

Motion graphs

Worked solutions

A-level Maths topic practice

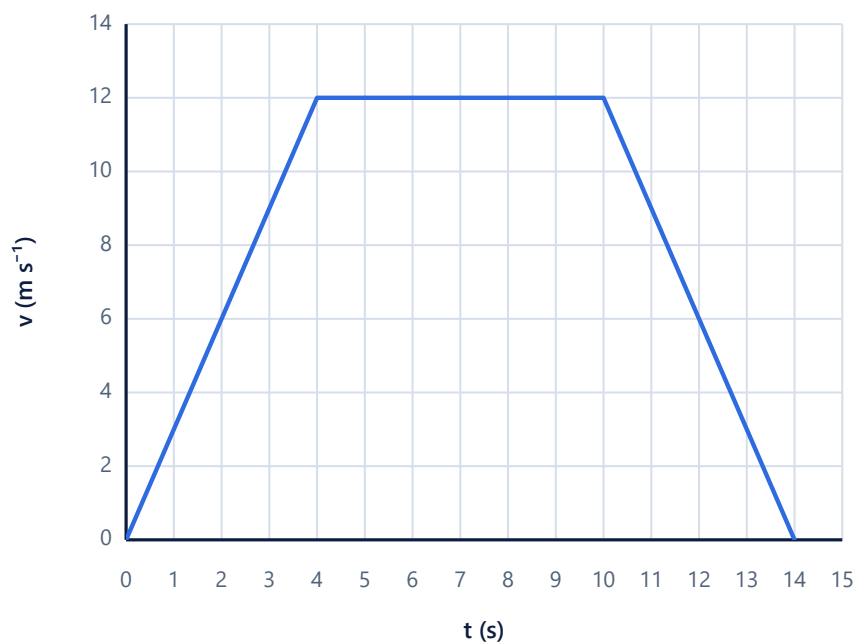


This topic online

1

The velocity-time graph shows the motion of a scooter along a straight road for 14 s.

- Find the acceleration of the scooter in the first 4 s.
- Find the deceleration of the scooter in the last 4 s.
- Find the total distance travelled by the scooter.



(4 marks)

a) Acceleration = gradient

$$a = \frac{12 - 0}{4 - 0}$$

$$= 3 \text{ m s}^{-2}$$

$$\boxed{3 \text{ m s}^{-2}}$$

b) Gradient from $t = 10$ to $t = 14$

$$\text{gradient} = \frac{0 - 12}{14 - 10}$$

$$= -3$$

$$\boxed{\text{deceleration } 3 \text{ m s}^{-2}}$$

c) Distance = area under the graph (trapezium)

parallel sides: 6 s (top) and 14 s (bottom), height 12

$$\text{distance} = \frac{1}{2}(6 + 14) \times 12$$

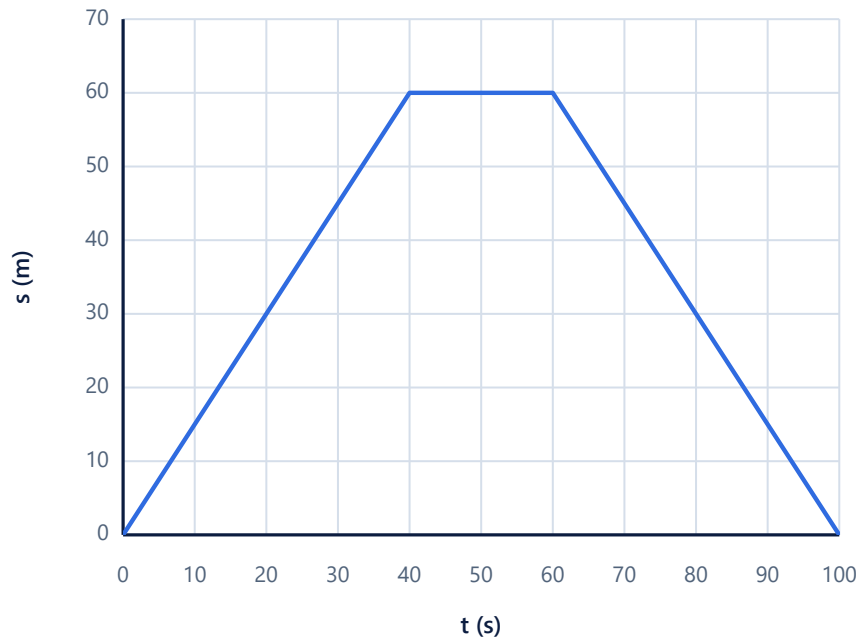
$$= 120 \text{ m}$$

$$\boxed{120 \text{ m}}$$

2

The displacement-time graph shows a dog running away from its owner along a straight path and then back again.

