



- 1 A particle P moves along a straight line. At time t seconds, $t \geq 0$, the displacement of P from a fixed point O on the line is x metres, where

$$x = 2t^3 - 9t^2 + 12t + 1$$

- Find an expression for the velocity of P at time t seconds.
- Find the two times at which P is instantaneously at rest.
- Find the acceleration of P when $t = 3$
- Find the displacement of P from O when $t = 2$

(6 marks)

a) Differentiate x to get v

$$v = \frac{dx}{dt}$$

$$v = 6t^2 - 18t + 12$$

b) At rest when $v = 0$

$$\begin{array}{rcl} 6t^2 - 18t + 12 & = & 0 \\ \div 6 & & \\ t^2 - 3t + 2 & = & 0 \quad \div 6 \\ (t-1)(t-2) & = & 0 \end{array}$$

$$t = 1 \text{ and } t = 2$$

c) Differentiate v to get a

$$a = \frac{dv}{dt} = 12t - 18$$

$$a = 12(3) - 18$$

$$a = 18$$

$$18 \text{ m s}^{-2}$$

d) Substitute $t = 2$ into x

$$x = 2(2)^3 - 9(2)^2 + 12(2) + 1$$

$$x = 16 - 36 + 24 + 1$$

$$x = 5$$

$$5 \text{ m}$$

2

A particle moves along the x -axis. At time t seconds the velocity of the particle is $v \text{ m s}^{-1}$ in the positive x direction, where

$$v = 3t^2 - 4t + 2, \quad t \geq 0$$

When $t = 0$ the particle is at the origin O .

- Find the displacement of the particle from O when $t = 3$
- Find the acceleration of the particle when $t = 3$
- Show that the particle is never at rest.

(6 marks)

a) Integrate v to get x

$$x = \int (3t^2 - 4t + 2) dt$$

$$x = t^3 - 2t^2 + 2t + c$$

$x = 0$ when $t = 0$, so $c = 0$

$$x = 3^3 - 2(3)^2 + 2(3)$$

$$x = 27 - 18 + 6$$

$$x = 15$$

$$\boxed{15 \text{ m}}$$

b) Differentiate v to get a

$$a = 6t - 4$$

$$a = 6(3) - 4 = 14$$

$$\boxed{14 \text{ m s}^{-2}}$$

c) At rest means $v = 0$. Use the discriminant

$$b^2 - 4ac = (-4)^2 - 4(3)(2)$$

$$b^2 - 4ac = 16 - 24 = -8$$

$-8 < 0$, so $3t^2 - 4t + 2 = 0$ has no real roots

v is never 0, so the particle is never at rest

3

A particle moves in a straight line. At time t seconds its acceleration is $a \text{ m s}^{-2}$, where

$$a = 6 - 2t, \quad t \geq 0$$

When $t = 0$ the velocity of the particle is 4 m s^{-1} .

- Find an expression for the velocity of the particle at time t seconds.
- Find the maximum velocity of the particle.
- Find the time at which the particle comes to instantaneous rest, giving your answer to 3 significant figures.

(6 marks)

a) Integrate a to get v

$$v = \int (6 - 2t) dt$$

$$v = 6t - t^2 + c$$

$$v = 4 \text{ when } t = 0, \text{ so } c = 4$$

$$v = 4 + 6t - t^2$$

b) Maximum velocity when $\frac{dv}{dt} = a = 0$

$$6 - 2t = 0$$

$$t = 3$$

$$v = 4 + 6(3) - 3^2$$

$$v = 13$$

$$13 \text{ m s}^{-1}$$

c) At rest when $v = 0$

$$4 + 6t - t^2 = 0$$

$$t^2 - 6t - 4 = 0$$

Quadratic formula

$$t = \frac{6 \pm \sqrt{(36 + 16)}}{2}$$

$$t = 3 \pm \sqrt{13}$$

$t \geq 0$, so reject $3 - \sqrt{13}$ (negative)

