



- 1 A factory packs light bulbs in boxes of 20. The random variable X is the number of faulty bulbs in a box.

- a. State two conditions needed for X to be modelled by a binomial distribution.
b. Sam deals 5 cards, without replacement, from a standard pack of 52 playing cards. The random variable H is the number of hearts dealt. Explain why H cannot be modelled by $B(5, 0.25)$.

(3 marks)

a) Binomial needs: fixed n , two outcomes, independent, constant p

Bulbs are faulty independently of each other

Each bulb has the same probability of being faulty

independent, constant probability of being faulty

b) Check constant probability

$$P(\text{1st card is a heart}) = \frac{13}{52}$$

If the 1st card is a heart:

$$P(\text{2nd card is a heart}) = \frac{12}{51}$$

The probability changes from card to card

without replacement, p is not constant (trials not independent)

2

The random variable $X \sim B(12, 0.3)$. Find

- a. $P(X = 4)$
- b. $P(X \leq 3)$
- c. $P(X > 5)$

giving each answer to 4 decimal places.

(3 marks)

a) Use $P(X = r) = {}^nC_r p^r (1-p)^{n-r}$

$$P(X = 4) = {}^{12}C_4 \times 0.3^4 \times 0.7^8$$

$$P(X = 4) = 495 \times 0.0081 \times 0.05764801$$

$$P(X = 4) = 0.2311$$

$$\boxed{0.2311}$$

b) Cumulative: binomial cdf on the calculator

$$P(X \leq 3) = 0.4925$$

$$\boxed{0.4925}$$

c) $X > 5$ means $X \geq 6$, so use the complement

$$P(X > 5) = 1 - P(X \leq 5)$$

$$P(X > 5) = 1 - 0.8822$$

$$P(X > 5) = 0.1178$$

$$\boxed{0.1178}$$

3

The random variable $X \sim B(20, 0.4)$. Find, to 4 decimal places,

- a. $P(X \geq 10)$
- b. $P(5 < X \leq 9)$

(4 marks)

a) $X \geq 10$ is everything except $X \leq 9$

$$P(X \geq 10) = 1 - P(X \leq 9)$$

$$P(X \geq 10) = 1 - 0.7553$$

$$P(X \geq 10) = 0.2447$$

$$\boxed{0.2447}$$

b) $5 < X \leq 9$ means $X = 6, 7, 8, 9$

$$P(5 < X \leq 9) = P(X \leq 9) - P(X \leq 5)$$

$$P(5 < X \leq 9) = 0.7553 - 0.1256$$

$$P(5 < X \leq 9) = 0.6297$$

$$\boxed{0.6297}$$

4

A café finds that 15% of its customers order a drink with oat milk. On one morning a random sample of 25 customers is taken. The random variable X is the number of these customers who order a drink with oat milk.

- Write down a suitable distribution to model X .
- Find the probability that exactly 4 of the customers order a drink with oat milk.
- Find the probability that at most 2 of the customers order a drink with oat milk.
- Find the probability that more than 5 of the customers order a drink with oat milk.

(5 marks)

a) $n = 25$ customers, $p = 0.15$

$$X \sim B(25, 0.15)$$

b) Exactly 4

$$P(X = 4) = {}^{25}C_4 \times 0.15^4 \times 0.85^{21}$$

$$P(X = 4) = 0.2110$$

$$0.2110$$

c) At most 2 means $X \leq 2$

$$P(X \leq 2) = 0.2537$$

$$0.2537$$

d) More than 5 means $X \geq 6$

$$P(X > 5) = 1 - P(X \leq 5)$$

$$P(X > 5) = 1 - 0.8385$$

$$P(X > 5) = 0.1615$$

$$0.1615$$

5

In a town, 45% of learner drivers pass their driving test at the first attempt. A random sample of 10 learner drivers from the town is taken.

- Find the probability that exactly 6 of them pass at the first attempt.
- Find the probability that at least 4 of them pass at the first attempt.
- Priya says the same model could be used for 10 friends who all took lessons with the same instructor. Give one reason why the model may not be suitable for them.

