

Connected particles

Worked solutions

A-level Maths topic practice

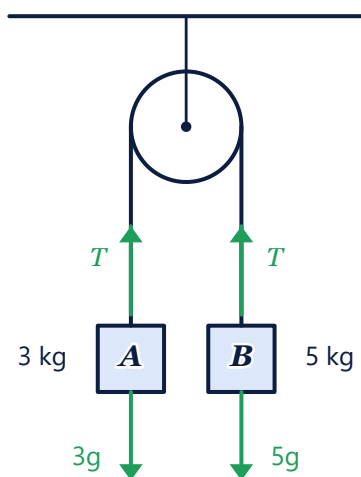


This topic online

1

Two particles A and B , of mass 3 kg and 5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed to a ceiling. The particles hang with the string taut and the hanging parts of the string vertical, as shown in the diagram. The system is released from rest.

- Find the acceleration of A .
- Find the tension in the string.
- Find the magnitude of the force exerted on the pulley by the string.



(6 marks)

a) B is heavier: B moves down, A moves up

Inextensible string: same acceleration a . Light string: same tension T

$F = ma$ for A (up positive)

$$T - 3g = 3a$$

$F = ma$ for B (down positive)

$$5g - T = 5a$$

Add the two equations to remove T

$$2g = 8a$$

$$a = \frac{2 \times 9.8}{8}$$

$$a = 2.45$$

$$\boxed{2.45 \text{ m s}^{-2}}$$

b) Substitute into the equation for A

$$T = 3g + 3a$$

$$T = 3(9.8 + 2.45)$$

$$T = 36.75$$

$$36.8 \text{ N}$$

c) Two vertical strands both pull the pulley down

$$\text{force} = 2T$$

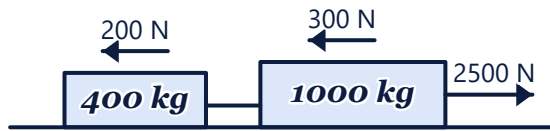
$$\text{force} = 2 \times 36.75 = 73.5$$

$$73.5 \text{ N}$$

2

A car of mass 1000 kg tows a trailer of mass 400 kg along a straight horizontal road using a light horizontal tow bar. The engine of the car produces a driving force of 2500 N. The resistances to motion are constant forces of 300 N on the car and 200 N on the trailer, as shown in the diagram.

- Find the acceleration of the car and trailer.
- Find the tension in the tow bar.
- State how you have used the modelling assumption that the tow bar is light.



(6 marks)

a) Whole system: the tension is internal and cancels

$$2500 - 300 - 200 = 1400a$$

$$2000 = 1400a$$

÷1400

$$a = 1.4285... \quad \div 1400$$

$$\boxed{1.43 \text{ m s}^{-2}}$$

b) Trailer alone: T forwards, 200 N backwards

$$T - 200 = 400a$$

$$T - 200 = 400 \times \frac{10}{7}$$

$$T - 200 = 571.42...$$

$$T = 771.42...$$

Check with the car: $2500 - 300 - 771.4 = 1428.6 = 1000a$

$$\boxed{771 \text{ N}}$$

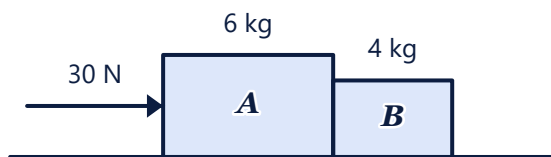
c) Light tow bar

Its mass is ignored, so the tension is the same at both ends

3

Two boxes A and B , of mass 6 kg and 4 kg respectively, rest side by side in contact on a smooth horizontal floor. A horizontal force of magnitude 30 N is applied to A , pushing it into B , as shown in the diagram. The boxes move together.

- Find the acceleration of the boxes.
- Find the magnitude of the force exerted on B by A .