$$t = 3 + \sqrt{13} = 6.605...$$

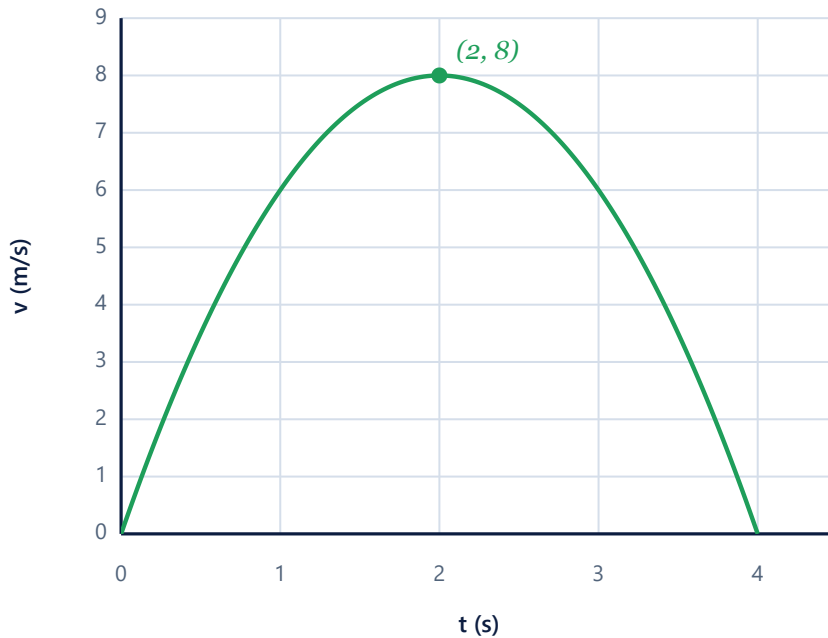
$$6.61 \text{ s}$$

4

A cyclist rides along a straight road between two sets of traffic lights. She starts from rest at the first set and stops at the second set. Her velocity t seconds after leaving the first set is $v \text{ m s}^{-1}$, where

$$v = 8t - 2t^2, \quad 0 \leq t \leq 4$$

- Sketch a velocity-time graph for the motion of the cyclist on the axes below.
- Find the maximum speed of the cyclist.
- Find the distance between the two sets of traffic lights.



(7 marks)

a) Factorise to find where $v = 0$

$$v = 2t(4 - t)$$

$v = 0$ at $t = 0$ and $t = 4$, n-shaped (see sketch)

b) Maximum when $\frac{dv}{dt} = 0$

$$\frac{dv}{dt} = 8 - 4t$$

$$8 - 4t = 0$$

$$t = 2$$

$$v = 8(2) - 2(2)^2 = 8$$

$$\boxed{8 \text{ m s}^{-1}}$$

c) $v \geq 0$ for the whole journey, so distance = area under the graph

$$s = \int (8t - 2t^2) dt$$

$$s = \left[4t^2 - \frac{2}{3}t^3 \right]$$

Limits: from 0 to 4

$$s = (4(4)^2 - \frac{2}{3}(4)^3) - 0$$

$$s = 64 - \frac{128}{3}$$

$$s = \frac{64}{3} = 21.33\ldots$$

21.3 m

5

A particle P moves along a straight line. At time t seconds the velocity of P is $v \text{ m s}^{-1}$, where

$$v = t^2 - 5t + 4, \quad t \geq 0$$

When $t = 0$, P is at the point O .

- Find the values of t at which P is instantaneously at rest.
- Find the displacement of P from O when $t = 5$
- Find the total distance travelled by P in the interval $0 \leq t \leq 5$

(7 marks)

a) At rest when $v = 0$

$$t^2 - 5t + 4 = 0$$

$$(t - 1)(t - 4) = 0$$

$$t = 1 \text{ and } t = 4$$

b) Integrate v to get x , with $x = 0$ when $t = 0$

$$x = \int (t^2 - 5t + 4) dt$$

$$x = \frac{t^3}{3} - \frac{5t^2}{2} + 4t$$

$$x(5) = \frac{125}{3} - \frac{125}{2} + 20$$

$$x(5) = -\frac{5}{6}$$

$$-\frac{5}{6} \text{ m (0.833 m on the negative side of } O)$$

c) P turns round at $t = 1$ and $t = 4$: find x there

$$x(1) = \frac{1}{3} - \frac{5}{2} + 4 = \frac{11}{6}$$

$$x(4) = \frac{64}{3} - 40 + 16 = -\frac{8}{3}$$

Add the distance of each stage (all positive)

