



- 1 For each situation, write down suitable null and alternative hypotheses in terms of p , and say whether the test is one-tailed or two-tailed.
- Harry thinks a coin is biased towards landing on heads. p is the probability the coin lands on heads.
 - A firm says 4% of the phone cases it makes are faulty. After a machine is serviced the manager believes the proportion has gone down. p is the proportion of faulty cases.
 - Records show 8% of households in a town have a pet rabbit. A vet thinks this has changed. p is the proportion of households with a pet rabbit.
- (3 marks)

H_0 = no change, H_1 = what they believe

a) "biased towards heads" means bigger than 0.5

$$H_0: p = 0.5$$

$$H_1: p > 0.5$$

one-tailed

b) "gone down" means smaller than 0.04

$$H_0: p = 0.04$$

$$H_1: p < 0.04$$

one-tailed

c) "changed" has no direction

$$H_0: p = 0.08$$

$$H_1: p \neq 0.08$$

two-tailed

2

Callum claims he can tell the difference between two brands of cola by taste. In a test he is given 10 cups, each filled at random with one of the two brands, and he identifies 9 of them correctly.

Test, at the 5% level of significance, whether there is evidence that Callum does better than guessing. State your hypotheses clearly.

(4 marks)

p = probability he identifies a cup correctly

$$H_0: p = 0.5$$

$$H_1: p > 0.5$$

Assume H_0 is true

$$X \sim B(10, 0.5)$$

Probability of a result this extreme or more

$$P(X \geq 9) = P(X = 9) + P(X = 10)$$

$$P(X \geq 9) = 10 \times 0.5^{10} + 0.5^{10}$$

$$P(X \geq 9) = \frac{11}{1024} = 0.0107$$

Compare with the significance level

$$0.0107 < 0.05, \text{ so reject } H_0$$

Evidence that Callum does better than guessing

3

Last year 30% of the students at a school walked to school. A new bus route has started and the head teacher believes that fewer students now walk. She takes a random sample of 20 students and finds that 2 of them walk to school.

Test, at the 5% level of significance, whether there is evidence to support the head teacher's belief. State your hypotheses clearly.

(5 marks)

p = proportion of students who walk

$$H_0: p = 0.3$$

$$H_1: p < 0.3$$

Assume H_0 is true

$$X \sim B(20, 0.3)$$

Lower tail: 2 or fewer

$$P(X \leq 2) = 0.0355$$

Compare with 5%

$$0.0355 < 0.05$$

Significant, so reject H_0

Evidence that fewer students now walk to school

4

A garden centre says that 20% of its tomato plants produce yellow tomatoes. Joy thinks the proportion is higher. She plans to test this using a random sample of 25 plants.

- Write down suitable hypotheses for her test.
- Find the critical region for the test at the 5% level of significance.
- Write down the actual significance level of the test.

(5 marks)

a) p = proportion with yellow tomatoes

$$H_0: p = 0.2$$

$$H_1: p > 0.2$$

b) Under H_0 , $X \sim B(25, 0.2)$

Upper tail: want $P(X \geq c) \leq 0.05$

$$P(X \geq 8) = 1 - P(X \leq 7) = 0.1091$$

more than 0.05

$$P(X \geq 9) = 1 - P(X \leq 8) = 0.0468$$

less than 0.05

Critical region $X \geq 9$

c) Actual significance level = $P(\text{in the critical region} \mid H_0)$

$$P(X \geq 9) = 0.0468$$

0.0468 (4.68%)

5

A survey company has found that 40% of adults in a city cycle at least once a week. A researcher believes this proportion is different for adults who live in the city centre. She will take a random sample of 30 adults who live in the city centre and test at the 10% level of significance.

- Write down suitable hypotheses.
- Find the critical region for the test. The probability in each tail should be less than 0.05
- Find the actual significance level of the test.

(6 marks)

a) "different" means two-tailed

$$H_0: p = 0.4$$

$$H_1: p \neq 0.4$$

b) Under H_0 , $X \sim B(30, 0.4)$

10% split into two tails of 5%

Lower tail

$$P(X \leq 7) = 0.0435$$

$$P(X \leq 8) = 0.0940 \text{ (too big)}$$

Upper tail

$$P(X \geq 17) = 1 - P(X \leq 16) = 0.0481$$

$$P(X \geq 16) = 1 - P(X \leq 15) = 0.0971 \text{ (too big)}$$

$$X \leq 7 \text{ or } X \geq 17$$

c) Add the two tails

$$\text{Actual significance level} = 0.0435 + 0.0481$$

$$\text{Actual significance level} = 0.0916$$

$$0.0916$$

6

A restaurant owner says that 20% of diners order a dessert. The new chef believes that since he changed the menu, the proportion of diners who order a dessert has increased. On one evening, 10 out of 30 randomly chosen diners order a dessert.

