



1

The events A and B are such that

$$P(A) = 0.4 \quad P(B) = 0.35 \quad P(A \cup B) = 0.6$$

- Find $P(A \cap B)$
- Find $P(A | B)$
- Determine whether or not A and B are independent. Show your working clearly.

(4 marks)

a) Addition rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.6 = 0.4 + 0.35 - P(A \cap B)$$

$$P(A \cap B) = 0.75 - 0.6$$

$$P(A \cap B) = 0.15$$

b) Conditional probability

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{0.15}{0.35}$$

$$P(A | B) = \frac{3}{7} = 0.429$$

c) Independent if $P(A \cap B) = P(A) \times P(B)$

$$P(A) \times P(B) = 0.4 \times 0.35$$

$$P(A) \times P(B) = 0.14$$

$$0.14 \neq 0.15$$

A and B are not independent

The table shows how the 200 students at a sixth form college travel to college.

	Bus	Car	Walk	Total
Year 12	38	22	30	90
Year 13	42	40	28	110
Total	80	62	58	200

A student is chosen at random.

- Find the probability that the student walks to college.
- Given that the student travels by car, find the probability that the student is in Year 13
- Find $P(\text{bus} \mid \text{Year 12})$

The event W is that the student walks and the event T is that the student is in Year 12

- Determine whether or not W and T are independent.

(5 marks)

a)

$$P(\text{walk}) = \frac{58}{200}$$

$$P(\text{walk}) = \frac{29}{100} = 0.29$$

b) Given car: only use the Car column (62)

$$P(\text{Year 13} \mid \text{car}) = \frac{40}{62}$$

$$P(\text{Year 13} \mid \text{car}) = \frac{20}{31} = 0.645$$

c) Given Year 12: only use the Year 12 row (90)

$$P(\text{bus} \mid \text{Year 12}) = \frac{38}{90}$$

$$P(\text{bus} \mid \text{Year 12}) = \frac{19}{45} = 0.422$$

d) Compare $P(W \cap T)$ with $P(W) \times P(T)$

$$P(W \cap T) = \frac{30}{200} = 0.15$$

$$P(W) = \frac{58}{200} = 0.29$$

$$P(T) = \frac{90}{200} = 0.45$$

$$P(W) \times P(T) = 0.29 \times 0.45$$

$$P(W) \times P(T) = 0.1305$$

$$0.1305 \neq 0.15$$

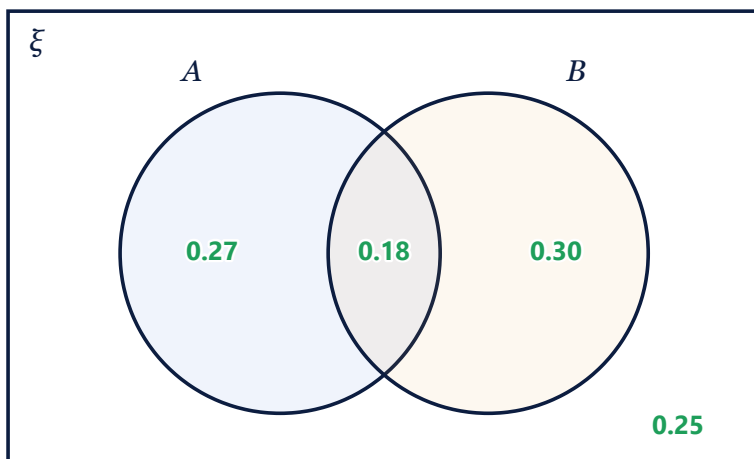
W and T are not independent

3

The events A and B are such that

$$P(A) = 0.45 \quad P(B | A) = 0.4 \quad P(A' \cap B') = 0.25$$

- Draw a Venn diagram to illustrate the events A and B , giving the probability for each region.
- Find $P(A | B)$
- Determine whether or not A and B are independent.