$$0 \text{ to } 1: \frac{11}{6} - 0 = \frac{11}{6}$$

$$1 \text{ to } 4: \frac{11}{6} - (-\frac{8}{3}) = \frac{27}{6}$$

$$4 \text{ to } 5: -\frac{5}{6} - (-\frac{8}{3}) = \frac{11}{6}$$

$$\text{distance} = \frac{11}{6} + \frac{27}{6} + \frac{11}{6}$$

$$\text{distance} = \frac{49}{6} = 8.166\dots$$

$$8.17 \text{ m}$$

6

A car moves along a straight road. It starts from rest at time $t = 0$. For $0 \leq t \leq 4$ its velocity is $v \text{ m s}^{-1}$, where

$$v = 6t - \frac{3}{4}t^2$$

For $t > 4$ the car moves at a constant speed equal to its speed when $t = 4$

- Show that the acceleration of the car is zero when $t = 4$
- Find the constant speed of the car for $t > 4$
- Find the distance travelled by the car in the first 10 seconds.

(7 marks)

a) Differentiate v to get a

$$a = 6 - \frac{3}{2}t$$

$$a = 6 - \frac{3}{2}(4)$$

$$a = 0$$

$$a = 0 \text{ when } t = 4, \text{ as required}$$

b) Substitute $t = 4$ into v

$$v = 6(4) - \frac{3}{4}(4)^2$$

$$v = 24 - 12 = 12$$

$$12 \text{ m s}^{-1}$$

c) First stage: $v \geq 0$, so integrate

$$s_1 = \int (6t - \frac{3}{4}t^2) dt$$

$$s_1 = [3t^2 - \frac{1}{4}t^3]$$

Limits: from 0 to 4

$$s_1 = 3(16) - \frac{1}{4}(64) = 32$$

Second stage: constant speed for $10 - 4 = 6 \text{ s}$

$$s_2 = 12 \times 6 = 72$$

$$\text{total} = 32 + 72$$

$$104 \text{ m}$$

7

A particle P moves along a straight line through the fixed point O . At time t seconds the displacement of P from O is x metres, where

$$x = 12t - t^3, \quad t \geq 0$$

- Find the greatest positive displacement of P from O .
- Find the exact value of t when P returns to O .
- Find the speed of P as it passes through O at this time.
- Find the total distance travelled by P in the first 4 seconds.

(8 marks)

a) Differentiate: greatest x when $v = 0$

$$v = 12 - 3t^2$$

$$12 - 3t^2 = 0$$

$$t^2 = 4$$

$$t = 2$$

$$x = 12(2) - 2^3 = 16$$

$$\boxed{16 \text{ m}}$$

b) Back at O when $x = 0$

$$t(12 - t^2) = 0$$

$$t^2 = 12 \quad (t \neq 0)$$

$$t = \sqrt{12} = 2\sqrt{3}$$

$$\boxed{t = 2\sqrt{3}}$$

c) Substitute $t^2 = 12$ into v

$$v = 12 - 3(12) = -24$$

$$\boxed{24 \text{ m s}^{-1}}$$

d) P turns round at $t = 2$, so split the journey there

$$x(0) = 0$$

$$x(2) = 16$$

$$x(4) = 48 - 64 = -16$$

$$\text{distance} = (16 - 0) + (16 - (-16))$$

$$\text{distance} = 16 + 32$$

$$\boxed{48 \text{ m}}$$

8

A particle P moves along a straight line. At time t seconds the acceleration of P is $(6t - 8) \text{ m s}^{-2}$. When $t = 0$, P passes through the fixed point O with velocity 4 m s^{-1} .

