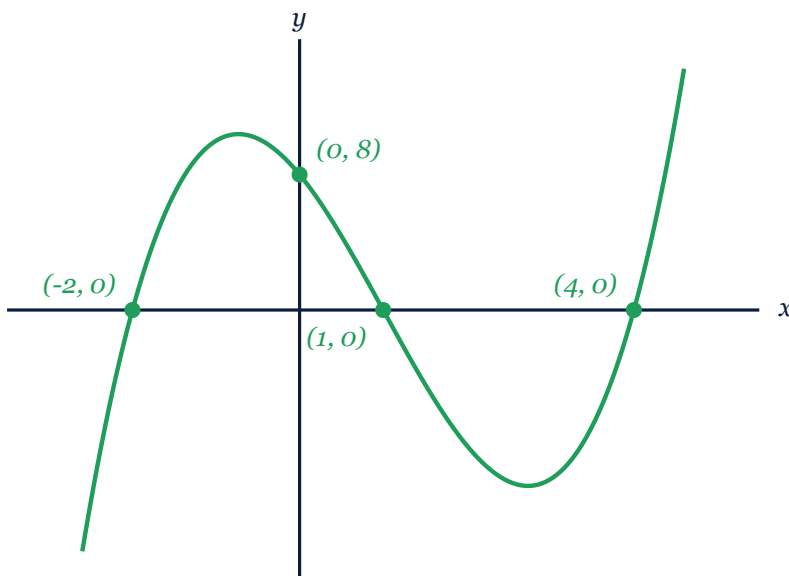




- 1 Sketch the curve with equation $y = (x + 2)(x - 1)(x - 4)$, showing the coordinates of the points where the curve meets the coordinate axes.



(3 marks)

Roots: set each bracket equal to zero

$$x = -2, x = 1, x = 4$$

y-intercept: put $x = 0$

$$y = (2)(-1)(-4)$$

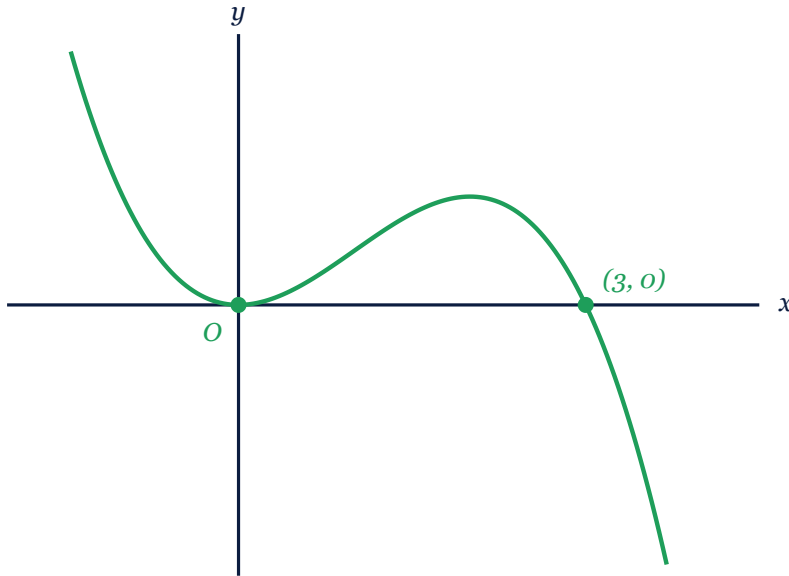
$$y = 8$$

Positive x^3 term: bottom left to top right

see the sketch: $(-2, 0), (1, 0), (4, 0), (0, 8)$

2

Sketch the curve with equation $y = x^2(3 - x)$, showing the coordinates of any points where the curve meets the coordinate axes.



