



1 The mass, X grams, of a chocolate bar made by a machine is modelled by $X \sim N(50, 4^2)$

- Find $P(X > 56)$
- Find $P(46 < X < 53)$
- Find the value of a such that $P(X < a) = 0.85$

(4 marks)

a) $X \sim N(50, 4^2)$, use the calculator

$$P(X > 56) = 0.0668$$

b)

$$P(46 < X < 53) = 0.6147$$

c) Inverse normal, area 0.85 to the left

Or standardise:

$$\frac{(a - 50)}{4} = 1.0364$$

$$a = 50 + 4 \times 1.0364$$

$$a = 54.15$$

2

The random variable $Z \sim N(0, 1^2)$

- Find $P(Z < 1.2)$
- Find $P(-0.5 < Z < 1.5)$
- Find the value of z such that $P(Z > z) = 0.05$
- Find the value of a such that $P(-a < Z < a) = 0.8$

(4 marks)

a)

$$P(Z < 1.2) = 0.8849$$

b)

$$P(-0.5 < Z < 1.5) = 0.6247$$

c) Area 0.95 to the left of z

$$P(Z < z) = 0.95$$

$$z = 1.6449$$

d) Symmetry: $1 - 0.8 = 0.2$ split into two tails

$$P(Z > a) = 0.1$$

$$P(Z < a) = 0.9$$

$$a = 1.2816$$

3

The heights, H cm, of the sunflowers in a field are normally distributed. The curve of the probability density function of H has points of inflection at $H = 150$ and $H = 190$

- Write down the mean and the standard deviation of H
- Write down, to the nearest whole number, the percentage of sunflowers with heights between 130 cm and 210 cm.
- Find $P(H > 200)$
- Find the interquartile range of the heights.

(6 marks)

a) Points of inflection are at $\mu - \sigma$ and $\mu + \sigma$

$$\mu = \frac{(150 + 190)}{2} = 170$$

$$\sigma = 190 - 170 = 20$$

mean 170 cm, s.d. 20 cm

b) $130 = \mu - 2\sigma$ and $210 = \mu + 2\sigma$

about 95%

c) $H \sim N(170, 20^2)$

$$P(H > 200) = 0.0668$$

d) Upper quartile: $P(Z < z) = 0.75$

$$z = 0.6745$$

$$UQ = 170 + 0.6745 \times 20 = 183.49$$

Lower quartile by symmetry

$$LQ = 170 - 0.6745 \times 20 = 156.51$$

$$IQR = 183.49 - 156.51$$

$$IQR = 27.0 \text{ cm}$$

4

A machine cuts wooden dowels. The length, L cm, of a dowel is normally distributed with mean 12 cm and standard deviation σ cm.

Given that 3% of the dowels are shorter than 11.82 cm,

- show that $\sigma = 0.096$ to 3 decimal places.
- Find the proportion of dowels with lengths between 11.9 cm and 12.15 cm.

(5 marks)

a) Standardise: $Z = \frac{(L - \mu)}{\sigma}$

$$P(L < 11.82) = 0.03$$

$$P\left(Z < \frac{(11.82 - 12)}{\sigma}\right) = 0.03$$

Inverse normal on Z , area 0.03 to the left

$$\frac{(11.82 - 12)}{\sigma} = -1.8808$$

$$\frac{-0.18}{\sigma} = -1.8808$$

$$\sigma = \frac{0.18}{1.8808}$$

$$\sigma = 0.0957$$

$$\sigma = 0.096 \text{ (3 d.p.)}$$

b) $L \sim N(12, 0.096^2)$

$$P(11.9 < L < 12.15) = 0.7921$$

5

The times, T minutes, taken by runners to complete a 5 km park run are modelled by a normal distribution with mean μ and standard deviation σ

20% of the runners take less than 25 minutes and 10% of the runners take more than 38 minutes.

- Find the value of μ and the value of σ . Show your working clearly.
- Find the probability that a randomly chosen runner takes between 28 and 32 minutes.

(7 marks)

a) Standardise each percentage

$$P\left(Z < \frac{(25 - \mu)}{\sigma}\right) = 0.2$$

$$\frac{(25 - \mu)}{\sigma} = -0.8416$$

$$25 - \mu = -0.8416\sigma$$

10% above 38, so 90% below

$$\frac{(38 - \mu)}{\sigma} = 1.2816$$

$$38 - \mu = 1.2816\sigma$$

Subtract the equations

$$13 = 2.1232\sigma$$

$$\sigma = 6.1229$$

$$\mu = 25 + 0.8416 \times 6.1229$$

$$\mu = 30.1532$$

$$\mu = 30.2, \sigma = 6.12$$

b) $T \sim N(30.15, 6.12^2)$

$$P(28 < T < 32) = 0.256$$

6

The masses, M grams, of apples from an orchard are normally distributed with mean 150 g and standard deviation 12 g. Apples with masses between 140 g and 165 g are sold to a supermarket.