- Find an expression for the displacement of P from O at time t seconds.
- Show that when P first returns to O it is instantaneously at rest.
- Find the greatest distance of P from O in the interval $0 \leq t \leq 2$

(8 marks)

a) Integrate a to get v

$$v = \int (6t - 8) dt$$

$$v = 3t^2 - 8t + c$$

$$v = 4 \text{ when } t = 0, \text{ so } c = 4$$

$$v = 3t^2 - 8t + 4$$

Integrate v to get x

$$x = t^3 - 4t^2 + 4t + k$$

$$x = 0 \text{ when } t = 0, \text{ so } k = 0$$

$$x = t^3 - 4t^2 + 4t$$

b) Factorise x to find when P is at O

$$x = t(t^2 - 4t + 4)$$

$$x = t(t - 2)^2$$

$x = 0$ again first at $t = 2$. Find v there

$$v = 3(2)^2 - 8(2) + 4$$

$$v = 12 - 16 + 4 = 0$$

P is at rest when it returns to O

c) Furthest from O when $v = 0$

$$(3t - 2)(t - 2) = 0$$

$$t = \frac{2}{3}$$

$$x = \frac{2}{3} \left(\frac{2}{3} - 2 \right)^2$$

$$x = \frac{2}{3} \times \frac{16}{9}$$

$$x = \frac{32}{27}$$

$$\frac{32}{27} \text{ m (1.19 m)}$$

9

A lift moves vertically upwards from rest at ground level. It comes to rest again at the top floor 6 seconds later. While moving, the velocity of the lift t seconds after it starts is $v \text{ m s}^{-1}$, where $v = kt(6 - t)$, $0 \leq t \leq 6$

and k is a positive constant. The top floor is 24 m above ground level.

- Find the value of k .
- Find the maximum speed of the lift.
- Find the magnitude of the acceleration of the lift when $t = 5$

(7 marks)

a) $v \geq 0$ throughout, so the distance is $\int v \, dt$

$$24 = \int k(6t - t^2) \, dt$$

$$24 = k \left[3t^2 - \frac{1}{3}t^3 \right]$$

Limits: from 0 to 6

$$24 = k \left(3(36) - \frac{1}{3}(216) \right)$$

$$24 = k(108 - 72)$$

$$24 = 36k$$

$$k = \frac{24}{36} = \frac{2}{3}$$

$$\boxed{k = \frac{2}{3}}$$

b) Maximum when $\frac{dv}{dt} = 0$

$$v = \frac{2}{3}(6t - t^2)$$

$$\frac{dv}{dt} = \frac{2}{3}(6 - 2t) = 0$$

$$t = 3$$

$$v = \frac{2}{3}(3)(6 - 3) = 6$$

$$\boxed{6 \text{ m s}^{-1}}$$

c) Substitute $t = 5$ into a

$$a = \frac{2}{3}(6 - 10)$$

$$a = -\frac{8}{3} = -2.666\dots$$

$$\boxed{2.67 \text{ m s}^{-2}}$$

10

Two cyclists, Priya and Tom, ride along a straight cycle track. At time $t = 0$ Priya passes a marker post O with speed 2 m s^{-1} , and she moves with constant acceleration 2 m s^{-2} . At the same moment Tom starts from rest at O and rides in the same direction. Tom's velocity at time t seconds is $3t^2 \text{ m s}^{-1}$. The model is valid for $0 \leq t \leq 3$

- Find, in terms of t , the distance of each cyclist from O .
- Find the time at which Tom catches up with Priya, and how far they are from O at that moment.
- Find the time at which the two cyclists have the same speed, giving your answer to 3 significant figures.

(8 marks)

a) Priya: constant acceleration, $u = 2$, $a = 2$, use $s = ut + \frac{1}{2}at^2$

$$s = 2t + \frac{1}{2}(2)t^2$$

$$s = t^2 + 2t$$

Tom: variable velocity, so integrate ($s = 0$ when $t = 0$)

$$s = \int 3t^2 dt$$

$$s = t^3$$

$$\boxed{\text{Priya } t^2 + 2t, \text{ Tom } t^3}$$

b) They meet when the distances are equal

$$t^3 = t^2 + 2t$$

$$t^3 - t^2 - 2t = 0$$

$$t(t - 2)(t + 1) = 0$$

$t > 0$, so $t = 2$

$$s = 2^3 = 8$$

$$\boxed{t = 2 \text{ s, } 8 \text{ m from } O}$$

c) Priya's speed $v = u + at$, Tom's speed $3t^2$

$$3t^2 = 2 + 2t$$

$$3t^2 - 2t - 2 = 0$$

$$t = \frac{2 \pm \sqrt{4 + 24}}{6}$$

$$t = \frac{1 + \sqrt{7}}{3} \quad (t > 0)$$

$$t = 1.215...$$

$$\boxed{1.22 \text{ s}}$$

11

A train leaves a station from rest at time $t = 0$. For the first 30 seconds its acceleration is $a \text{ m s}^{-2}$, where

$$a = 0.6 - 0.02t, \quad 0 \leq t \leq 30$$

- Find the speed of the train when $t = 30$
- Find the distance travelled by the train in the first 30 seconds.