(4 marks)

a) Both boxes together, total mass 10 kg

$$\begin{array}{rcl} 30 & = & 10a \\ \div 10 & & \div 10 \\ a & = & 3 \end{array}$$

$$\boxed{3\text{ m s}^{-2}}$$

b) B alone: the only horizontal force on B is R from A

$$R = 4a$$

$$R = 4 \times 3$$

$$R = 12$$

Check with A : $30 - 12 = 18 = 6 \times 3$

$$\boxed{12\text{ N}}$$

4

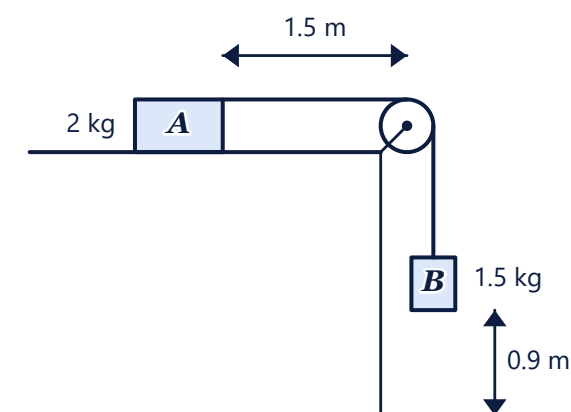
A particle A of mass 2 kg rests on a smooth horizontal table. It is attached to one end of a light inextensible string. The string passes over a small smooth pulley fixed at the edge of the table, and its other end is attached to a particle B of mass 1.5 kg which hangs freely. A is 1.5 m from the pulley and B is 0.9 m above the floor, as shown in the diagram.

The system is released from rest with the string taut.

- Find the acceleration of A and the tension in the string.
- Find the speed of B as it hits the floor.

B does not rebound.

- Find the time from the moment B hits the floor until A reaches the pulley.



(9 marks)

a) $F = ma$ for A (horizontal, smooth table)

$$T = 2a$$

$F = ma$ for B (down positive)

$$1.5g - T = 1.5a$$

Add to remove T

$$1.5g = 3.5a$$

$$14.7 = 3.5a$$

$\div 3.5$

$$a = 4.2$$

$\div 3.5$

$$T = 2 \times 4.2 = 8.4$$

$$a = 4.2\text{ m s}^{-2}, T = 8.4\text{ N}$$

b) B : $u = 0$, $a = 4.2$, $s = 0.9$, use $v^2 = u^2 + 2as$

$$v^2 = 0^2 + 2(4.2)(0.9)$$

$$v^2 = 7.56$$

$$v = 2.749\dots$$

$$2.75 \text{ m s}^{-1}$$

c) String goes slack, no friction: A moves at constant speed
A has already moved 0.9 m, so the distance left is

$$1.5 - 0.9 = 0.6$$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{time} = \frac{0.6}{2.749...}$$

$$\text{time} = 0.2182...$$

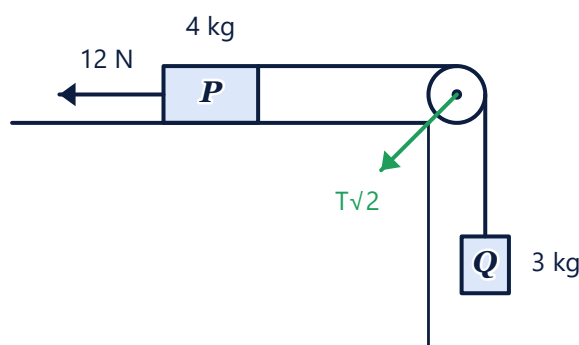
$$0.218 \text{ s}$$

5

A particle P of mass 4 kg lies on a rough horizontal table. P is attached to one end of a light inextensible string which passes over a small smooth pulley fixed at the edge of the table. The other end of the string is attached to a particle Q of mass 3 kg which hangs freely below the pulley. The part of the string between P and the pulley is horizontal.

The system is released from rest with the string taut. As P moves, the table exerts a constant resistance of magnitude 12 N on P , as shown in the diagram.

- Find the acceleration of P .
- Find the tension in the string.
- Find the magnitude and direction of the resultant force exerted on the pulley by the string.



(8 marks)

a) $F = ma$ for P (towards the pulley positive)

$$T - 12 = 4a$$

$F = ma$ for Q (down positive)

$$3g - T = 3a$$

Add to remove T

$$3g - 12 = 7a$$

$$29.4 - 12 = 7a$$

$$17.4 = 7a$$

$\div 7$

$$a = 2.4857... \quad \div 7$$

$$\boxed{2.49 \text{ m s}^{-2}}$$

b) Substitute into the equation for P

$$T = 12 + 4 \times 2.4857...$$

$$T = 21.942...$$

$$\boxed{21.9 \text{ N}}$$

c) The pulley feels T horizontally and T vertically down

$$\text{force} = \sqrt{T^2 + T^2}$$

$$\text{force} = T\sqrt{2}$$

$$\text{force} = 21.942... \times \sqrt{2} = 31.03...$$

Equal components, so the angle is 45°

31.0 N at 45° below the horizontal

6

Two particles P and Q have masses 1.2 kg and m kg respectively. They are attached to the ends of a light inextensible string which passes over a small smooth peg. The particles hang freely with the string taut and the hanging parts of the string vertical.

