



1 Find the exact value of

a. $32^{\frac{3}{5}}$

b. $(\frac{16}{81})^{-\frac{3}{4}}$

(3 marks)

a)

Root first (denominator), then power (numerator)

$$32^{3/5} = ({}^5\sqrt{32})^3$$

$$= 2^3$$

8

b)

Negative power: flip the fraction

$$(\frac{16}{81})^{-3/4} = (\frac{81}{16})^{3/4}$$

$$= (\frac{3}{2})^3$$

$\frac{27}{8}$

2 Write $\sqrt{75} + \sqrt{48} - \sqrt{12}$ in the form $k\sqrt{3}$, where k is an integer.

(2 marks)

Take out the biggest square factor of each

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$

$$\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$$

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$$

$$\sqrt{75} + \sqrt{48} - \sqrt{12} = 5\sqrt{3} + 4\sqrt{3} - 2\sqrt{3}$$

$7\sqrt{3}$

3

Simplify fully $(9x^4)^{\frac{1}{2}} \times 2x^{-3}$

(2 marks)

The power $1/2$ applies to the 9 and to the x^4

$$(9x^4)^{1/2} = 9^{1/2} \times x^{4 \times 1/2}$$

$$= 3x^2$$

Multiply: add the powers

$$3x^2 \times 2x^{-3} = 6x^{2+(-3)}$$

$$= 6x^{-1}$$

$$\boxed{\frac{6}{x}}$$

4

Write $\frac{12}{3 - \sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

(3 marks)

Multiply top and bottom by $3 + \sqrt{5}$

$$\frac{12}{3 - \sqrt{5}} = \frac{12(3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})}$$

Difference of two squares on the bottom

$$(3 - \sqrt{5})(3 + \sqrt{5}) = 9 - 5 = 4$$

$$\frac{12(3 + \sqrt{5})}{4} = 3(3 + \sqrt{5})$$

$$\boxed{9 + 3\sqrt{5}}$$

5

Write $\frac{5 + 2\sqrt{3}}{2 + \sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are integers.

You must show all stages of your working.

(3 marks)

Multiply top and bottom by $2 - \sqrt{3}$

$$\frac{5 + 2\sqrt{3}}{2 + \sqrt{3}} = \frac{(5 + 2\sqrt{3})(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}$$

Expand the top

$$(5 + 2\sqrt{3})(2 - \sqrt{3}) = 10 - 5\sqrt{3} + 4\sqrt{3} - 2 \times 3$$

$$= 4 - \sqrt{3}$$

Expand the bottom

$$(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$$

$$\frac{4 - \sqrt{3}}{1} = 4 - \sqrt{3}$$

$$\boxed{4 - \sqrt{3}}$$

6

a. Write 8^{x+1} in the form 2^{ax+b} , where a and b are integers.b. Hence solve the equation $8^{x+1} = 4^{2x-3}$

(4 marks)

a)

$$8 = 2^3$$

$$8^{x+1} = (2^3)^{x+1}$$

$$\boxed{2^{3x+3}}$$

b)

Write the right-hand side as a power of 2 too

$$4^{2x-3} = (2^2)^{2x-3} = 2^{4x-6}$$

$$2^{3x+3} = 2^{4x-6}$$

Same base, so the powers are equal

$$3x + 3 = 4x - 6$$

$$\begin{array}{ccc} -3x + 6 & 9 = x & -3x + 6 \end{array}$$

$$\boxed{x = 9}$$

7

Write $\frac{8x^3 - 3\sqrt{x}}{2x}$ in the form $ax^p + bx^q$, where a , b , p and q are constants.

(3 marks)

 $\sqrt{x} = x^{1/2}$, then split into two fractions

$$\frac{8x^3 - 3\sqrt{x}}{2x} = \frac{8x^3}{2x} - \frac{3x^{1/2}}{2x}$$

Divide: subtract the powers

$$\frac{8x^3}{2x} = 4x^2$$

$$\frac{3x^{1/2}}{2x} = \frac{3}{2}x^{1/2-1} = \frac{3}{2}x^{-1/2}$$

$$\boxed{4x^2 - \frac{3}{2}x^{-1/2}}$$

$$a = 4, p = 2, b = -\frac{3}{2}, q = -\frac{1}{2}$$

8

Given that $y = 3^x$,a. write 9^x in terms of y ,b. write 3^{x+1} in terms of y .c. Hence solve the equation $9^x - 4 \times 3^{x+1} + 27 = 0$

(5 marks)

a)

$$9^x = (3^2)^x = (3^x)^2$$

$$\boxed{y^2}$$

b)

$$3^{x+1} = 3^x \times 3^1$$

$$\boxed{3y}$$

c)

Replace with y

$$y^2 - 4 \times 3y + 27 = 0$$

$$y^2 - 12y + 27 = 0$$

$$(y - 3)(y - 9) = 0$$

$$y = 3 \text{ or } y = 9$$

Go back to x

$$3^x = 3 \text{ so } x = 1$$

$$3^x = 9 \text{ so } x = 2$$

$$\boxed{x = 1 \text{ or } x = 2}$$

9

Solve the equation

$$\sqrt{5}x = \sqrt{3}(x + 2)$$

giving your answer in the form $a + \sqrt{b}$, where a and b are integers.