The train then travels at this speed for 2 minutes. It then decelerates uniformly at 0.25 m s^{-2} until it comes to rest at the next station.

- Find the distance between the two stations.
- Find the average speed of the train between the two stations.

(9 marks)

a) Variable acceleration: integrate a to get v

$$v = \int (0.6 - 0.02t) dt$$

$$v = 0.6t - 0.01t^2 + c$$

From rest, so $c = 0$

$$v = 0.6(30) - 0.01(30)^2$$

$$v = 18 - 9 = 9$$

$$\boxed{9 \text{ m s}^{-1}}$$

b) $v \geq 0$, so integrate v

$$s_1 = \int (0.6t - 0.01t^2) dt$$

$$s_1 = \left[0.3t^2 - \frac{0.01}{3}t^3 \right]$$

Limits: from 0 to 30

$$s_1 = 0.3(900) - \frac{0.01}{3}(27000)$$

$$s_1 = 270 - 90 = 180$$

$$\boxed{180 \text{ m}}$$

c) Constant speed for 2 minutes = 120 s

$$s_2 = 9 \times 120 = 1080$$

Uniform deceleration: now suvat. $u = 9$, $v = 0$, $a = -0.25$

$$0 = 9^2 + 2(-0.25)s_3$$

$$0.5s_3 = 81$$

$$s_3 = 162$$

$$\text{total} = 180 + 1080 + 162$$

$$\boxed{1422 \text{ m}}$$

d) Time to stop: $v = u + at$

$$0 = 9 - 0.25t$$

$$t = 36$$

$$\text{total time} = 30 + 120 + 36 = 186$$

$$\text{average speed} = \frac{1422}{186}$$

$$\text{average speed} = 7.645...$$

7.65 m s^{-1}

12

A particle P moves along the x -axis. At time t seconds the velocity of P is $v \text{ m s}^{-1}$ in the positive x direction, where

$$v = t^2 - 6t + 8, \quad t \geq 0$$

- Find the interval of time during which P is moving in the negative x direction.
- Find the values of t at which the speed of P is 3 m s^{-1} , and explain why there are only two such values.
- Find the total distance travelled by P in the interval $0 \leq t \leq 5$

(9 marks)

a) Negative direction when $v < 0$

$$= t^2 - 6t + 8 < 0$$

$$= (t - 2)(t - 4) < 0$$

U-shaped, so negative between the roots

$$2 < t < 4$$

b) Speed 3 means $v = 3$ or $v = -3$

$$t^2 - 6t + 8 = 3$$

$$t^2 - 6t + 5 = 0$$

$$(t - 1)(t - 5) = 0$$

$$t = 1 \text{ or } t = 5$$

Now try $v = -3$

$$t^2 - 6t + 11 = 0$$

$$b^2 - 4ac = 36 - 44 = -8 < 0$$

No real roots: complete the square to see why

$$v = (t - 3)^2 - 1 \geq -1$$

$$t = 1 \text{ and } t = 5 \text{ only, since } v \text{ is never below } -1$$

c) Integrate v , taking $x = 0$ when $t = 0$

$$x = \frac{t^3}{3} - 3t^2 + 8t$$

P turns round at $t = 2$ and $t = 4$

$$x(2) = \frac{8}{3} - 12 + 16 = \frac{20}{3}$$

$$x(4) = \frac{64}{3} - 48 + 32 = \frac{16}{3}$$

$$x(5) = \frac{125}{3} - 75 + 40 = \frac{20}{3}$$

Add the distance of each stage

$$\text{distance} = \frac{20}{3} + \left(\frac{20}{3} - \frac{16}{3}\right) + \left(\frac{20}{3} - \frac{16}{3}\right)$$

$$\text{distance} = \frac{20}{3} + \frac{4}{3} + \frac{4}{3}$$

$$\text{distance} = \frac{28}{3} = 9.333\ldots$$

9.33 m