0.2 = 0$
- Hence find an approximate value for α , giving your answer to 4 decimal places and explaining your choice of root.
- Given that the true value of α is 0.10579 to 5 decimal places, find the percentage error in using $1 - \sqrt{0.8}$ as an approximation for α .

(6 marks)

a)

$$\sin \theta \approx \theta \text{ and } \cos \theta \approx 1 - \frac{\theta^2}{2}$$

$$\theta + 1 - \frac{\theta^2}{2} = 1.1$$

$$\theta - \frac{\theta^2}{2} - 0.1 = 0$$

Multiply by -2

$$\theta^2 - 2\theta + 0.2 = 0$$

b)

$$\theta = \frac{2 \pm \sqrt{4 - 0.8}}{2}$$

$$= 1 \pm \sqrt{0.8}$$

$$1 + \sqrt{0.8} = 1.894...$$

This root is not small, so the approximations do not hold

$$\alpha \approx 1 - \sqrt{0.8}$$

$$= 0.105572...$$

$$\alpha \approx 0.1056$$

c)

$$\% \text{ error} = \frac{0.10579 - 0.105572...}{0.10579} \times 100$$

$$= 0.203...$$

$$0.20\%$$

9

The diagram shows a sector OAB of a circle with centre O and radius 6 cm. Angle AOB = 0.8 radians. The tangent to the circle at A meets the line OB extended at the point C. The shaded region is bounded by AC, CB and the arc AB.

- Find the length of AC.
- Find the area of the shaded region.
- Find the perimeter of the shaded region.

Give each answer to 3 significant figures.

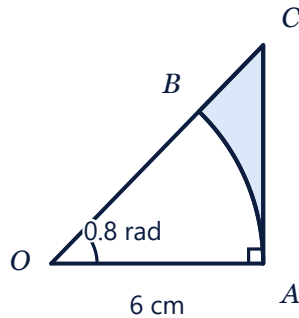


Diagram NOT accurately drawn

(7 marks)

a)

Tangent $\perp$ radius, so triangle OAC has a right angle at A

$$\begin{aligned} AC &= 6 \tan 0.8 \\ &= 6.1778... \end{aligned}$$

$$\boxed{6.18 \text{ cm}}$$

b)

Shaded = triangle OAC - sector OAB

$$\begin{aligned} \text{triangle} &= \frac{1}{2} \times 6 \times 6.1778... \\ &= 18.533... \end{aligned}$$

$$\begin{aligned} \text{sector} &= \frac{1}{2} \times 6^2 \times 0.8 \\ &= 14.4 \end{aligned}$$

$$\begin{aligned} \text{shaded} &= 18.533... - 14.4 \\ &= 4.1334... \end{aligned}$$

$$4.13 \text{ cm}^2$$

c)

$$OC = \frac{6}{\cos 0.8}$$

$$= 8.6119...$$

$$BC = 8.6119... - 6$$

$$= 2.6119...$$

$$\text{arc } AB = 6 \times 0.8$$

$$= 4.8$$

$$P = 6.1778... + 2.6119... + 4.8$$

$$= 13.589...$$

$$13.6 \text{ cm}$$

10

The points A and B lie on a circle with centre O and radius 5 cm, and angle AOB = θ radians, where $0 < \theta < \pi$. The chord AB divides the sector OAB into the triangle OAB and a segment. The area of the segment is equal to the area of the triangle OAB.

- Show that $\theta = 2 \sin \theta$
- Show that this equation has a root between 1.8 and 2.0
- Given that the root is $\theta = 1.8955$ to 4 decimal places, find the area of the segment, giving your answer to 3 significant figures.

(7 marks)

a)

$$\text{triangle} = \frac{1}{2} r^2 \sin \theta$$

$$\text{segment} = \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta$$

$$\frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta = \frac{1}{2} r^2 \sin \theta$$

Divide by $\frac{1}{2} r^2$

$$\theta - \sin \theta = \sin \theta$$

$$\boxed{\theta = 2 \sin \theta}$$

b)

$$f(\theta) = \theta - 2 \sin \theta$$

$$f(1.8) = 1.8 - 2 \sin 1.8 = -0.1477...$$

$$f(2.0) = 2.0 - 2 \sin 2.0 = 0.1814...$$

Sign change and f is continuous on $[1.8, 2.0]$

$$\boxed{\text{so there is a root between 1.8 and 2.0}}$$

c)