- Find the velocity of the dog in the first 40 s.
- Describe the motion of the dog between $t = 40$ and $t = 60$
- Find the velocity of the dog between $t = 60$ and $t = 100$
- Find the average speed and the average velocity of the dog for the whole 100 s.



(4 marks)

a) Velocity = gradient of s-t graph

$$v = \frac{60}{40}$$

$$= 1.5 \text{ m s}^{-1}$$

$$\boxed{1.5 \text{ m s}^{-1}}$$

b) Horizontal line

Displacement stays 60 m, so the dog is at rest

$\boxed{\text{at rest, 60 m away}}$

c) Gradient from $t = 60$ to $t = 100$

$$v = \frac{0 - 60}{100 - 60}$$

$$= -1.5 \text{ m s}^{-1}$$

$$\boxed{-1.5 \text{ m s}^{-1}}$$

d) Distance out and back

$$\text{total distance} = 60 + 60$$

$$= 120 \text{ m}$$

$$\begin{aligned}\text{average speed} &= \frac{120}{100} \\ &= 1.2 \text{ m s}^{-1}\end{aligned}$$

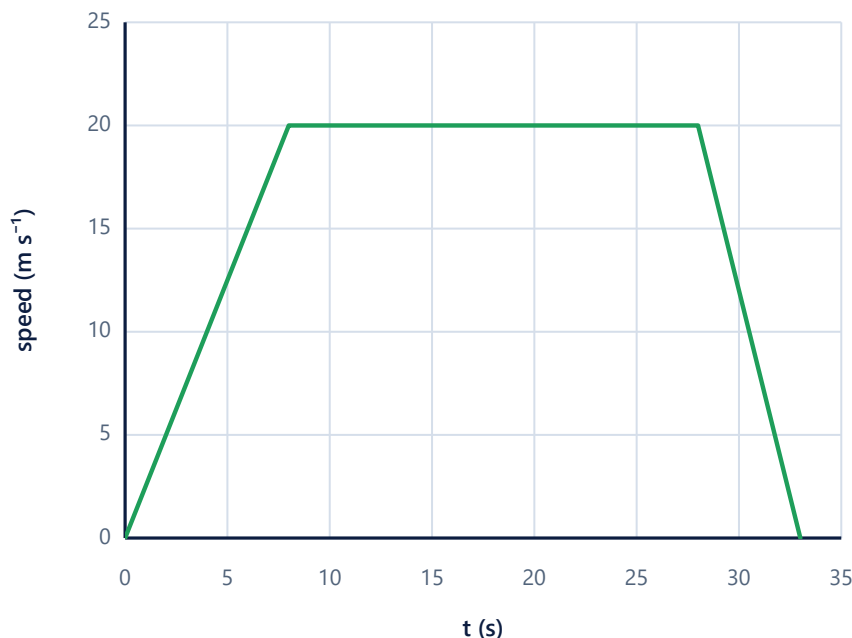
$$\text{Final displacement} = 0$$

$$\begin{aligned}\text{average velocity} &= \frac{0}{100} \\ &= 0\end{aligned}$$

1.2 m s^{-1} and 0

- 3 A car starts from rest and accelerates uniformly to 20 m s^{-1} in 8 s. It travels at 20 m s^{-1} for 20 s and then decelerates uniformly to rest in 5 s.

- Sketch a speed-time graph for the journey on the axes.
- Find the total distance travelled.



(4 marks)

a) Times: 8 s, then to $8 + 20 = 28$ s, then to $28 + 5 = 33$ s

line up to (8, 20), flat to (28, 20), down to (33, 0)