(6 marks)

a) Multiplication rule

$$P(A \cap B) = P(B | A) \times P(A)$$

$$P(A \cap B) = 0.4 \times 0.45$$

$$P(A \cap B) = 0.18$$

$$P(A \text{ only}) = 0.45 - 0.18$$

$$P(A \text{ only}) = 0.27$$

All four regions add to 1

$$P(B \text{ only}) = 1 - 0.27 - 0.18 - 0.25$$

$$P(B \text{ only}) = 0.30$$

A only 0.27, $A \cap B$ 0.18, B only 0.30, outside 0.25

b)

$$P(B) = 0.18 + 0.30 = 0.48$$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{0.18}{0.48}$$

$$P(A | B) = 0.375$$

c)

$$P(A) \times P(B) = 0.45 \times 0.48$$

$$P(A) \times P(B) = 0.216$$

$$0.216 \neq 0.18$$

A and B are not independent

4

When Leila serves in tennis, her first serve is in with probability 0.6. If her first serve is in, she wins the point with probability 0.7

If her first serve is out, she takes a second serve. Her second serve is in with probability 0.9 and, if it is in, she wins the point with probability 0.5. If her second serve is out, she loses the point.

- Find the probability that Leila wins a point when she serves.
- Given that Leila wins a point, find the probability that her first serve was in.
- Given that Leila loses a point, find the probability that both of her serves were out.

(6 marks)

a) Two ways to win the point

$$P(\text{1st in and win}) = 0.6 \times 0.7 = 0.42$$

$$P(\text{1st out, 2nd in, win}) = 0.4 \times 0.9 \times 0.5 = 0.18$$

$$P(\text{win}) = 0.42 + 0.18$$

$$P(\text{win}) = 0.6$$

b) Conditional probability

$$P(\text{1st in} \mid \text{win}) = \frac{P(\text{1st in} \cap \text{win})}{P(\text{win})}$$

$$P(\text{1st in} \mid \text{win}) = \frac{0.42}{0.6}$$

$$P(\text{1st in} \mid \text{win}) = 0.7$$

c)

$$P(\text{lose}) = 1 - 0.6 = 0.4$$

$$P(\text{both out}) = 0.4 \times 0.1 = 0.04$$

Both out means she must lose the point

$$P(\text{both out} \mid \text{lose}) = \frac{0.04}{0.4}$$

$$P(\text{both out} \mid \text{lose}) = 0.1$$

5

The Venn diagram, where p , q and r are probabilities, shows the events A , B and C and their associated probabilities.

a. State a pair of mutually exclusive events.

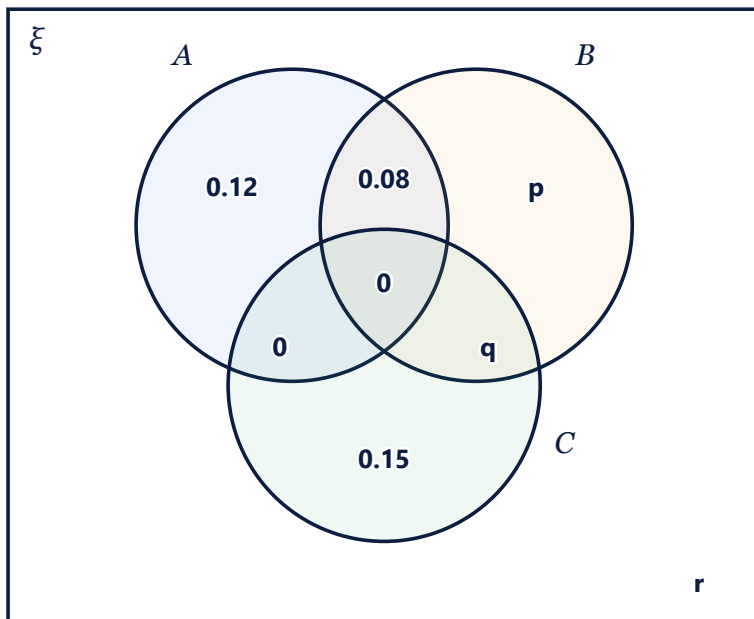
Given that $P(B) = 0.4$ and that the events B and C are independent,

b. find the value of q

c. find the value of p and the value of r

d. Find $P(A \mid B')$

e. Find $P(B \cap C' \mid A \cup C)$



(8 marks)