(4 marks)

Expand the bracket

$$\sqrt{5}x = \sqrt{3}x + 2\sqrt{3}$$

Get the x terms together and factorise

$$\sqrt{5}x - \sqrt{3}x = 2\sqrt{3}$$

$$x(\sqrt{5} - \sqrt{3}) = 2\sqrt{3}$$

$$x = \frac{2\sqrt{3}}{\sqrt{5} - \sqrt{3}}$$

Rationalise: $x(\sqrt{5} + \sqrt{3})$ top and bottom

$$x = \frac{2\sqrt{3}(\sqrt{5} + \sqrt{3})}{5 - 3}$$

$$= \frac{2\sqrt{3}(\sqrt{5} + \sqrt{3})}{2}$$

$$= \sqrt{3} \times \sqrt{5} + \sqrt{3} \times \sqrt{3}$$

$$= \sqrt{15} + 3$$

$$\boxed{x = 3 + \sqrt{15}}$$

10

Given that

$$2^x \times 4^y = \frac{1}{8} \quad \text{and} \quad \frac{27^x}{9^y} = 2187$$

find the value of x and the value of y .

(5 marks)

First equation: everything as a power of 2

$$2^x \times (2^2)^y = 2^{-3}$$

$$2^{x+2y} = 2^{-3}$$

$$x + 2y = -3$$

Second equation: everything as a power of 3

$$2187 = 3^7$$

$$\frac{(3^3)^x}{(3^2)^y} = 3^7$$

$$3^{3x-2y} = 3^7$$

$$3x - 2y = 7$$

Add the two equations to eliminate y

$$4x = 4$$

$$x = 1$$

Substitute back

$$1 + 2y = -3$$

$$y = -2$$

$$x = 1, y = -2$$

11

A rectangular tile has area 14 cm^2 . Its width is $(4 - \sqrt{2}) \text{ cm}$.

- Find the length of the tile, giving your answer in the form $(a + b\sqrt{2}) \text{ cm}$, where a and b are integers.
- Show that the diagonal of the tile is exactly 6 cm .

(6 marks)

a)

$$\text{length} = \frac{14}{4 - \sqrt{2}}$$

Rationalise: $\times (4 + \sqrt{2})$ top and bottom

$$= \frac{14(4 + \sqrt{2})}{(4 - \sqrt{2})(4 + \sqrt{2})}$$

$$= \frac{14(4 + \sqrt{2})}{16 - 2}$$

$$= \frac{14(4 + \sqrt{2})}{14}$$

$$\boxed{(4 + \sqrt{2}) \text{ cm}}$$

b)

Pythagoras

$$d^2 = (4 - \sqrt{2})^2 + (4 + \sqrt{2})^2$$

$$(4 - \sqrt{2})^2 = 16 - 8\sqrt{2} + 2 = 18 - 8\sqrt{2}$$

$$(4 + \sqrt{2})^2 = 16 + 8\sqrt{2} + 2 = 18 + 8\sqrt{2}$$

$$d^2 = 18 - 8\sqrt{2} + 18 + 8\sqrt{2}$$

$$d^2 = 36$$

$$d = 6$$

$$\boxed{\text{The diagonal is } 6 \text{ cm}}$$

a. Show that, for any positive integer n ,

$$\frac{1}{\sqrt{n} + \sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$$

b. Hence find the exact value of

$$\frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{24} + \sqrt{25}}$$

(5 marks)

a)

Multiply top and bottom by $\sqrt{n+1} - \sqrt{n}$

$$\frac{1}{\sqrt{n} + \sqrt{n+1}} = \frac{\sqrt{n+1} - \sqrt{n}}{(\sqrt{n+1} + \sqrt{n})(\sqrt{n+1} - \sqrt{n})}$$

Difference of two squares on the bottom

$$(\sqrt{n+1})^2 - (\sqrt{n})^2 = n+1 - n = 1$$

$$\frac{1}{\sqrt{n} + \sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$$

as required

b)

Use part a) on each term, $n = 1$ to 24

$$(\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + \dots + (\sqrt{25} - \sqrt{24})$$

Everything cancels except the first and last

$$\text{sum} = \sqrt{25} - \sqrt{1}$$

$$= 5 - 1$$

4