(see the graph)

b) Distance = area under the graph

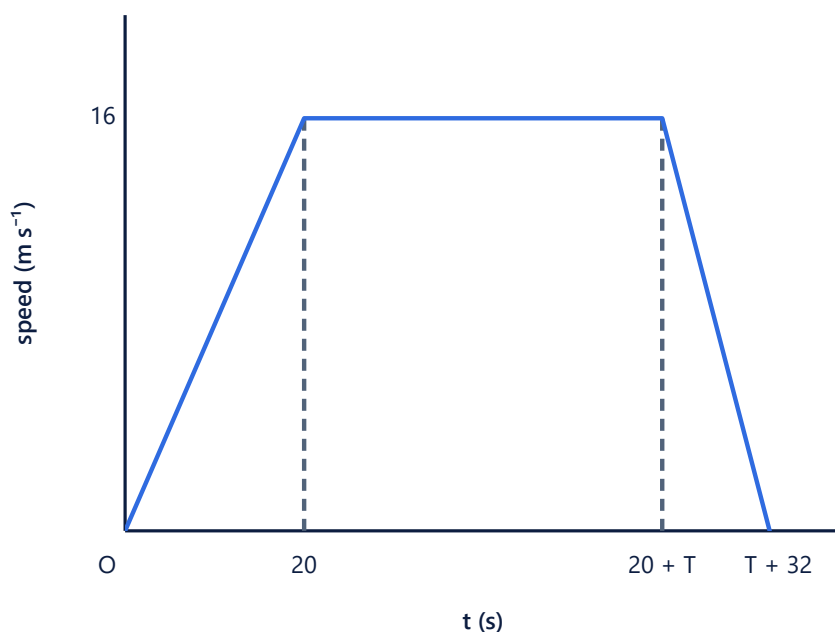
$$\begin{aligned}\text{distance} &= \frac{1}{2} \times 8 \times 20 + 20 \times 20 + \frac{1}{2} \times 5 \times 20 \\ &= 80 + 400 + 50 \\ &= 530 \text{ m}\end{aligned}$$

530 m

4

A train starts from rest at a station and accelerates uniformly to 16 m s^{-1} in 20 s. It then travels at 16 m s^{-1} for T seconds and then decelerates uniformly to rest at the next station in 12 s. The speed-time graph is shown. The distance between the stations is 1600 m.

Find the value of T .



(5 marks)

Distance = area of the trapezium

top = T, bottom = T + 32, height = 16

$$\frac{1}{2}(T + T + 32) \times 16 = 1600$$

$$8(2T + 32) = 1600$$

$$16T + 256 = 1600$$

$$16T = 1344$$

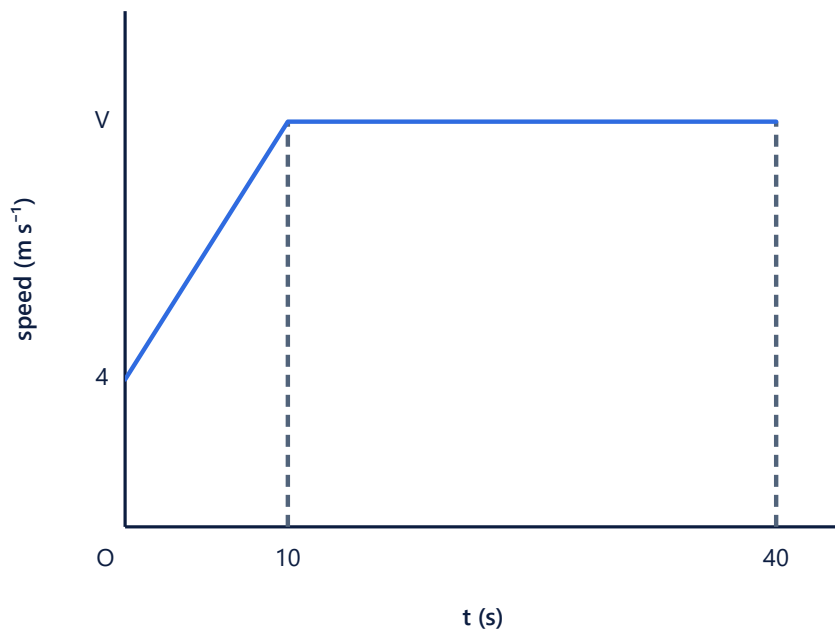
$$T = 84$$

$T = 84$

5

A cyclist passes a point A with speed 4 m s^{-1} . She accelerates uniformly for 10 s to a speed of $V \text{ m s}^{-1}$, and then travels at this speed for a further 30 s , when she reaches the point B . The distance AB is 405 m . The speed-time graph is shown.

- Find the value of V .
- Find the acceleration of the cyclist in the first 10 s .