$$a) P(A \cap C) = 0$$

A and C

$$b) B \text{ and } C \text{ independent: } P(B \cap C) = P(B) \times P(C)$$

$$P(B \cap C) = q$$

$$P(C) = 0.15 + q$$

$$q = 0.4(0.15 + q)$$

$$q = 0.06 + 0.4q$$

$$0.6q = 0.06$$

$$q = 0.1$$

$$c) \text{ Use } P(B) = 0.4$$

$$0.08 + p + q = 0.4$$

$$0.08 + p + 0.1 = 0.4$$

$$p = 0.22$$

All probabilities add to 1

$$r = 1 - (0.12 + 0.08 + 0.22 + 0.1 + 0.15)$$

$$r = 1 - 0.67$$

$$p = 0.22, r = 0.33$$

d)

$$p(A \cap B') = 0.12$$

$$p(B') = 1 - 0.4 = 0.6$$

$$p(A | B') = \frac{0.12}{0.6}$$

$$p(A | B') = 0.2$$

e) The part of $B \cap C'$ that lies inside $A \cup C$

$$p(A \cup C) = 0.12 + 0.08 + 0.1 + 0.15 = 0.45$$

$$p(B \cap C' \cap (A \cup C)) = 0.08$$

$$p(B \cap C' | A \cup C) = \frac{0.08}{0.45}$$

$$p(B \cap C' | A \cup C) = \frac{8}{45} = 0.178$$

6

For the events A and B ,

$$P(A) = 0.3 \quad P(B | A) = 0.5 \quad P(A | B) = 0.25$$

- Find $P(B)$
- Find $P(A \cup B)$
- Find $P(A' | B')$

(6 marks)

a) Find $P(A \cap B)$ first

$$P(A \cap B) = P(B | A) \times P(A)$$

$$P(A \cap B) = 0.5 \times 0.3 = 0.15$$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$0.25 = \frac{0.15}{P(B)}$$

$$P(B) = \frac{0.15}{0.25}$$

$$P(B) = 0.6$$

b) Addition rule

$$P(A \cup B) = 0.3 + 0.6 - 0.15$$

$$P(A \cup B) = 0.75$$

c) $A' \cap B'$ is outside both events

$$P(A' \cap B') = 1 - 0.75 = 0.25$$

$$P(B') = 1 - 0.6 = 0.4$$

$$P(A' | B') = \frac{0.25}{0.4}$$

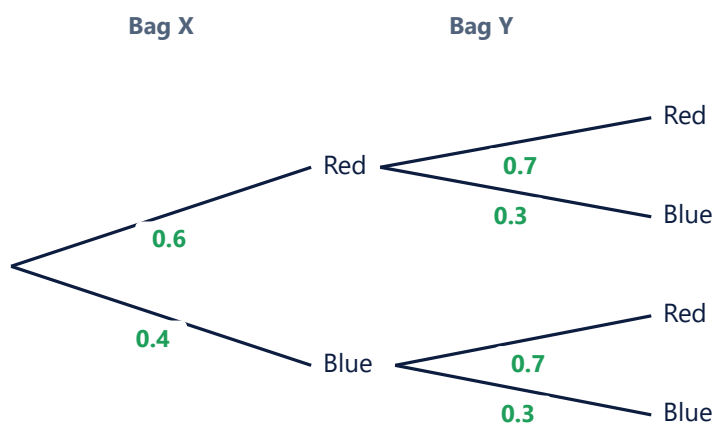
$$P(A' | B') = 0.625$$

7

Hamza has two bags of beads. Bag X contains 6 red beads and 4 blue beads. Bag Y contains only red beads and blue beads.

Hamza takes a bead at random from bag X and then a bead at random from bag Y. The probability that both beads are blue is 0.12

- Find the probability of taking a blue bead from bag Y
- Complete the tree diagram.
- Find the probability that the two beads are different colours.
- Given that the two beads are different colours, find the probability that the bead from bag X was red.



