



- 1 Expand $(2 + x)^4$ fully, simplifying each term.

(3 marks)

Pascal's triangle row for $n = 4$: 1, 4, 6, 4, 1

$$\begin{aligned}(2 + x)^4 &= 2^4 + 4(2^3)x + 6(2^2)x^2 + 4(2)x^3 + x^4 \\ &= 16 + 32x + 24x^2 + 8x^3 + x^4\end{aligned}$$

$$\boxed{16 + 32x + 24x^2 + 8x^3 + x^4}$$

- 2 Find the first four terms, in ascending powers of x , of the binomial expansion of $(1 - 3x)^6$. Give each term in its simplest form.

(3 marks)

$${}^6C_1 = 6, {}^6C_2 = 15, {}^6C_3 = 20$$

$$\begin{aligned}(1 - 3x)^6 &= 1 + 6(-3x) + 15(-3x)^2 + 20(-3x)^3 + \dots \\ &= 1 - 18x + 15(9x^2) + 20(-27x^3) + \dots \\ &= 1 - 18x + 135x^2 - 540x^3 + \dots\end{aligned}$$

$$\boxed{1 - 18x + 135x^2 - 540x^3}$$

- 3 Find the coefficient of x^3 in the binomial expansion of $(3 + 2x)^7$

(2 marks)

The x^3 term: ${}^7C_3 (3)^4 (2x)^3$

$${}^7C_3 = 35$$

$$\begin{aligned}\text{coefficient} &= 35 \times 3^4 \times 2^3 \\ &= 35 \times 81 \times 8 \\ &= 22680\end{aligned}$$

$$\boxed{22680}$$

4

In the binomial expansion of $(1 + kx)^9$, where k is a positive constant, the coefficient of x^2 is 252

Find the exact value of k .

(3 marks)

The x^2 term: ${}^9C_2 (kx)^2$

$${}^9C_2 = 36$$

$$36k^2 = 252$$

$$k^2 = 7$$

$k > 0$, so take the positive root

$$k = \sqrt{7}$$

5

a. Find the first four terms, in ascending powers of x , of the binomial expansion of $(2 - \frac{x}{4})^8$. Give each term in its simplest form.

b. Use your expansion, with a suitable value of x , to find an estimate for 1.975^8 . Give your answer to 3 decimal places.

(6 marks)

$$a) {}^8C_1 = 8, {}^8C_2 = 28, {}^8C_3 = 56$$

$$\begin{aligned} (2 - \frac{x}{4})^8 &= 2^8 + 8(2^7)(-\frac{x}{4}) + 28(2^6)(-\frac{x}{4})^2 + 56(2^5)(-\frac{x}{4})^3 + \dots \\ &= 256 + 8(128)(-\frac{x}{4}) + 28(64)(\frac{x^2}{16}) + 56(32)(-\frac{x^3}{64}) + \dots \\ &= 256 - 256x + 112x^2 - 28x^3 + \dots \end{aligned}$$

$$256 - 256x + 112x^2 - 28x^3$$

b) Choose x to make the bracket 1.975

$$2 - \frac{x}{4} = 1.975$$

$$\frac{x}{4} = 0.025$$

$$x = 0.1$$

Substitute $x = 0.1$ into the expansion

$$\begin{aligned} 1.975^8 &\approx 256 - 256(0.1) + 112(0.1)^2 - 28(0.1)^3 \\ &= 256 - 25.6 + 1.12 - 0.028 \\ &= 231.492 \end{aligned}$$

$$231.492$$

6

In the binomial expansion of $(a + 3x)^6$, where a is a non-zero constant, the coefficient of x is equal to the coefficient of x^2

Find the value of a .

(4 marks)

Write down the two terms

$$x \text{ term} = 6a^5(3x) = 18a^5x$$

$$x^2 \text{ term} = 15a^4(3x)^2 = 135a^4x^2$$

Coefficients are equal

$$18a^5 = 135a^4$$

$a \neq 0$, so divide by $9a^4$

$$2a = 15$$

$$a = \frac{15}{2}$$

$$\boxed{a = \frac{15}{2}}$$

7

In the binomial expansion of $(1 + 2x)^n$, where n is a positive integer, the coefficient of x^2 is 180

Find the value of n .