(6 marks)

a) Distance = area under the graph

$$\begin{aligned} \text{trapezium} &= \frac{1}{2}(4 + V) \times 10 \\ &= 20 + 5V \end{aligned}$$

$$\text{rectangle} = 30V$$

$$20 + 5V + 30V = 405$$

$$35V = 385$$

$$V = 11$$

$$\boxed{V = 11}$$

b) Acceleration = gradient

$$a = \frac{11 - 4}{10}$$

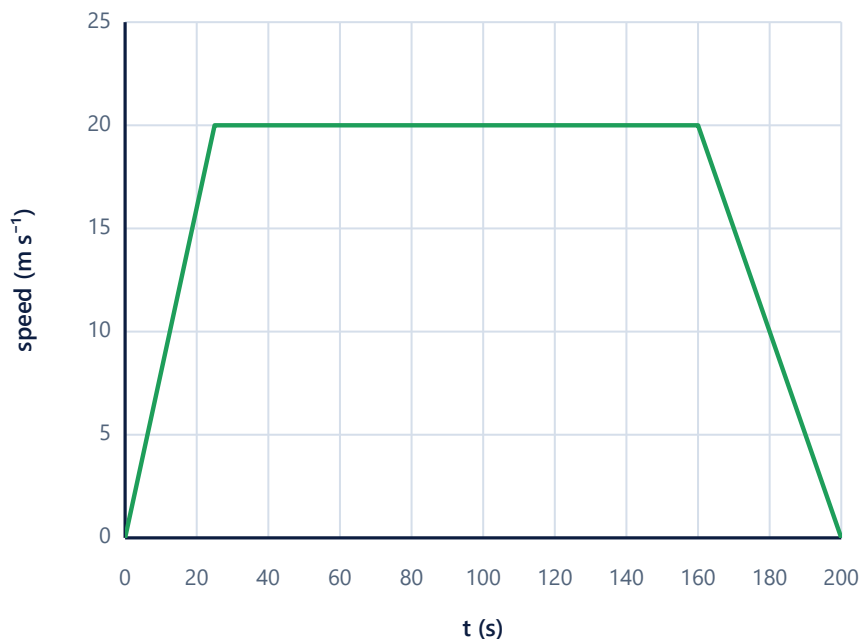
$$= 0.7 \text{ m s}^{-2}$$

$$\boxed{0.7 \text{ m s}^{-2}}$$

6

A tram starts from rest and accelerates uniformly at 0.8 m s^{-2} until it reaches a speed of 20 m s^{-1} . It travels at this speed and then decelerates uniformly at 0.5 m s^{-2} until it stops. The whole journey takes 200 s.

- Sketch a speed-time graph for the journey.
- Find the time for which the tram travels at constant speed.
- Find the total distance travelled.



(6 marks)

Time for each sloping part = change in speed $\div$ acceleration

$$\begin{aligned} \text{time accelerating} &= \frac{20}{0.8} \\ &= 25 \text{ s} \end{aligned}$$

$$\begin{aligned} \text{time decelerating} &= \frac{20}{0.5} \\ &= 40 \text{ s} \end{aligned}$$

a) Up to (25, 20), flat, down from (160, 20) to (200, 0)

(see the graph)

b) Constant speed time

$$200 - 25 - 40 = 135 \text{ s}$$

135 s

c) Area under the graph

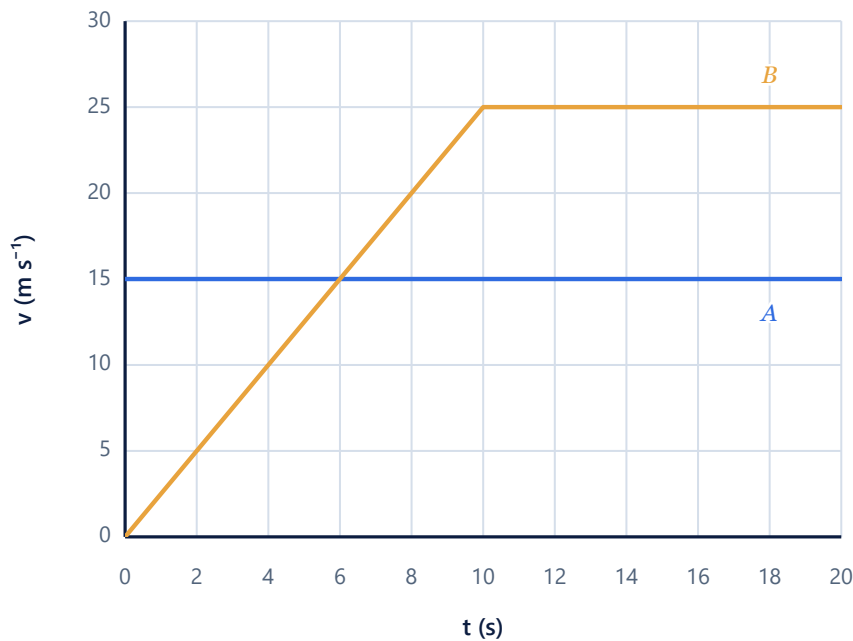
$$\begin{aligned} \text{distance} &= \frac{1}{2} \times 25 \times 20 + 135 \times 20 + \frac{1}{2} \times 40 \times 20 \\ &= 250 + 2700 + 400 \\ &= 3350 \text{ m} \end{aligned}$$

