



1 Solve the equation $2x^2 - 7x - 15 = 0$

(2 marks)

Factorise: find two numbers that multiply to $2 \times -15 = -30$ and add to -7
 -10 and 3

$$2x^2 - 10x + 3x - 15 = 0$$

$$2x(x - 5) + 3(x - 5) = 0$$

$$(2x + 3)(x - 5) = 0$$

Set each bracket equal to zero

$$x = 5$$

$$x = -\frac{3}{2}$$

$$x = 5 \text{ or } x = -\frac{3}{2}$$

2 By completing the square, solve $x^2 + 6x - 4 = 0$, giving your answers in the form $a \pm \sqrt{b}$, where a and b are integers.

(3 marks)

Complete the square: halve the 6

$$(x + 3)^2 - 9 - 4 = 0$$

$$(x + 3)^2 - 13 = 0$$

$$(x + 3)^2 = 13$$

Square root both sides, remember $\pm$

$$x + 3 = \pm\sqrt{13}$$

$$x = -3 \pm \sqrt{13}$$

$$x = -3 \pm \sqrt{13}$$

3 Solve $3x^2 - 5x - 1 = 0$, giving your answers to 3 significant figures.

(3 marks)

Use the quadratic formula with $a = 3$, $b = -5$, $c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{(25 - 4(3)(-1))}}{6}$$

$$x = \frac{5 \pm \sqrt{37}}{6}$$

$$x = \frac{5 + 6.0828}{6} = 1.8471...$$

$$x = \frac{5 - 6.0828}{6} = -0.18046...$$

$$x = 1.85 \text{ or } x = -0.180$$

4 Solve the simultaneous equations

$$y = x + 3$$

$$x^2 + y^2 = 29$$

(5 marks)

Substitute the linear equation into the quadratic

$$x^2 + (x + 3)^2 = 29$$

$$x^2 + x^2 + 6x + 9 = 29$$

$$2x^2 + 6x - 20 = 0$$

$$\div 2 \quad x^2 + 3x - 10 = 0 \quad \div 2$$

$$(x + 5)(x - 2) = 0$$

$$x = 2 \text{ or } x = -5$$

Find each y from $y = x + 3$

$$x = 2$$

$$y = 2 + 3 = 5$$

$$x = -5$$

$$y = -5 + 3 = -2$$

$$x = 2, y = 5 \text{ or } x = -5, y = -2$$

5 Solve the simultaneous equations

$$x + 2y = 7$$

$$x^2 - xy + y^2 = 7$$

You must show all stages of your working.

(6 marks)

Make x the subject of the linear equation

$$x = 7 - 2y$$

Substitute into the quadratic

$$\begin{aligned}(7 - 2y)^2 - (7 - 2y)y + y^2 &= 7 \\ 49 - 28y + 4y^2 - 7y + 2y^2 + y^2 &= 7 \\ 7y^2 - 35y + 49 &= 7 \\ 7y^2 - 35y + 42 &= 0 \\ \div 7 \quad y^2 - 5y + 6 &= 0 \quad \div 7 \\ (y - 2)(y - 3) &= 0 \\ y &= 2 \text{ or } y = 3\end{aligned}$$

Substitute back into $x = 7 - 2y$

$$y = 2$$

$$x = 7 - 4 = 3$$

$$y = 3$$

$$x = 7 - 6 = 1$$

$$x = 3, y = 2 \text{ or } x = 1, y = 3$$

6 Using the substitution $u = \sqrt{x}$, or otherwise, solve $x - 7\sqrt{x} + 10 = 0$

(4 marks)

Let $u = \sqrt{x}$, so $x = u^2$

$$u^2 - 7u + 10 = 0$$

$$(u - 2)(u - 5) = 0$$

$$u = 2 \text{ or } u = 5$$

Change back: $\sqrt{x} = u$, so square

$$\sqrt{x} = 2 \Rightarrow x = 4$$

$$\sqrt{x} = 5 \Rightarrow x = 25$$

$$x = 4 \text{ or } x = 25$$

7 Solve the equation $2^{2x} - 9(2^x) + 8 = 0$

(4 marks)

Spot the hidden quadratic: $2^{2x} = (2^x)^2$

Let $y = 2^x$

$$y^2 - 9y + 8 = 0$$

$$(y - 1)(y - 8) = 0$$

$$y = 1 \text{ or } y = 8$$

$$2^x = 1 \Rightarrow x = 0$$

$$2^x = 8 = 2^3 \Rightarrow x = 3$$

$$x = 0 \text{ or } x = 3$$

8 The line l has equation $y = 2x + k$, where k is a constant. The curve C has equation $y = x^2 - 4x + 7$

- Show that the x coordinates of any points where l meets C satisfy $x^2 - 6x + (7 - k) = 0$
- Given that l is a tangent to C , find the value of k and the coordinates of the point where l touches C .