Test, at the 5% level of significance, whether there is evidence to support the chef's belief. State your hypotheses clearly.

(5 marks)

p = proportion of diners who order dessert

$$H_0: p = 0.2$$

$$H_1: p > 0.2$$

Assume H_0 is true

$$X \sim B(30, 0.2)$$

Upper tail: 10 or more

$$P(X \geq 10) = 1 - P(X \leq 9)$$

$$P(X \geq 10) = 1 - 0.9389$$

$$P(X \geq 10) = 0.0611$$

Compare with 5%

$$0.0611 > 0.05$$

Not significant, so do not reject H_0

Insufficient evidence that more diners now order dessert

7

A national survey found that 15% of Year 10 students play a musical instrument. Mr Okafor believes the proportion at his school is different. In a random sample of 40 Year 10 students from his school, 11 play a musical instrument.

- Test, at the 5% level of significance, whether there is evidence to support Mr Okafor's belief. State your hypotheses clearly.
- Explain how the conclusion would change if Mr Okafor had believed that the proportion at his school was higher than 15%.

(6 marks)

a) "different" means two-tailed

$$H_0: p = 0.15$$

$$H_1: p \neq 0.15$$

Assume H_0 is true

$$X \sim B(40, 0.15)$$

Expected $40 \times 0.15 = 6$, so 11 is in the upper tail

$$P(X \geq 11) = 1 - P(X \leq 10)$$

$$P(X \geq 11) = 1 - 0.9701$$

$$P(X \geq 11) = 0.0299$$

Two-tailed: compare with $2.5\% = 0.025$

$$0.0299 > 0.025$$

Not significant, do not reject H_0

Insufficient evidence the proportion is different

b) One-tailed: compare with 0.05

$$0.0299 < 0.05$$

Significant, so reject H_0

Evidence that the proportion is higher

8

A delivery firm finds that 8% of its parcels arrive late. It changes its routes and wants to know if the proportion of late parcels has gone down. A random sample of 60 parcels is taken after the change.

- Using a 5% level of significance, find the critical region for a suitable test.
- State the actual significance level of the test.
- In the sample, 1 parcel arrives late. State, with a reason, the conclusion of the test.

(6 marks)

a) p = proportion of parcels that arrive late

$$H_0: p = 0.08$$

$$H_1: p < 0.08$$

Under H_0 , $X \sim B(60, 0.08)$

Lower tail: want $P(X \leq c) \leq 0.05$

$$P(X \leq 0) = 0.0067$$

$$P(X \leq 1) = 0.0418$$

$$P(X \leq 2) = 0.1317 \text{ (too big)}$$

Critical region $X \leq 1$

b) Actual significance level = $P(X \leq 1)$

0.0418

c) Is the observed value in the critical region?

$X = 1$ is in $X \leq 1$

Significant, so reject H_0

Evidence that fewer parcels arrive late

9

A quiz app says that a player who is guessing gets each question right with probability 0.25. Zainab thinks that she does better than guessing. She answers 15 questions and X is the number she gets right.

- Find the critical region for a test of $H_0: p = 0.25$ against $H_1: p > 0.25$ at the 5% level of significance.
- Find the probability of rejecting H_0 when it is in fact true.
- Zainab gets 7 questions right. Find $P(X \geq 7)$ and hence explain whether this result is significant at the 5% level and at the 10% level.

(7 marks)

a) Under H_0 , $X \sim B(15, 0.25)$

Upper tail: want $P(X \geq c) \leq 0.05$

$$P(X \geq 7) = 1 - P(X \leq 6) = 0.0566$$

more than 0.05

$$P(X \geq 8) = 1 - P(X \leq 7) = 0.0173$$

less than 0.05

Critical region $X \geq 8$

b) H_0 is rejected when X is in the critical region

$$P(X \geq 8 \mid p = 0.25) = 0.0173$$

0.0173

c)

$$P(X \geq 7) = 1 - P(X \leq 6)$$

$$P(X \geq 7) = 1 - 0.9434 = 0.0566$$

At 5%

$0.0566 > 0.05$: not significant

At 10%

$0.0566 < 0.10$: significant

Evidence she beats guessing at 10%, but not at 5%

10

A cereal company says that 30% of its boxes contain a free toy. Ella believes the proportion is lower. She will buy n boxes and test $H_0: p = 0.3$ against $H_1: p < 0.3$ at the 5% level of significance.