3350 m

7

Car A moves along a straight road at a constant speed of 15 m s^{-1} . At time $t = 0$ it passes car B, which is at rest. At that instant B sets off in the same direction, accelerating uniformly to 25 m s^{-1} in 10 s and then keeping this speed. The velocity-time graphs of both cars are shown.

- Find the time at which the cars have the same speed.
- Find the greatest distance by which A is ahead of B.
- Find the time at which B overtakes A.



(7 marks)

a) Acceleration of B = gradient

$$a = \frac{25}{10}$$

$$= 2.5 \text{ m s}^{-2}$$

$$2.5t = 15$$

$$t = 6 \text{ s}$$

$$t = 6 \text{ s}$$

b) A is furthest ahead when the speeds are equal

$$A: 15 \times 6 = 90 \text{ m}$$

$$B: \frac{1}{2} \times 6 \times 15 = 45 \text{ m}$$

$$\text{gap} = 90 - 45$$

$$= 45 \text{ m}$$

$$45 \text{ m}$$

c) Check $t = 10$ first

$$A: 15 \times 10 = 150 \text{ m}$$

$$B: \frac{1}{2} \times 10 \times 25 = 125 \text{ m}$$

B is still behind, so it overtakes after $t = 10$

$$125 + 25(t - 10) = 15t$$

$$25t - 125 = 15t$$

$$10t = 125$$

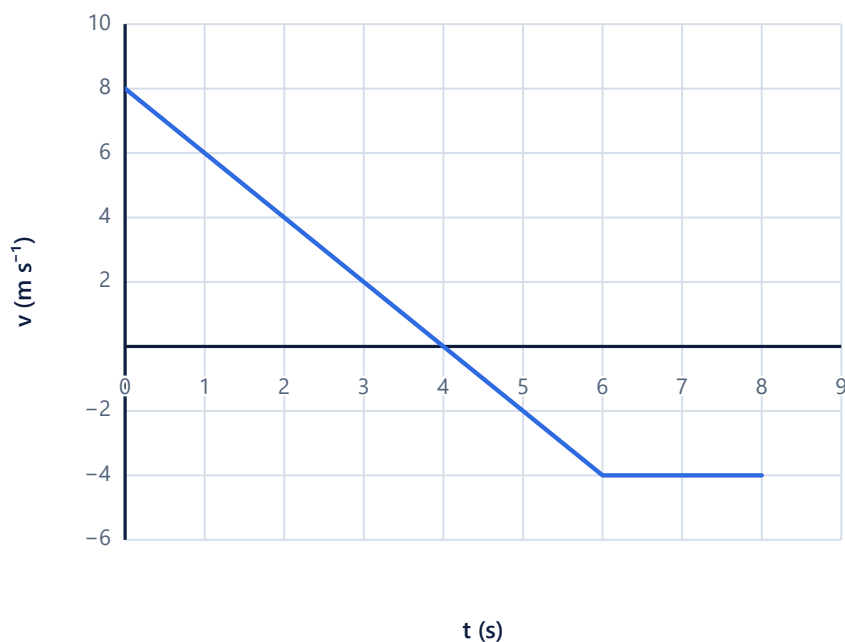
$$t = 12.5 \text{ s}$$

$t = 12.5 \text{ s}$

8

The velocity-time graph shows the motion of a particle moving along a straight line for 8 s.

- Find the acceleration of the particle in the first 6 s.
- Find the displacement of the particle from its starting point after 8 s.
- Find the total distance travelled by the particle in the 8 s.