(7 marks)

a) Let $y = P(\text{blue from bag Y})$

$$P(\text{blue from X}) = \frac{4}{10} = 0.4$$

$$0.4 \times y = 0.12$$

$$y = 0.3$$

b) Bag Y does not depend on bag X

X: R 0.6, B 0.4. Y: R 0.7, B 0.3 on both branches

c) Red then blue, or blue then red

$$P(\text{different}) = 0.6 \times 0.3 + 0.4 \times 0.7$$

$$P(\text{different}) = 0.18 + 0.28$$

$$P(\text{different}) = 0.46$$

d) Conditional probability

$$P(X \text{ red} \mid \text{different}) = \frac{0.18}{0.46}$$

$$P(X \text{ red} \mid \text{different}) = \frac{9}{23} = 0.391$$

8

The events A and B are independent, with $P(A) > P(B)$.

$$P(A \cap B) = 0.12 \quad P(A \cup B) = 0.58$$

a. Find $P(A)$ and $P(B)$

b. Find $P(A' | B)$

c. Prove that, for any two independent events A and B , the events A and B' are also independent.

(7 marks)

a) Let $P(A) = a$, $P(B) = b$

Independent: $P(A \cap B) = P(A) \times P(B)$

$$ab = 0.12$$

Addition rule

$$a + b - ab = 0.58$$

$$a + b - 0.12 = 0.58$$

$$a + b = 0.7$$

$$b = 0.7 - a$$

$$a(0.7 - a) = 0.12$$

$$a^2 - 0.7a + 0.12 = 0$$

$$(a - 0.4)(a - 0.3) = 0$$

$$a = 0.4 \text{ or } a = 0.3$$

$P(A) > P(B)$, so take the larger value for $P(A)$

$$P(A) = 0.4, P(B) = 0.3$$

b) Independent, so knowing B does not change A'

$$P(A' | B) = P(A')$$

$$P(A' | B) = 1 - 0.4$$

$$P(A' | B) = 0.6$$

c) A is split into $A \cap B$ and $A \cap B'$

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$P(A \cap B') = P(A) - P(A)P(B)$$

$$P(A \cap B') = P(A)(1 - P(B))$$

$$P(A \cap B') = P(A)P(B')$$

So A and B' are independent

9

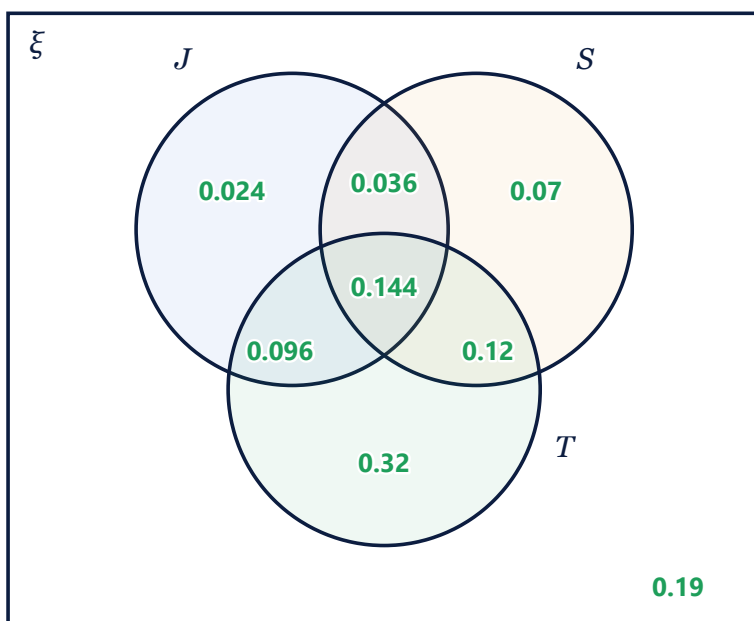
A leisure centre has 500 members. The table shows the age group of each member and whether or not they live in the town.

	Lives in town	Lives outside town
Junior	120	30
Adult	160	90
Senior	60	40

Of the members, 60% of juniors, 30% of adults and 20% of seniors swim, wherever they live.

A member is chosen at random. The event J is that the member is a junior, S that the member swims and T that the member lives in the town.