(5 marks)

Define the model

X = number who pass first time

$$X \sim B(10, 0.45)$$

a) Exactly 6

$$P(X = 6) = {}^{10}C_6 \times 0.45^6 \times 0.55^4$$

$$P(X = 6) = 210 \times 0.45^6 \times 0.55^4$$

$$P(X = 6) = 0.1596$$

0.1596

b) At least 4 means $X \geq 4$

$$P(X \geq 4) = 1 - P(X \leq 3)$$

$$P(X \geq 4) = 1 - 0.2660$$

$$P(X \geq 4) = 0.7340$$

0.7340

c) Think about the conditions

Same instructor, so the probability of passing is unlikely to be 0.45 for these friends and their results may not be independent

p may not be 0.45 / not independent

6

The random variable $X \sim B(30, 0.25)$.

- Find $P(4 \leq X < 10)$.
- Find the largest value of k such that $P(X \leq k) < 0.1$
- Find the smallest value of m such that $P(X \geq m) < 0.05$

(6 marks)

a) $4 \leq X < 10$ means $X = 4, 5, \dots, 9$

$$P(4 \leq X < 10) = P(X \leq 9) - P(X \leq 3)$$

$$P(4 \leq X < 10) = 0.8034 - 0.0374$$

$$P(4 \leq X < 10) = 0.7660$$

$$\boxed{0.7660}$$

b) Look along the cumulative values

$$P(X \leq 4) = 0.0979$$

less than 0.1

$$P(X \leq 5) = 0.2026$$

more than 0.1, too big

$$\boxed{k = 4}$$

c) $P(X \geq m) = 1 - P(X \leq m - 1)$

$$P(X \geq 12) = 1 - P(X \leq 11) = 0.0507$$

more than 0.05, not small enough

$$P(X \geq 13) = 1 - P(X \leq 12) = 0.0216$$

less than 0.05

$$\boxed{m = 13}$$

7

On any school day, the probability that a particular bus arrives late is 0.2, independently of other days.

- Find the probability that the bus is late on exactly 2 of the 5 days in one week.
- Find the smallest number of school days, n , for which the probability that the bus is late on at least one of the days is greater than 0.95

(5 marks)

$$a) X \sim B(5, 0.2)$$

$$P(X = 2) = {}^5C_2 \times 0.2^2 \times 0.8^3$$

$$P(X = 2) = 10 \times 0.04 \times 0.512$$

$$P(X = 2) = 0.2048$$

$$\boxed{0.2048}$$

$$b) \text{At least one} = 1 - P(\text{none})$$

$$P(\text{at least one late}) = 1 - 0.8^n$$

$$1 - 0.8^n > 0.95$$

$$0.8^n < 0.05$$

Take logs; $\ln 0.8$ is negative so the sign flips

$$n \ln 0.8 < \ln 0.05$$

$$n > \frac{\ln 0.05}{\ln 0.8}$$

$$n > 13.4...$$

Check

$$0.8^{13} = 0.0550$$

$$0.8^{14} = 0.0440$$

$$\boxed{n = 14}$$

8

Lily rolls a fair six-sided die 8 times and records X , the number of sixes.

a. Find $P(X = 2)$.

b. Lily repeats this experiment once a day for 12 days. Find the probability that she gets exactly 2 sixes on at least 3 of the days.

(6 marks)

$$a) X \sim B(8, \frac{1}{6})$$

$$P(X = 2) = {}^8C_2 \times (\frac{1}{6})^2 \times (\frac{5}{6})^6$$

$$P(X = 2) = 28 \times \frac{1}{36} \times \frac{15625}{46656}$$

$$P(X = 2) = 0.2605$$

$$\boxed{0.2605}$$

b) New variable: count the days

Y = number of days with exactly 2 sixes

$$Y \sim B(12, 0.2605)$$

Keep the full value of $P(X = 2)$ on the calculator

$$P(Y \geq 3) = 1 - P(Y \leq 2)$$

$$P(Y \geq 3) = 1 - 0.3589$$

$$P(Y \geq 3) = 0.6411$$

$$\boxed{0.6411}$$

9

A spinner lands on red with probability p on each spin. It is spun 5 times and X is the number of times it lands on red.

Given that $P(X = 0) = 0.07776$

- show that $p = 0.4$
- find $P(X = 2)$
- find $P(X \geq 4)$.