- Find the proportion of apples that are sold to the supermarket.
- An apple is chosen at random from those sold to the supermarket. Find the probability that its mass is more than 155 g.

(5 marks)

$$a) M \sim N(150, 12^2)$$

$$P(140 < M < 165) = 0.6920$$

b) Conditional probability

$$P(M > 155 \mid 140 < M < 165) = \frac{P(155 < M < 165)}{P(140 < M < 165)}$$

$M > 155$ and $140 < M < 165$ overlap in $155 < M < 165$

$$P(155 < M < 165) = 0.2328$$

$$P(M > 155 \mid 140 < M < 165) = \frac{0.2328}{0.6920}$$

$$P(M > 155 \mid 140 < M < 165) = 0.336$$

7

The mass, F grams, of flour in a bag filled by a machine is modelled by $F \sim N(1010, 8^2)$

- A bag with less than 1000 g cannot be sold and is emptied and refilled, at a loss of 30p.
- A bag with between 1000 g and 1025 g is sold at a profit of 40p.
- A bag with more than 1025 g is sold at a profit of 25p.

- Find the proportion of bags that cannot be sold.
- Find the expected profit for every 1000 bags filled. Give your answer to the nearest pound.

(6 marks)

$$a) F \sim N(1010, 8^2)$$

$$P(F < 1000) = 0.1056$$

b) Probability of each outcome

$$P(1000 < F < 1025) = 0.8640$$

$$P(F > 1025) = 0.0304$$

Expected profit per bag in pounds

$$E = -0.30 \times 0.1056 + 0.40 \times 0.8640 + 0.25 \times 0.0304$$

$$E = -0.0317 + 0.3456 + 0.0076$$

$$E = £0.3215 \text{ per bag}$$

$$1000 \text{ bags: } 1000 \times 0.3215 = £321.49$$

$$£321$$

8

Kemi's journey time to work, J minutes, is modelled by $J \sim N(42, 6^2)$. Her journeys on different days are independent.

- Find the probability that a journey takes more than 50 minutes.
- Find the probability that, in 20 working days, more than 2 of her journeys take more than 50 minutes.
- Kemi wants to arrive on time on 95% of days. Find how many minutes before her start time she should leave, giving your answer to the nearest minute.

(7 marks)

$$a) J \sim N(42, 6^2)$$

$$P(J > 50) = 0.0912$$

$$b) X = \text{number of days with a journey over 50 minutes}$$

$$X \sim B(20, 0.0912)$$

$$P(X > 2) = 1 - P(X \leq 2)$$

$$P(X > 2) = 1 - 0.7267$$

$$P(X > 2) = 0.273$$

$$c) \text{ Need } P(J < t) = 0.95$$

$$\frac{(t - 42)}{6} = 1.6449$$

$$t = 42 + 6 \times 1.6449$$

$$t = 51.87$$

Round up so that at least 95% of journeys fit

Leave 52 minutes before her start time

9

At a café, the temperature of a cup of tea when served, C °C, is modelled by $C \sim N(72, 4^2)$. The waiting time, W minutes, for a cup of tea is modelled by $W \sim N(4.5, 1.2^2)$. A customer is happy if the tea is served at between 65 °C and 80 °C and the waiting time is less than 6 minutes.

- Find the probability that a cup of tea is served at between 65 °C and 80 °C.
- State a necessary assumption about C and W for the manager to calculate the probability that a customer is happy.
- Find the probability that a randomly chosen customer is happy.
- Two customers order tea separately. Find the probability that at least one of them is happy.

(7 marks)

a) $C \sim N(72, 4^2)$

$$P(65 < C < 80) = 0.9372$$

b)

C and W are independent

c) $W \sim N(4.5, 1.2^2)$

$$P(W < 6) = 0.8944$$

Independent, so multiply

$$P(\text{happy}) = 0.9372 \times 0.8944$$

$$P(\text{happy}) = 0.8382$$

$$P(\text{happy}) = 0.838$$

d) Complement: neither is happy

$$P(\text{at least one}) = 1 - (1 - 0.8382)^2$$

$$P(\text{at least one}) = 1 - 0.1618^2$$

$$P(\text{at least one}) = 0.974$$

10

A teacher models the time, T minutes, that students take to finish a short online quiz by $T \sim N(6, 3^2)$

- Find $P(T < 0)$
- Find $P(T < 1 \mid T > 0)$
- Using your answer to part (a), explain why this normal distribution may not be a good model for T

The teacher refines the model by only including values of $T > 0$

- Find the median time to finish the quiz using the refined model.