(6 marks)

a) Gradient from $t = 0$ to $t = 6$

$$a = \frac{-4 - 8}{6}$$

$$= -2 \text{ m s}^{-2}$$

$$\boxed{-2 \text{ m s}^{-2}}$$

b) The graph crosses the axis at $t = 4$

$$\text{area above} = \frac{1}{2} \times 4 \times 8$$

$$= 16$$

$$\text{area below} = \frac{1}{2} \times 2 \times 4 + 2 \times 4$$

$$= 12$$

Area below the axis is negative displacement

$$\text{displacement} = 16 - 12$$

$$= 4 \text{ m}$$

$$\boxed{4 \text{ m}}$$

c) Distance adds both areas

$$\text{distance} = 16 + 12$$

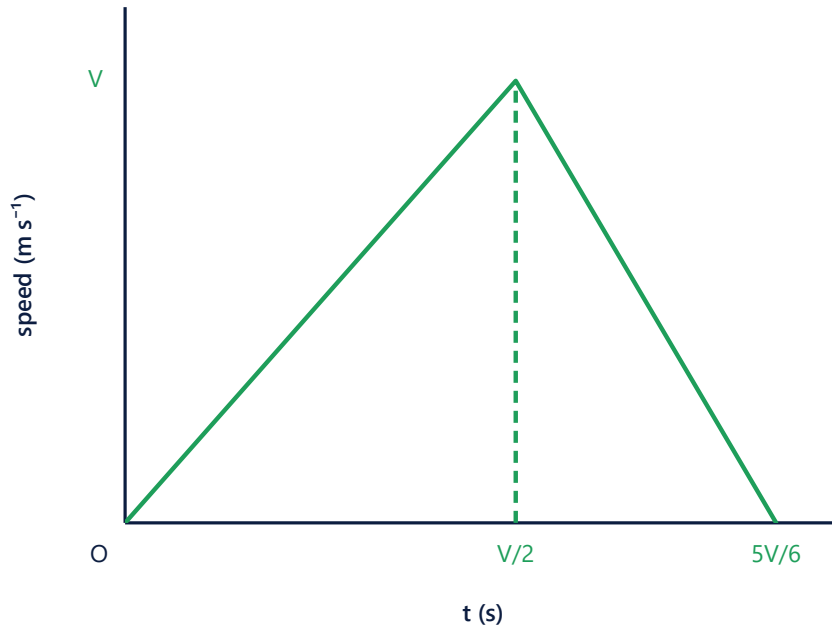
$$\boxed{28 \text{ m}}$$

$$= 28 \text{ m}$$

9

A car starts from rest at A and accelerates at 2 m s^{-2} until it reaches a speed of $V \text{ m s}^{-1}$. It then immediately decelerates at 3 m s^{-2} until it stops at B . The distance AB is 240 m .

- Sketch a speed-time graph for the motion.
- Find, in terms of V , the total time from A to B .
- Find the value of V and the total time.



(6 marks)

a) Triangle: up to V , then straight down to 0
(see the graph)

b) Time = change in speed $\div$ acceleration

$$\text{time accelerating} = \frac{V}{2}$$

$$\text{time braking} = \frac{V}{3}$$

$$\begin{aligned} \text{total} &= \frac{V}{2} + \frac{V}{3} \\ &= \frac{5V}{6} \end{aligned}$$