(6 marks)

a) $X = 0$ means no reds in 5 spins

$$P(X = 0) = (1 - p)^5$$

$$(1 - p)^5 = 0.07776$$

Fifth root of both sides

$$1 - p = 0.07776^{\frac{1}{5}}$$

$$1 - p = 0.6$$

$$p = 0.4$$

$$p = 0.4 \text{ as required}$$

b) $X \sim B(5, 0.4)$

$$P(X = 2) = {}^5C_2 \times 0.4^2 \times 0.6^3$$

$$P(X = 2) = 10 \times 0.16 \times 0.216$$

$$P(X = 2) = 0.3456$$

$$0.3456$$

c) $X \geq 4$ means $X = 4$ or 5

$$P(X = 4) = {}^5C_4 \times 0.4^4 \times 0.6 = 0.0768$$

$$P(X = 5) = 0.4^5 = 0.01024$$

$$P(X \geq 4) = 0.0768 + 0.01024$$

$$P(X \geq 4) = 0.08704$$

$$0.0870$$

10

A seed company sells packets of 30 sunflower seeds. Each seed germinates with probability 0.9, independently of the others.

- Find the probability that at least 25 seeds in a packet germinate.
- Kwame buys 6 packets. Find the probability that exactly 4 of his packets have at least 25 seeds that germinate.

(6 marks)

a) X = number of seeds that germinate

$$X \sim B(30, 0.9)$$

$$P(X \geq 25) = 1 - P(X \leq 24)$$

$$P(X \geq 25) = 1 - 0.0732$$

$$P(X \geq 25) = 0.9268$$

0.9268

b) Y = number of packets with at least 25 germinating

$$Y \sim B(6, 0.9268)$$

$$P(Y = 4) = {}^6C_4 \times 0.9268^4 \times 0.0732^2$$

Use the unrounded value from a)

$$P(Y = 4) = 0.0593$$

0.0593

11

The random variable $X \sim B(n, 0.25)$. Given that $P(X = 1) = P(X = 2)$

a. show that $n = 7$

b. find $P(X \leq 1)$.

(6 marks)

a) Write both probabilities with ${}^nC_r$

$$P(X = 1) = n \times 0.25 \times 0.75^{n-1}$$

$$P(X = 2) = \frac{n(n-1)}{2} \times 0.25^2 \times 0.75^{n-2}$$

Set them equal

$$n \times 0.25 \times 0.75^{n-1} = \frac{n(n-1)}{2} \times 0.25^2 \times 0.75^{n-2}$$

Divide both sides by $n \times 0.25 \times 0.75^{n-2}$

$$0.75 = \frac{n-1}{2} \times 0.25$$

$$0.75 = \frac{n-1}{8}$$

$$n-1 = 6$$

$$n = 7$$

$n = 7$ as required

b) $X \sim B(7, 0.25)$

$$P(X \leq 1) = P(X = 0) + P(X = 1)$$

$$P(X \leq 1) = 0.75^7 + 7 \times 0.25 \times 0.75^6$$

$$P(X \leq 1) = 0.13348... + 0.31146...$$

$$P(X \leq 1) = 0.4449$$

0.4449

12

Ana is an archer. Each arrow she shoots hits the gold ring with probability 0.35, independently of other arrows. She shoots arrows in rounds of 8 arrows, and X is the number of golds in a round.

- Find $P(X \geq 3)$.
- Find the probability that she gets exactly 2 golds in a round.
- Ana shoots 3 rounds. Find the probability that she gets at least 10 golds altogether.
- Find the probability that she gets at least 3 golds in each of the 3 rounds.

(7 marks)

a) $X \sim B(8, 0.35)$

$$P(X \geq 3) = 1 - P(X \leq 2)$$

$$P(X \geq 3) = 1 - 0.4278$$

$$P(X \geq 3) = 0.5722$$

0.5722

b) Exactly 2 golds

$$P(X = 2) = {}^8C_2 \times 0.35^2 \times 0.65^6$$

$$P(X = 2) = 0.2587$$

0.2587

c) 3 rounds = 24 independent arrows

$$T = \text{total golds}, T \sim B(24, 0.35)$$

$$P(T \geq 10) = 1 - P(T \leq 9)$$

$$P(T \geq 10) = 1 - 0.6866$$

$$P(T \geq 10) = 0.3134$$

0.3134

d) Rounds are independent, so multiply

$$P(\text{each round} \geq 3) = 0.5722^3$$

$$P(\text{each round} \geq 3) = 0.1873$$

0.1873