(8 marks)

a) $T \sim N(6, 3^2)$

$$P(T < 0) = 0.0228$$

b) Conditional probability

$$P(T < 1 \mid T > 0) = \frac{P(0 < T < 1)}{P(T > 0)}$$

$$P(0 < T < 1) = 0.0250$$

$$P(T > 0) = 1 - 0.0228 = 0.9772$$

$$P(T < 1 \mid T > 0) = \frac{0.0250}{0.9772}$$

$$P(T < 1 \mid T > 0) = 0.0256$$

c)

Times cannot be negative, but $P(T < 0)$ is not zero

d) Median m : half of the $T > 0$ probability lies below m

$$P(0 < T < m) = 0.5 \times P(T > 0)$$

$$P(T < m) - 0.0228 = 0.5 \times 0.9772$$

$$P(T < m) = 0.5114$$

Inverse normal

$$\frac{(m - 6)}{3} = 0.0285$$

$$m = 6 + 3 \times 0.0285$$

$$m = 6.09 \text{ minutes}$$

11

A machine fills cups of soup. The volume, V ml, in a cup is normally distributed with mean μ ml and standard deviation 5 ml. Given that 2.5% of cups contain more than 340 ml,

a. find the value of μ

A cup is rejected if it contains less than 320 ml or more than 338 ml.

b. Find the probability that a randomly chosen cup is rejected.

c. A random sample of 12 cups is taken. Find the probability that fewer than 2 of them are rejected.

d. The machine is adjusted so that only 1% of cups contain more than 340 ml. The standard deviation is unchanged. Find the new value of the mean.

(9 marks)

a) 2.5% above 340, so 97.5% below

$$\frac{(340 - \mu)}{5} = 1.9600$$

$$\mu = 340 - 5 \times 1.9600$$

$$\mu = 330.2$$

b) $V \sim N(330.2, 5^2)$

$$P(V < 320) = 0.0207$$

$$P(V > 338) = 0.0594$$

$$P(\text{rejected}) = 0.0207 + 0.0594$$

$$P(\text{rejected}) = 0.0801$$

c) R = number rejected out of 12

$$R \sim B(12, 0.0801)$$

$$P(R < 2) = P(R = 0) + P(R = 1)$$

$$P(R < 2) = 0.9199^{12} + 12 \times 0.0801 \times 0.9199^{11}$$

$$P(R < 2) = 0.3674 + 0.3837$$

$$P(R < 2) = 0.751$$

d) 1% above 340, so 99% below

$$\frac{(340 - \mu)}{5} = 2.3263$$

$$\mu = 340 - 5 \times 2.3263$$

$$\mu = 328.4 \text{ ml}$$

12

A display board uses 4 bulbs. The lifetime, L hours, of a bulb is modelled by $L \sim N(800, 60^2)$. The display stops working as soon as any one of its bulbs fails. The lifetimes of the bulbs are independent.

A display is fitted with 4 new bulbs.

- Find the probability that a bulb lasts more than 750 hours.
- Find the probability that the display is still working after 750 hours.
- Given that the display is still working after 750 hours, find the probability that it is still working after 800 hours.
- Find the time, to the nearest hour, by which there is a 10% chance that the display has stopped working.

(9 marks)

a) $L \sim N(800, 60^2)$

$$P(L > 750) = 0.7977$$

b) All four bulbs must last more than 750 hours

$$P = 0.7977^4$$

$$P = 0.405$$

c) For one bulb: $P(L > 800 \mid L > 750)$

$$P(L > 800 \mid L > 750) = \frac{P(L > 800)}{P(L > 750)}$$

$$P(L > 800 \mid L > 750) = \frac{0.5}{0.7977} = 0.6268$$

All four bulbs

$$P = 0.6268^4$$

$$P = 0.154$$

d) Display still working at time t with probability 0.9

$$P(L > t)^4 = 0.9$$

$$P(L > t) = 0.9^{1/4} = 0.9740$$

$$P(L < t) = 0.0260$$

Inverse normal

$$\frac{(t - 800)}{60} = -1.9432$$

$$t = 800 + 60 \times (-1.9432)$$

$$t = 683.4$$

$$t = 683 \text{ hours}$$