- Find the probability that the member is an adult who lives outside the town.
- Complete the Venn diagram for the events J , S and T , giving the probability for each region.
- Find $P(S \mid T)$
- Find $P(J \mid S)$
- Find $P([S \cup T]')$



(9 marks)

a)

$$P(\text{adult outside town}) = \frac{90}{500}$$

0.18

b) Count members in each region

$$J \cap S \cap T: 0.6 \times 120 = 72$$

$$J \cap S \cap T': 0.6 \times 30 = 18$$

$$J \cap S' \cap T: 120 - 72 = 48$$

$$J \text{ only: } 30 - 18 = 12$$

$$S \cap T \text{ only: } 0.3 \times 160 + 0.2 \times 60 = 60$$

$$S \text{ only: } 0.3 \times 90 + 0.2 \times 40 = 35$$

$$T \text{ only: } (160 - 48) + (60 - 12) = 160$$

$$\text{outside: } (90 - 27) + (40 - 8) = 95$$

$$\text{Check: } 72 + 18 + 48 + 12 + 60 + 35 + 160 + 95 = 500$$

Divide each by 500

$$J \cap S \cap T \ 0.144, J \cap S \text{ only } 0.036, J \cap T \text{ only } 0.096, J \text{ only } 0.024, S \cap T \text{ only } 0.12, S \text{ only } 0.07, T \text{ only } 0.32, \text{ outside } 0.19$$

c) Given T: 340 members live in town

$$P(S | T) = \frac{(72 + 60)}{340}$$

$$P(S | T) = \frac{132}{340}$$

$$P(S | T) = \frac{33}{85} = 0.388$$

d) Given S

$$\text{swimmers} = 72 + 18 + 60 + 35 = 185$$

$$P(J | S) = \frac{(72 + 18)}{185}$$

$$P(J | S) = \frac{18}{37} = 0.486$$

e) Regions outside both S and T

$$P([S \cup T]') = 0.024 + 0.19$$

$$P([S \cup T]') = 0.214$$

A company estimates that 15% of the emails it receives are spam. Its filter labels a spam email as spam with probability 0.9 and labels a genuine email as spam with probability 0.05

- Find the probability that a randomly chosen email is labelled as spam.
- Given that an email is labelled as spam, find the probability that it is actually spam.
- Two emails that have been labelled as spam are chosen at random. Find the probability that exactly one of them is genuine.
- The company says that the filter is "right 90% of the time". Use your answer to part (b) to comment on this statement.

(7 marks)

a) S = spam, L = labelled as spam

$$P(L) = P(S \cap L) + P(S' \cap L)$$

$$P(L) = 0.15 \times 0.9 + 0.85 \times 0.05$$

$$P(L) = 0.135 + 0.0425$$

$$P(L) = 0.1775$$

b) Conditional probability

$$P(S | L) = \frac{P(S \cap L)}{P(L)}$$

$$P(S | L) = \frac{0.135}{0.1775}$$

$$P(S | L) = 0.76056...$$

$$P(S | L) = 0.761$$

c) For a labelled email $P(\text{genuine}) = 1 - 0.76056 = 0.23944$

Genuine then spam, or spam then genuine

$$P(\text{exactly one genuine}) = 2 \times 0.76056 \times 0.23944$$

$$P(\text{exactly one genuine}) = 0.364$$

d)

Only 76% of emails labelled spam are spam

About 1 in 4 labelled emails is genuine, so the claim is misleading

11

The events A , B and C are such that B and C are mutually exclusive. The Venn diagram, where x and y are probabilities, shows the probability of each region.

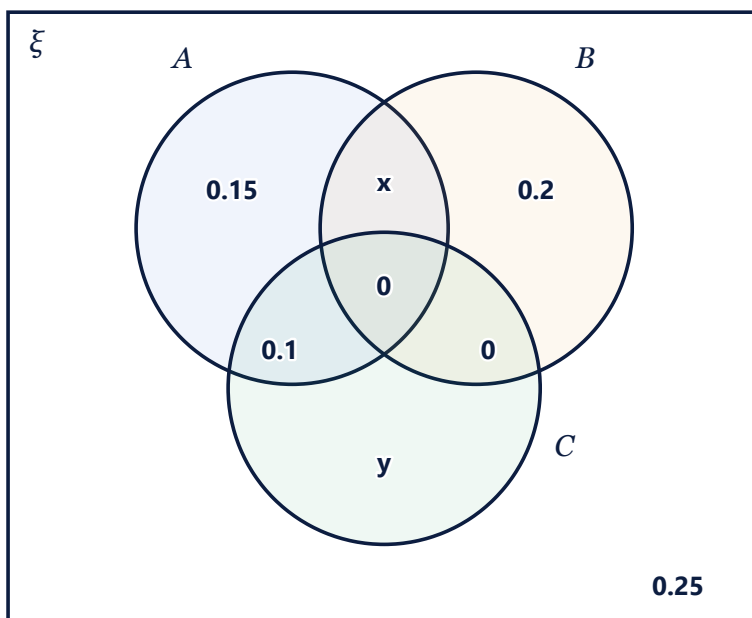
a. Find the greatest possible value of $P(B | A)$