- Find the smallest value of n for which finding no toys at all would lead Ella to reject H_0 .
- Ella buys 9 boxes. Show that the critical region is $X = 0$ and find the actual significance level.
- Ella finds 1 toy in her 9 boxes. State the conclusion of her test in context.

(7 marks)

a) No toys in n boxes, under H_0

$$P(X = 0) = 0.7^n$$

Need this to be in the 5% tail

$$0.7^n < 0.05$$

Take logs; $\ln 0.7$ is negative so the sign flips

$$n > \frac{\ln 0.05}{\ln 0.7}$$

$$n > 8.39...$$

Check

$$0.7^8 = 0.0576$$

$$0.7^9 = 0.0404$$

$$n = 9$$

b) $X \sim B(9, 0.3)$

$$P(X = 0) = 0.7^9 = 0.0404$$

less than 0.05

$$P(X \leq 1) = 0.1960$$

more than 0.05, so 1 is not in the critical region

Critical region $X = 0$, actual significance level 0.0404

c) $X = 1$ is not in the critical region

Not significant, do not reject H_0

Insufficient evidence that fewer than 30% of boxes have a toy

11

Mia wants to find out whether a coin is biased. She will toss it 20 times and record X , the number of heads.

- Write down suitable hypotheses.
- Using a 5% level of significance, find the critical region. The probability in each tail should be as close as possible to 2.5% without going over.
- Find the actual significance level of the test.
- Mia gets 14 heads. She says "this shows the coin is biased". Comment on her statement.

(7 marks)

a) "biased" in either direction, two-tailed

$$H_0: p = 0.5$$

$$H_1: p \neq 0.5$$

b) Under H_0 , $X \sim B(20, 0.5)$

2.5% in each tail

Lower tail

$$P(X \leq 5) = 0.0207$$

$$P(X \leq 6) = 0.0577 \text{ (too big)}$$

Upper tail (symmetric since $p = 0.5$)

$$P(X \geq 15) = 1 - P(X \leq 14) = 0.0207$$

$$P(X \geq 14) = 1 - P(X \leq 13) = 0.0577 \text{ (too big)}$$

$$X \leq 5 \text{ or } X \geq 15$$

c) Add the two tails

$$\text{Actual significance level} = 0.0207 + 0.0207$$

$$\text{Actual significance level} = 0.0414$$

$$0.0414$$

d) Is 14 in the critical region?

14 is not in $X \geq 15$

Not significant, do not reject H_0

Insufficient evidence the coin is biased, so Mia is wrong

A garden centre claims that 85% of its tulip bulbs flower in the first spring after planting.

- a. Assuming the claim is true, find the probability that at least 10 bulbs in a pack of 12 flower.

Olu thinks that fewer than 85% of the bulbs flower. He plants a random sample of 50 bulbs to test the claim at the 5% level of significance.

- b. Write down suitable hypotheses and find the critical region for his test.

- c. State the actual significance level of the test.

- d. In the spring, 37 of Olu's 50 bulbs flower. State the conclusion of his test in context.

- e. Give one reason why the bulbs in a single garden might not flower independently of each other.

(9 marks)

a) Y = number in a pack of 12 that flower

$$Y \sim B(12, 0.85)$$

$$P(Y \geq 10) = 1 - P(Y \leq 9)$$

$$P(Y \geq 10) = 1 - 0.2642$$

$$P(Y \geq 10) = 0.7358$$

$$\boxed{0.7358}$$

b) p = proportion of bulbs that flower

$$H_0: p = 0.85$$

$$H_1: p < 0.85$$

Under H_0 , $X \sim B(50, 0.85)$

Lower tail: want $P(X \leq c) \leq 0.05$

$$P(X \leq 37) = 0.0301$$

less than 0.05

$$P(X \leq 38) = 0.0628$$

more than 0.05

$$\boxed{\text{Critical region } X \leq 37}$$

c) Actual significance level

$$P(X \leq 37) = 0.0301$$

$$\boxed{0.0301}$$

d) Is the observed value in the critical region?

$$37 \text{ is in } X \leq 37$$

Significant, so reject H_0

$$\boxed{\text{Evidence that fewer than 85\% of the bulbs flower}}$$

e) Think about independence

Bulbs in one garden share soil, frost and pests

one bulb flowering may be linked to another