The system is released from rest and Q moves downwards with acceleration 2.8 m s^{-2} .

- Find the tension in the string.
- Show that $m = 2.16$
- Find the magnitude of the force exerted on the peg by the string.
- State where in your working you have used the fact that the peg is smooth.

(7 marks)

a) P moves up with $a = 2.8$. $F = ma$ for P (up positive)

$$T - 1.2g = 1.2 \times 2.8$$

$$T = 1.2(9.8 + 2.8)$$

$$T = 15.12$$

15.1 N

b) $F = ma$ for Q (down positive)

$$mg - T = 2.8m$$

$$9.8m - 15.12 = 2.8m$$

$$7m = 15.12$$

$$\div 7 \quad m = 2.16 \quad \div 7$$

$m = 2.16$ as required

c) Both strands pull the peg vertically down

$$\text{force} = 2T$$

$$\text{force} = 2 \times 15.12 = 30.24$$

30.2 N

d) Smooth peg

The same T is used in the equations for P and Q

7

A car of mass 1200 kg is towing a trailer of mass 600 kg along a straight horizontal road. The car and trailer are joined by a light rigid horizontal tow bar. The resistances to motion are constant forces of 400 N on the car and 200 N on the trailer.

The car and trailer are moving at 20 m s^{-1} when the driver applies the brakes. The brakes produce a constant braking force of $F \text{ N}$ on the car only. The car and trailer come to rest in a distance of 80 m.

- Find the deceleration of the car and trailer.
- Find the value of F .
- Find the magnitude of the force in the tow bar and state whether it is a tension or a thrust.
- Explain why the trailer could not have been towed with a rope during the braking.

(8 marks)

a) $u = 20, v = 0, s = 80$, use $v^2 = u^2 + 2as$

$$0 = 20^2 + 2a(80)$$

$$-400 = 160a$$

$$a = -2.5$$

$$2.5 \text{ m s}^{-2}$$

b) Whole system, backwards positive

$$F + 400 + 200 = 1800 \times 2.5$$

$$F + 600 = 4500$$

-600

$$F = 3900$$

-600

$$F = 3900$$

c) Trailer alone, backwards positive. Let the tow bar push back with S

$$200 + S = 600 \times 2.5$$

$$200 + S = 1500$$

-200

$$S = 1300$$

-200

$S > 0$, so the bar pushes: it is a thrust

Check with the car: $3900 + 400 - 1300 = 3000 = 1200 \times 2.5$

$$1300 \text{ N, a thrust}$$

d) Why not a rope

A rope can only pull (tension), it cannot push,

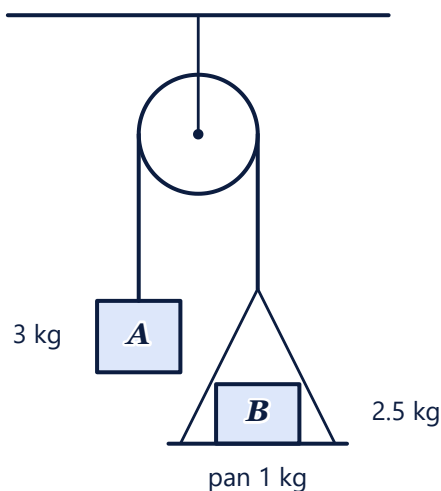
so it would go slack and the trailer would run into the car

8

A particle A of mass 3 kg is attached to one end of a light inextensible string. The string passes over a small smooth fixed pulley and its other end is attached to a scale pan of mass 1 kg. A block B of mass 2.5 kg rests in the scale pan, as shown in the diagram. The hanging parts of the string are vertical.

The system is released from rest.

- Find the acceleration of A .
- Find the tension in the string.
- Find the magnitude of the force exerted on the scale pan by B .