(5 marks)

a) Set the equations equal

$$x^2 - 4x + 7 = 2x + k$$

$$x^2 - 6x + 7 - k = 0$$

$$x^2 - 6x + (7 - k) = 0$$

shown

b) Tangent: one repeated root, so $b^2 - 4ac = 0$

$$(-6)^2 - 4(1)(7 - k) = 0$$

$$36 - 28 + 4k = 0$$

$$8 + 4k = 0$$

$$k = -2$$

Solve the quadratic with $k = -2$

$$x^2 - 6x + 9 = 0$$

$$(x - 3)^2 = 0$$

$$x = 3$$

$$y = 2(3) - 2 = 4$$

$$k = -2, \text{ touches at } (3, 4)$$

9

Solve $\frac{2}{x-1} + \frac{3}{x+2} = 1$, giving your answers in simplest surd form.

(4 marks)

Multiply every term by $(x-1)(x+2)$

$$2(x+2) + 3(x-1) = (x-1)(x+2)$$

$$2x + 4 + 3x - 3 = x^2 + x - 2$$

$$5x + 1 = x^2 + x - 2$$

$$0 = x^2 - 4x - 3$$

Does not factorise: complete the square

$$(x-2)^2 - 4 - 3 = 0$$

$$(x-2)^2 = 7$$

$$x-2 = \pm\sqrt{7}$$

$$x = 2 \pm \sqrt{7}$$

Neither value is 1 or -2, so both are valid

$$x = 2 \pm \sqrt{7}$$

10

A rectangular allotment plot in Leeds has a perimeter of 46 m. The diagonal of the plot is 17 m long. Find the length and the width of the plot.

(5 marks)

Let the sides be l and w metres

$$2l + 2w = 46$$

$$\div 2 \quad l + w = 23 \quad \div 2$$

Pythagoras on the diagonal

$$l^2 + w^2 = 17^2 = 289$$

Substitute $w = 23 - l$

$$l^2 + (23-l)^2 = 289$$

$$l^2 + 529 - 46l + l^2 = 289$$

$$2l^2 - 46l + 240 = 0$$

$$\div 2 \quad l^2 - 23l + 120 = 0$$

$$(l-15)(l-8) = 0$$

$$l = 15 \text{ or } l = 8$$

$$w = 8 \text{ or } w = 15$$

$$15 \text{ m by } 8 \text{ m}$$

11

Find all the real solutions of the equation $(x^2 - 3x)^2 - 2(x^2 - 3x) - 8 = 0$

(5 marks)

$$\text{Let } u = x^2 - 3x$$

$$u^2 - 2u - 8 = 0$$

$$(u - 4)(u + 2) = 0$$

$$u = 4 \text{ or } u = -2$$

$$u = 4$$

$$x^2 - 3x = 4$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4 \text{ or } x = -1$$

$$u = -2$$

$$x^2 - 3x = -2$$

$$x^2 - 3x + 2 = 0$$

$$(x - 1)(x - 2) = 0$$

$$x = 1 \text{ or } x = 2$$

$x = -1, 1, 2 \text{ or } 4$

12

Ayesha chips a golf ball up a straight hillside. Relative to the point where the ball is struck, the path of the ball is modelled by

$$y = 2 + 0.8x - 0.05x^2$$

where y m is the height and x m is the horizontal distance. The hillside is modelled by the line $y = 0.2x$

- Find the horizontal distance travelled by the ball before it lands on the hillside. Give your answer to 3 significant figures.
- Show that the vertical height of the ball above the hillside is $h = 2 + 0.6x - 0.05x^2$ metres, and hence find the greatest vertical height of the ball above the hillside.
- Give one limitation of the model.

(7 marks)

a) The ball lands where the two heights are equal

$$2 + 0.8x - 0.05x^2 = 0.2x$$

$$0.05x^2 - 0.6x - 2 = 0$$

$$\begin{array}{ccc} \times 20 & x^2 - 12x - 40 = 0 & \times 20 \end{array}$$

Quadratic formula

$$x = \frac{12 \pm \sqrt{(144 + 160)}}{2}$$

$$x = 6 \pm \sqrt{76}$$

$$x = 14.717... \text{ (the other root is negative)}$$

$$\boxed{14.7 \text{ m}}$$

b) Height above hillside = ball height - hillside height

$$h = 2 + 0.8x - 0.05x^2 - 0.2x$$

$$h = 2 + 0.6x - 0.05x^2$$

Complete the square

$$h = -0.05(x^2 - 12x) + 2$$

$$h = -0.05[(x - 6)^2 - 36] + 2$$

$$h = -0.05(x - 6)^2 + 1.8 + 2$$

$$h = 3.8 - 0.05(x - 6)^2$$

Greatest when $x = 6$

$$\boxed{3.8 \text{ m}}$$

c) Any one sensible limitation

ball modelled as a particle, or air resistance ignored