Given also that A and C are independent and that A and B are not mutually exclusive,

b. find the value of x and the value of y

c. find $P(A \cup B | C')$

d. Use set notation to write down an event with probability 0.2



(8 marks)

Total probability is 1

$$0.15 + x + 0.2 + 0.1 + y + 0.25 = 1$$

$$x + y = 0.3$$

a)

$$P(A) = 0.15 + x + 0.1 = 0.25 + x$$

$$P(B | A) = \frac{x}{(0.25 + x)}$$

This increases as x increases, and the largest x can be is 0.3

$$P(B | A) = \frac{0.3}{0.55}$$

$$\text{greatest } P(B | A) = \frac{6}{11} = 0.545$$

b) A and C independent: $P(A \cap C) = P(A) \times P(C)$

$$P(A \cap C) = 0.1$$

$$P(C) = 0.1 + y = 0.4 - x$$

$$(0.25 + x)(0.4 - x) = 0.1$$

$$0.1 + 0.15x - x^2 = 0.1$$

$$x^2 - 0.15x = 0$$

$$x(x - 0.15) = 0$$

A and B not mutually exclusive, so $x \neq 0$

$$x = 0.15$$

$$y = 0.3 - 0.15$$

$$x = 0.15, y = 0.15$$

c) Regions of $A \cup B$ outside C

$$P(C') = 1 - (0.1 + 0.15) = 0.75$$

$$P((A \cup B) \cap C') = 0.15 + 0.15 + 0.2 = 0.5$$

$$P(A \cup B | C') = \frac{0.5}{0.75}$$

$$P(A \cup B | C') = \frac{2}{3}$$

d) 0.2 is in B but not in A

$$B \cap A'$$

12

A factory makes bike lights on three machines. Machine A makes 50% of the lights, machine B makes 30% and machine C makes 20%.

Of the lights made on machine A, 2% are faulty, and of the lights made on machine C, 5% are faulty. The percentage of lights made on machine B that are faulty is $k\%$.

Given that a randomly chosen faulty light was made on machine B with probability 0.36,

- show that $k = 3.75$
- find the probability that a randomly chosen light is faulty.

The lights are packed in boxes of 40, chosen at random from all the lights made.

- Find the probability that a box contains more than 2 faulty lights.
- A box is found to contain exactly one faulty light. Find the probability that this light was made on machine A.

(9 marks)

a) Let $d = \frac{k}{100} = P(\text{faulty} | B)$

$$P(F) = 0.5 \times 0.02 + 0.3d + 0.2 \times 0.05$$

$$P(F) = 0.02 + 0.3d$$

$$P(B | F) = \frac{0.3d}{(0.02 + 0.3d)}$$

$$\frac{0.3d}{(0.02 + 0.3d)} = 0.36$$

$$0.3d = 0.0072 + 0.108d$$

$$0.192d = 0.0072$$

$$d = 0.0375$$

$$k = 3.75$$

b)

$$P(F) = 0.02 + 0.3 \times 0.0375$$

$$P(F) = 0.03125$$

c) $X = \text{number of faulty lights in a box}$

$$= X \sim B(40, 0.03125)$$

$$P(X > 2) = 1 - P(X \leq 2)$$

$$P(X > 2) = 1 - 0.87118$$

$$P(X > 2) = 0.129$$

d) The faulty light is one faulty light chosen at random

$$P(A | F) = \frac{0.5 \times 0.02}{0.03125}$$

$$P(A | F) = \frac{0.01}{0.03125}$$

$$P(A | F) = 0.32$$