(4 marks)

The x^2 term: $nC2 (2x)^2$

$$nC2 = \frac{n(n-1)}{2}$$

$$\frac{n(n-1)}{2} \times 4 = 180$$

$$2n(n-1) = 180$$

$$n^2 - n = 90$$

$$n^2 - n - 90 = 0$$

$$(n-10)(n+9) = 0$$

n must be a positive integer, reject $n = -9$

$$\boxed{n = 10}$$

8

a. Find the first four terms, in ascending powers of x , of the binomial expansion of $(2 - x)^6$ b. Hence find the coefficient of x^3 in the expansion of $(1 + 4x)(2 - x)^6$

(5 marks)

$$a) {}^6C_1 = 6, {}^6C_2 = 15, {}^6C_3 = 20$$

$$\begin{aligned}(2 - x)^6 &= 2^6 + 6(2^5)(-x) + 15(2^4)(-x)^2 + 20(2^3)(-x)^3 + \dots \\ &= 64 - 192x + 240x^2 - 160x^3 + \dots\end{aligned}$$

$$\boxed{64 - 192x + 240x^2 - 160x^3}$$

b) Only two ways to make x^3

$$1 \times (-160x^3) = -160x^3$$

$$4x \times 240x^2 = 960x^3$$

$$\begin{aligned}\text{coefficient} &= -160 + 960 \\ &= 800\end{aligned}$$

$$\boxed{800}$$

9

Find the term independent of x in the binomial expansion of $(2x - \frac{3}{x})^6$

(4 marks)

$$\text{General term: } {}^6C_r (2x)^{6-r} \left(-\frac{3}{x}\right)^r$$

$$\text{Power of } x \text{ is } (6 - r) - r$$

$$6 - 2r = 0$$

$$r = 3$$

$$\begin{aligned}\text{term} &= {}^6C_3 (2x)^3 \left(-\frac{3}{x}\right)^3 \\ &= 20 \times 8x^3 \times \left(-\frac{27}{x^3}\right) \\ &= -4320\end{aligned}$$

$$\boxed{-4320}$$

10

The first three terms, in ascending powers of x , of the binomial expansion of $(1 + px)^n$ are

$$1 + 12x + 63x^2$$

where n is a positive integer and p is a constant.

Find the value of n and the value of p .

(5 marks)

Compare coefficients

$$np = 12$$

$$\frac{n(n-1)}{2} p^2 = 63$$

$$n(n-1)p^2 = 126$$

Square the first equation

$$n^2 p^2 = 144$$

Divide the two equations

$$\frac{n(n-1)p^2}{n^2 p^2} = \frac{126}{144}$$

$$\frac{n-1}{n} = \frac{7}{8}$$

$$8n - 8 = 7n$$

$$n = 8$$

$$p = \frac{12}{8} = \frac{3}{2}$$

$$\boxed{n = 8, p = \frac{3}{2}}$$

11

Priya invests £5000 in an account that pays $r\%$ interest per year, compounded annually. After 10 years the account holds $£5000(1 + 0.01r)^{10}$

- Use the first three terms of the binomial expansion of $(1 + y)^{10}$, in ascending powers of y , to estimate the amount in the account after 10 years when $r = 2$
- Explain why your answer to part (a) is less than the true amount.
- Find the percentage error in your estimate. Give your answer to 2 significant figures.

(5 marks)

$$a) 10C1 = 10, 10C2 = 45$$

$$(1 + y)^{10} \approx 1 + 10y + 45y^2$$

$$r = 2 \text{ so } y = 0.01 \times 2$$

$$y = 0.02$$

$$(1.02)^{10} \approx 1 + 10(0.02) + 45(0.02)^2$$

$$= 1 + 0.2 + 0.018$$

$$= 1.218$$

$$5000 \times 1.218 = 6090$$

$$\boxed{£6090}$$

b)

$y > 0$, so the terms left out ($120y^3 + \dots$) are all positive - the estimate is too small

c) True amount

$$5000 \times 1.02^{10} = 6094.97\dots$$

$$\text{error} = 6094.97 - 6090 = 4.97$$

$$\% \text{ error} = \frac{4.97}{6094.97} \times 100$$

$$= 0.0816\dots$$

$$\boxed{0.082\%}$$

12

- a. Find the first three terms, in ascending powers of x , of the binomial expansion of $(2 + kx)^5$, where k is a non-zero constant. Give each term in terms of k .
- b. Given that the coefficient of x^2 in the expansion of $(3 - x)(2 + kx)^5$ is 160, find the possible values of k .

(6 marks)

$$a) 5C_1 = 5, 5C_2 = 10$$

$$(2 + kx)^5 = 2^5 + 5(2^4)(kx) + 10(2^3)(kx)^2 + \dots$$

$$= 32 + 80kx + 80k^2x^2 + \dots$$

$$\boxed{32 + 80kx + 80k^2x^2}$$

$$b) \text{ Two ways to make } x^2$$

$$3 \times 80k^2x^2 = 240k^2x^2$$

$$-x \times 80kx = -80kx^2$$

$$240k^2 - 80k = 160$$

$$\text{Divide by 80}$$

$$3k^2 - k - 2 = 0$$

$$(3k + 2)(k - 1) = 0$$

$$\boxed{k = 1 \text{ or } k = -\frac{2}{3}}$$