

Addition formulae, double angles and $R \sin(x + a)$

A-level Pure Mathematics (Year 2) | Edexcel 9MA0 | 12 questions, 72 marks

A-level Maths topic practice



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1

Without using a calculator, find the exact value of

a. $\sin \frac{7\pi}{12}$, giving your answer in the form $\frac{\sqrt{a} + \sqrt{b}}{4}$

b. $\tan \frac{\pi}{12}$, giving your answer in the form $p + q\sqrt{3}$ where p and q are integers.

(4 marks)

Answer _____

2

Given that $\sin A = \frac{4}{5}$, where $\frac{\pi}{2} < A < \pi$, and $\cos B = \frac{5}{13}$, where $0 < B < \frac{\pi}{2}$, find the exact value of

a. $\sin 2A$

b. $\cos(A - B)$

(5 marks)

Answer _____

3

a. Express $5 \sin x - 2 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

b. Write down the maximum value of $5 \sin x - 2 \cos x$

c. Find the smallest positive value of x at which this maximum occurs, giving your answer to 3 significant figures.

(5 marks)

Answer _____

4

Prove that

$$\cot \theta - \tan \theta \equiv 2 \cot 2\theta, \quad \theta \neq \frac{n\pi}{2}, \quad n \in \mathbb{Z}$$

(3 marks)

Answer _____

5

In this question you must show all stages of your working.

Solve, for $0 \leq x < 2\pi$, the equation

$$\cos 2x = 3 \sin x - 1$$

Give your answers as exact multiples of π .

(5 marks)

Answer _____

6 Solve, for $0 \leq \theta < 360^\circ$, the equation

$$5 \sin 2\theta = 3 \cos \theta$$

Give your answers to one decimal place where appropriate.

(5 marks)

Answer _____

7 Solve, for $0 \leq \theta < \pi$, the equation

$$\tan\left(\theta + \frac{\pi}{4}\right) = 6 \tan \theta$$

Give your answers in radians to 3 decimal places.

(5 marks)

Answer _____

8 a. Show that $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$

b. Hence solve, for $0 < x < \pi$, the equation

$$\sin 3x = 2 \sin x$$

giving your answers as exact multiples of π .

(7 marks)

Answer _____

9

- a. Express $2 \sin \theta + 5 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$.
Give the exact value of R and give the value of α to 2 decimal places.
- b. Hence solve, for $0 \leq \theta < 360^\circ$, the equation
 $2 \sin \theta + 5 \cos \theta = 3$
 giving your answers to one decimal place.

(6 marks)

Answer _____

10

In this question you must show all stages of your working.

- a. Express $\sqrt{3} \sin x + \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, giving the exact values of R and α .
- b. Hence solve, for $0 \leq x < \pi$, the equation
 $\sqrt{3} \sin 2x + \cos 2x = 1$
- c. Find the maximum value and the minimum value of

$$\frac{1}{4 + \sqrt{3} \sin x + \cos x}$$

(8 marks)

Answer _____

11

The depth of water, D metres, at the entrance to a harbour is modelled by the equation

$$D = 5 + 1.2 \sin\left(\frac{\pi t}{6}\right) - 0.9 \cos\left(\frac{\pi t}{6}\right), \quad 0 \leq t < 12$$

where t is the time in hours after midnight and the angles are in radians.

- a. Express $1.2 \sin x - 0.9 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α to 4 decimal places.

- b. Using your answer to part (a), find the maximum depth of the water according to the model.
- c. Find the time when this maximum depth occurs, giving your answer to the nearest minute.
- d. A fishing boat can only pass through the entrance when the depth of water is at least 5.9 metres. Find, according to the model, the times between which the boat can pass through the entrance, giving your answers to the nearest minute.

(9 marks)

Answer _____

12

$$f(x) = 4 \sin^2 x + 6 \sin x \cos x, \quad 0 \leq x \leq \pi$$

a. Show that $f(x) = 2 + 3 \sin 2x - 2 \cos 2x$

b. Express $3 \sin 2x - 2 \cos 2x$ in the form $R \sin(2x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 4 decimal places.

c. Hence state the maximum value of $f(x)$ and find the value of x at which it occurs, giving x to 3 significant figures.

d. Find the set of values of the constant k for which the equation $f(x) = k$ has exactly two solutions in the interval $0 \leq x \leq \pi$

(10 marks)

Answer

End of questions | Total 72 marks