$$\boxed{\frac{5V}{6} \text{ s}}$$

c) Distance = area of the triangle

$$\frac{1}{2} \times \frac{5V}{6} \times V = 240$$

$$\frac{5V^2}{12} = 240$$

$$V^2 = 576$$

$$V = 24$$

$$\text{total time} = \frac{5 \times 24}{6}$$

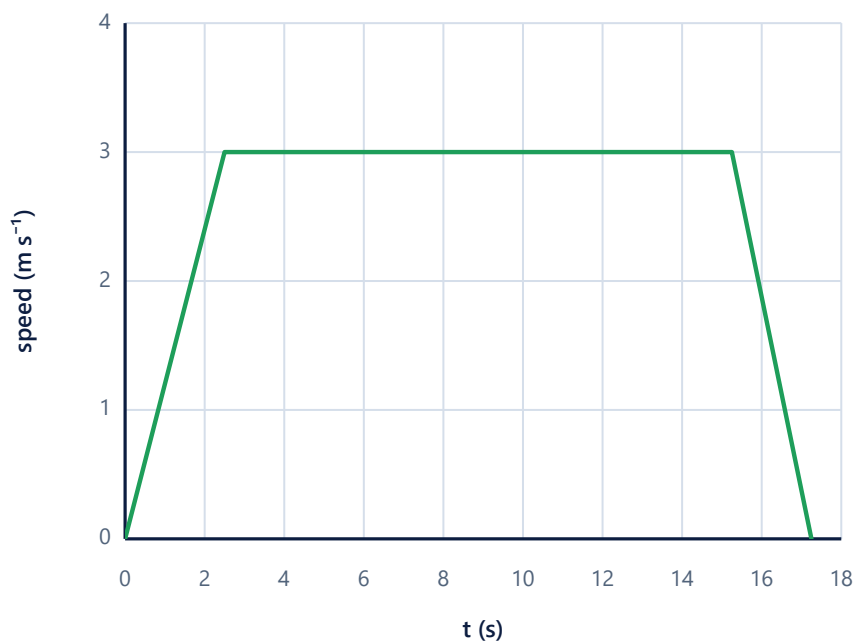
$$= 20 \text{ s}$$

$$V = 24, 20 \text{ s}$$

10

A lift rises from rest. It accelerates uniformly at 1.2 m s^{-2} for 2.5 s, then moves at constant speed, and then decelerates uniformly to rest in 2 s. The lift rises 45 m in total.

- Find the greatest speed of the lift.
- Sketch a speed-time graph for the motion of the lift.
- Find the time for which the lift moves at constant speed, and the total time for the journey.



(6 marks)

$$a) v = u + at$$

$$v = 0 + 1.2 \times 2.5$$

$$= 3 \text{ m s}^{-1}$$

$$\boxed{3 \text{ m s}^{-1}}$$

b) Up to 3 at $t = 2.5$, flat, then down to 0 in 2 s

(see the graph)

c) Let the constant speed time be c . Area = 45

$$\frac{1}{2} \times 2.5 \times 3 + 3c + \frac{1}{2} \times 2 \times 3 = 45$$

$$3.75 + 3c + 3 = 45$$

$$3c = 38.25$$

$$c = 12.75 \text{ s}$$

$$\text{total} = 2.5 + 12.75 + 2$$

$$= 17.25 \text{ s}$$

$$\boxed{12.75 \text{ s and } 17.25 \text{ s}}$$

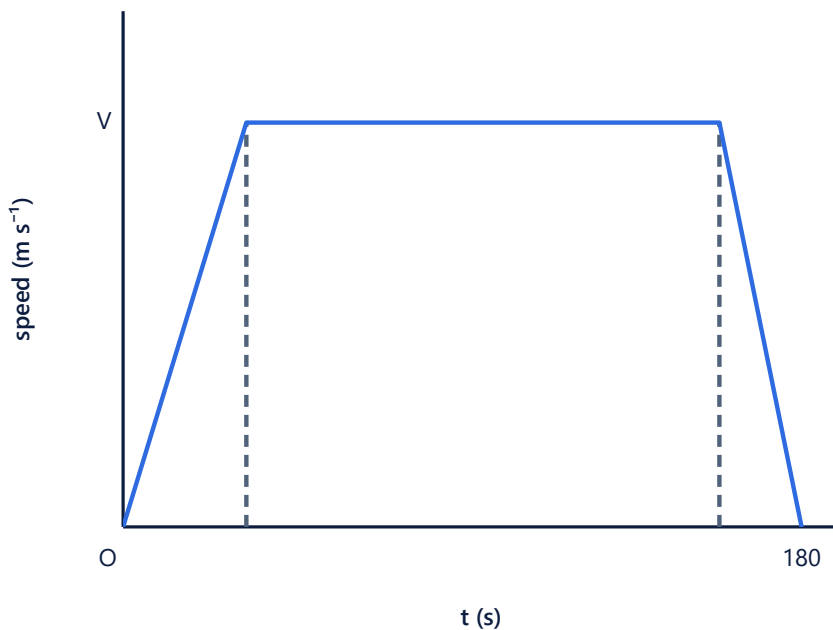
11

A train travels between two stations 3000 m apart. It starts from rest and accelerates uniformly at 0.6 m s^{-2} until it reaches a speed of $V \text{ m s}^{-1}$. It travels at this speed and then decelerates uniformly at 0.9 m s^{-2} until it comes to rest at the second station. The whole journey takes 180 s. The speed-time graph is shown.

a. Show that the total time spent accelerating and decelerating is $\frac{25V}{9}$ seconds.

b. Show that $25V^2 - 3240V + 54000 = 0$

c. Find the value of V , explaining why you reject the other solution.