Segment = triangle

$$A = \frac{1}{2} \times 5^2 \times \sin 1.8955$$

$$= 12.5 \times 0.94774...$$

$$= 11.846...$$

$$\boxed{11.8 \text{ cm}^2}$$

11

A flower bed is in the shape of the region between two arcs of circles with the same centre O , as shown in the diagram. The inner arc has radius r metres and the outer arc has radius $2r$ metres. Both arcs subtend an angle of θ radians at O , and $\theta > 1$

The perimeter of the flower bed is 26 m and its area is 36 m^2 .

a. Show that $r^2\theta = 24$

b. Show that $r^2 - 13r + 36 = 0$

c. Find the value of r and the value of θ , explaining why you reject one solution.

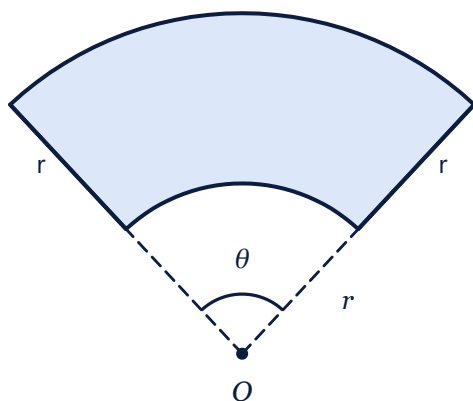


Diagram NOT accurately drawn

(7 marks)

a)

Area = large sector - small sector

$$A = \frac{1}{2}(2r)^2\theta - \frac{1}{2}r^2\theta$$

$$= 2r^2\theta - \frac{1}{2}r^2\theta$$

$$= \frac{3}{2}r^2\theta$$

$$\frac{3}{2}r^2\theta = 36$$

$$\boxed{r^2\theta = 24}$$

b)

Perimeter: two arcs and two straight edges of length r

$$P = 2r\theta + r\theta + 2r$$

$$3r\theta + 2r = 26$$

From (a), $r\theta = \frac{24}{r}$

$$\frac{72}{r} + 2r = 26$$

$$72 + 2r^2 = 26r$$

$$r^2 - 13r + 36 = 0$$

c)

$$(r - 4)(r - 9) = 0$$

$$r = 4: \theta = \frac{24}{16} = 1.5$$

$$r = 9: \theta = \frac{24}{81} = 0.296...$$

$\theta > 1$, so reject $r = 9$

$$r = 4, \theta = 1.5$$

12

A pendulum consists of a small bob on a light string of length L metres. When the string makes an angle θ radians with the vertical, the bob has moved a distance s metres along its circular path from the lowest point, and it is h metres above the lowest point, where

$$h = L(1 - \cos \theta)$$

a. Write down s in terms of L and θ .

b. Show that, when θ is small, $h \approx \frac{s^2}{2L}$

A pendulum with $L = 2$ is pulled back until the string makes an angle of 8° with the vertical.

c. Use the approximation in part (b) to estimate h , and find the percentage error compared with the exact value of h . Give your answers to 2 significant figures.

d. Explain why the approximation becomes less accurate as the angle increases.

(8 marks)

a)

Arc length = radius $\times$ angle

$$s = L\theta$$

b)

$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$

$$h \approx L\left(1 - \left(1 - \frac{\theta^2}{2}\right)\right)$$

$$= \frac{L\theta^2}{2}$$

$$\theta = \frac{s}{L}$$

$$h \approx \frac{L}{2} \times \frac{s^2}{L^2}$$

$$h \approx \frac{s^2}{2L}$$

c)

Convert to radians first

$$\theta = 8 \times \frac{\pi}{180}$$

$$= 0.139626\dots$$

$$s = 2 \times 0.139626\dots$$

$$= 0.279252\dots$$

$$h \approx \frac{0.279252\dots^2}{4}$$

$$= 0.0194955\dots$$

$$h \approx 0.019 \text{ m}$$

$$\text{exact } h = 2(1 - \cos 8^\circ)$$

$$= 0.0194638\dots$$

$$\% \text{ error} = \frac{0.0194955\dots - 0.0194638\dots}{0.0194638\dots} \times 100$$

$$= 0.1626\dots$$

0.16%

d)

$\cos \theta \approx 1 - \frac{\theta^2}{2}$ ignores terms that grow quickly as θ gets bigger