(8 marks)

a) Pan + B = 3.5 kg > 3 kg, so the pan goes down and A goes up

$F = ma$ for A (up positive)

$$T - 3g = 3a$$

$F = ma$ for the pan and B together (down positive)

$$3.5g - T = 3.5a$$

Add to remove T

$$0.5g = 6.5a$$

$$a = \frac{0.5 \times 9.8}{6.5}$$

$$a = 0.7538...$$

$$\boxed{0.754 \text{ m s}^{-2}}$$

b) Substitute into the equation for A

$$T = 3(9.8 + 0.7538...)$$

$$T = 31.66...$$

$$31.7 \text{ N}$$

c) B alone: weight $2.5g$ down, reaction R from the pan up

$$2.5g - R = 2.5a$$

$$R = 2.5(9.8 - 0.7538\dots)$$

$$R = 22.61\dots$$

Newton's third law: B pushes on the pan with the same force

$$22.6 \text{ N}$$

9

A lift of mass 400 kg is raised and lowered by a vertical cable. A parcel of mass 8 kg rests on the floor of the lift. At one moment the normal reaction of the floor on the parcel has magnitude 70 N .

- Find the magnitude and direction of the acceleration of the lift at this moment.
- Find the tension in the cable at this moment.
- Ella says the lift must be moving downwards. Explain why she may be wrong.

(7 marks)

a) Parcel: weight $8g = 78.4 \text{ N} > 70 \text{ N}$, so a is downwards

$F = ma$ for the parcel (down positive)

$$\begin{array}{rcl} 8g - 70 & = & 8a \\ 78.4 - 70 & = & 8a \\ 8.4 & = & 8a \\ \div 8 & & \div 8 \\ a & = & 1.05 \end{array}$$

$$1.05 \text{ m s}^{-2} \text{ downwards}$$

b) Lift and parcel together, 408 kg (down positive)

$$408g - T = 408 \times 1.05$$

$$T = 408(9.8 - 1.05)$$

$$T = 408 \times 8.75$$

$$T = 3570$$

$$3570 \text{ N}$$

c) Acceleration is not the direction of motion

The lift could be moving upwards

and slowing down

10

A car of mass 900 kg tows a boat trailer of mass 600 kg along a straight horizontal road using a light inextensible horizontal tow rope. The resistances to motion are constant forces of 250 N on the car and 350 N on the trailer. The car and trailer start from rest and accelerate at 0.8 m s^{-2} .

- Find the driving force produced by the engine of the car.
- Find the tension in the tow rope.

After 10 seconds the tow rope breaks. The driving force and the resistances do not change.

- Find the distance travelled by the trailer from the moment the rope breaks until it comes to rest.
- Find the distance travelled by the car in the same time.

(9 marks)

a) Whole system: tension is internal

$$\begin{array}{rcl}
 0 - 250 - 350 & = & 1500 \times 0.8 \\
 0 - 600 & = & 1200 \\
 +600 & & \\
 0 & = & 1800 \quad +600
 \end{array}$$

1800 N

b) Trailer alone

$$\begin{array}{rcl}
 T - 350 & = & 600 \times 0.8 \\
 T - 350 & = & 480 \\
 +350 & & \\
 T & = & 830 \quad +350
 \end{array}$$