(9 marks)

a) Time = change in speed $\div$ acceleration

$$\text{accelerating: } \frac{V}{0.6} = \frac{5V}{3}$$

$$\text{decelerating: } \frac{V}{0.9} = \frac{10V}{9}$$

$$\begin{aligned} \text{total} &= \frac{15V}{9} + \frac{10V}{9} \\ &= \frac{25V}{9} \end{aligned}$$

as required

b) Time at constant speed

$$\text{constant speed time} = 180 - \frac{25V}{9}$$

Distance = area: rectangle + the two triangles

$$V\left(180 - \frac{25V}{9}\right) + \frac{1}{2} \times \frac{25V}{9} \times V = 3000$$

$$180V - \frac{25V^2}{9} + \frac{25V^2}{18} = 3000$$

$$180V - \frac{25V^2}{18} = 3000$$

Multiply by 18

$$3240V - 25V^2 = 54000$$

$$25V^2 - 3240V + 54000 = 0$$

as required

c) Quadratic formula

$$V = \frac{3240 \pm \sqrt{(3240^2 - 4 \times 25 \times 54000)}}{50}$$

$$= \frac{3240 \pm \sqrt{5097600}}{50}$$

$$V = 19.64 \text{ or } V = 109.96$$

Check $V = 109.96$

$$\frac{25 \times 109.96}{9} = 305 \text{ s}$$

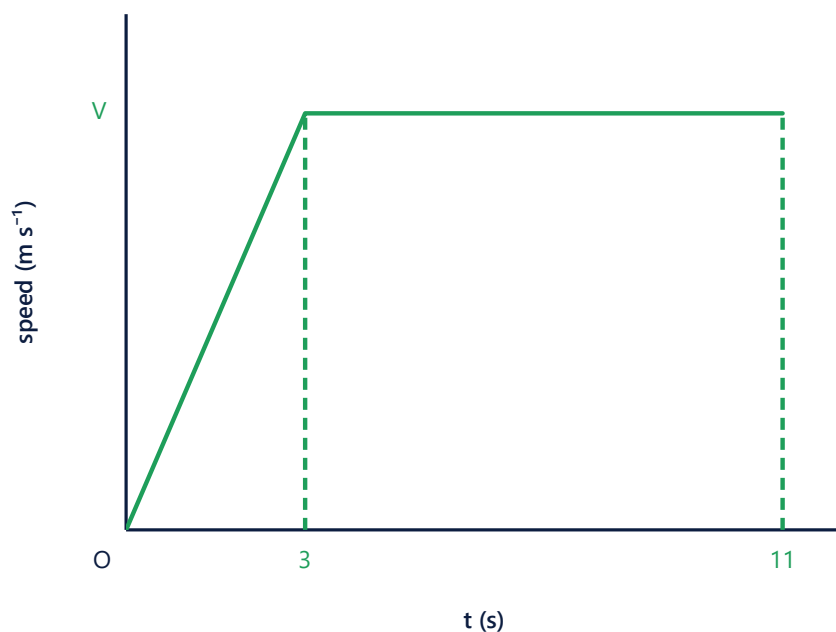
$305 > 180$, impossible

$V = 19.6$

12

In a model of a 100 m race, a sprinter starts from rest, accelerates uniformly for 3 s to a speed of $V \text{ m s}^{-1}$, and then runs at constant speed $V \text{ m s}^{-1}$ to finish the race in 11 s.

- Sketch a speed-time graph for the model.
- Find the value of V .
- Find the acceleration of the sprinter in the first 3 s.
- Give two reasons why the model may not be realistic.



(8 marks)

a) Up to V at $t = 3$, then flat until $t = 11$

(see the graph)

b) Distance = area under the graph

$$\frac{1}{2} \times 3 \times V + (11 - 3) \times V = 100$$

$$1.5V + 8V = 100$$

$$9.5V = 100$$

$$V = 10.526...$$

$$V = 10.5$$

c) Acceleration = gradient

$$a = \frac{10.526}{3}$$

$$= 3.508...$$

$$3.51 \text{ m s}^{-2}$$

d) Criticise the model

Acceleration would not really be constant

Speed would not jump suddenly to constant at $t = 3$

A real sprinter slows down near the finish