830 N

c) Speed when the rope breaks: $v = u + at$

$$v = 0 + 0.8 \times 10 = 8$$

Trailer: only its resistance acts

$$-350 = 600a$$

$$a = -\frac{7}{12}$$

$u = 8, v = 0$, use $v^2 = u^2 + 2as$

$$0 = 8^2 + 2\left(-\frac{7}{12}\right)s$$

$$\frac{7}{6}s = 64$$

$$s = 54.857...$$

54.9 m

d) Time for the trailer to stop: $v = u + at$

$$0 = 8 - \frac{7}{12}t$$

$$t = \frac{96}{7} = 13.71\dots$$

Car alone: driving force and its own resistance

$$1800 - 250 = 900a$$

$$a = 1.7222\dots$$

$u = 8$, $t = 13.71\dots$, use $s = ut + \frac{1}{2}at^2$

$$s = 8(13.71\dots) + \frac{1}{2}(1.7222\dots)(13.71\dots)^2$$

$$s = 109.71\dots + 161.95\dots$$

$$s = 271.67\dots$$

272 m

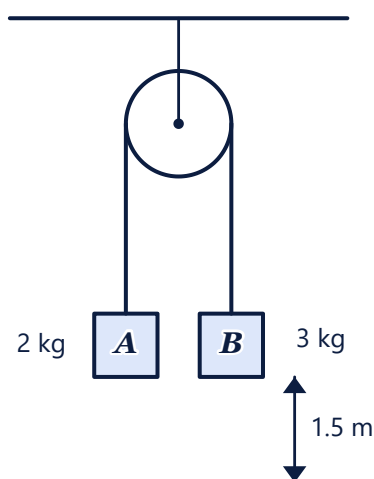
11

Particles A and B , of mass 2 kg and 3 kg respectively, are attached to the ends of a light inextensible string which passes over a small smooth fixed pulley. The particles hang with the string taut and the hanging parts vertical. B is 1.5 m above the ground, as shown in the diagram. The system is released from rest.

- Find the tension in the string while both particles are moving.
- Find the speed of B as it reaches the ground.

B does not rebound. A does not reach the pulley.

- Find the greatest height of A above its starting position.
- Find the time from release until A reaches its greatest height.



(9 marks)

a) $F = ma$ for A (up positive)

$$T - 2g = 2a$$

$F = ma$ for B (down positive)

$$3g - T = 3a$$

Add to remove T

$$g = 5a$$

$$a = 1.96$$

$$T = 2(9.8 + 1.96)$$

$$T = 23.52$$

$$\boxed{23.5\text{ N}}$$

b) B : $u = 0$, $a = 1.96$, $s = 1.5$, use $v^2 = u^2 + 2as$

$$v^2 = 0^2 + 2(1.96)(1.5)$$

$$v^2 = 5.88$$

$$v = 2.424...$$

$$\boxed{2.42 \text{ m s}^{-1}}$$

c) A has risen 1.5 m. String now slack: A moves freely under gravity

$$u = 2.424..., v = 0, a = -9.8$$

$$0 = 2.424... - 2(9.8)s$$

$$19.6s = 2.424...$$

$$s = 0.1237...$$

$$\text{height} = 1.5 + 0.1237...$$

$$\boxed{1.62 \text{ m}}$$

d) Time for each stage, $v = u + at$

$$t_1 = \frac{2.424...}{9.8} = 0.2474...$$

$$t_2 = \frac{2.424...}{9.8} = 0.2474...$$

$$\text{total} = 0.2474... + 0.2474...$$

$$\text{total} = 0.4948...$$

$$\boxed{0.49 \text{ s}}$$

12

Amara, of mass 60 kg, stands on a set of bathroom scales on the floor of a lift. The scales measure the normal reaction on Amara, in newtons. The lift has mass 640 kg, including the scales, and is moved by a vertical cable.

The lift starts from rest and moves upwards. It accelerates uniformly to a speed of 3 m s^{-1} in 2 seconds, travels at 3 m s^{-1} for 5 seconds, then decelerates uniformly to rest in 3 seconds.

- Find the reading on the scales during each of the three stages.
- Find the greatest tension in the cable during the journey.
- Find the total distance travelled by the lift.

(9 marks)

a) Stage 1: $v = u + at$

$$3 = 0 + 2a$$

$$a = 1.5$$

$F = ma$ for Amara (up positive)

$$R - 60g = 60 \times 1.5$$

$$R = 60(9.8 + 1.5) = 678$$

Stage 2: constant speed, $a = 0$

$$R = 60g = 588$$

Stage 3: $v = u + at$

$$0 = 3 + 3a$$

$$a = -1$$

$$R - 60g = 60 \times (-1)$$

$$R = 60(9.8 - 1) = 528$$

$$\boxed{678 \text{ N}, 588 \text{ N}, 528 \text{ N}}$$

b) Greatest tension when the upward acceleration is greatest: stage 1

Lift and Amara together: $640 + 60 = 700 \text{ kg}$

$$T - 700g = 700 \times 1.5$$

$$T = 700(9.8 + 1.5)$$

$$T = 7910$$

$$\boxed{7910 \text{ N}}$$

c) Distance in each stage: $s = \frac{1}{2}(u + v)t$

$$s = \frac{1}{2}(0 + 3)(2) + 3 \times 5 + \frac{1}{2}(3 + 0)(3)$$

$$s = 3 + 15 + 4.5$$

$$\boxed{22.5 \text{ m}}$$