(3 marks)

Roots

$$x^2 = 0 \Rightarrow x = 0 \text{ (repeated: the curve touches)}$$

$$3 - x = 0 \Rightarrow x = 3 \text{ (crosses)}$$

Shape: the x^3 term is $-x^3$

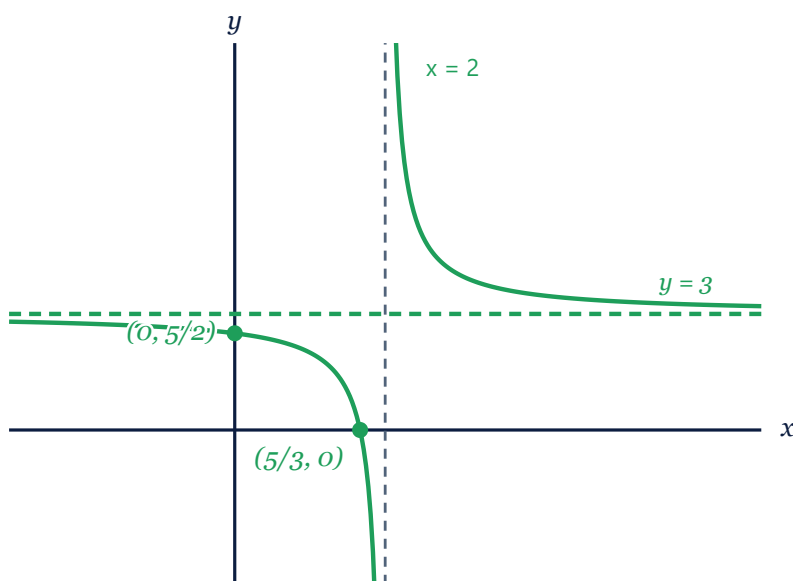
negative cubic: starts top left, ends bottom right

y-intercept: $x = 0$ gives $y = 0$

see the sketch: touches at $(0, 0)$, crosses at $(3, 0)$

3

Sketch the curve with equation $y = \frac{1}{x-2} + 3$. On your sketch show the equations of the asymptotes and the coordinates of the points where the curve crosses the coordinate axes.



(4 marks)

Translation of $y = \frac{1}{x}$: 2 right and 3 up

asymptotes: $x = 2$ and $y = 3$

y-intercept: $x = 0$

$$y = \frac{1}{-2} + 3 = \frac{5}{2}$$

x-intercept: $y = 0$

$$\frac{1}{x-2} = -3$$

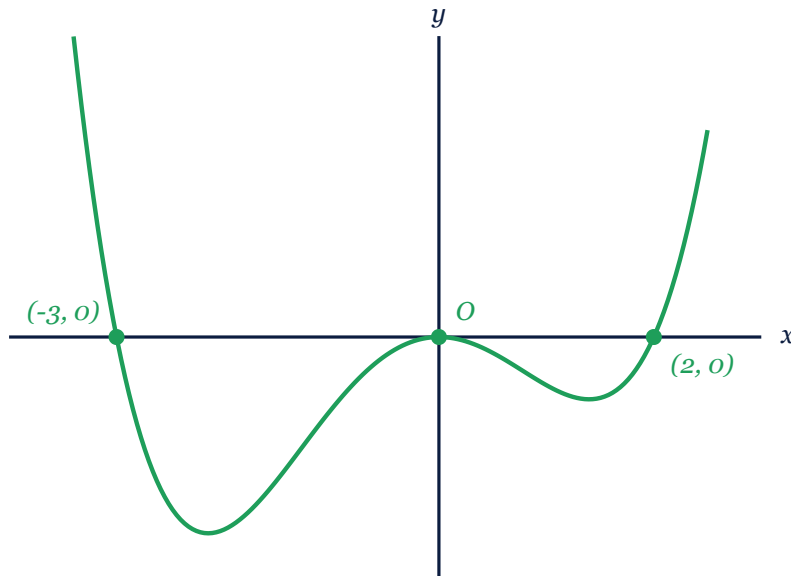
$$x - 2 = -\frac{1}{3}$$

$$x = \frac{5}{3}$$

$$x = 2, y = 3, (0, \frac{5}{2}), (\frac{5}{3}, 0)$$

4

Sketch the curve with equation $y = x^2(x - 2)(x + 3)$, showing the coordinates of the points where the curve meets the coordinate axes.



(3 marks)

Roots

$x = 0$ (repeated: touches)

$x = 2$ and $x = -3$ (cross)

Highest power: $x^2 \times x \times x = x^4$, positive

both ends of the curve point upwards

The curve goes up to touch at O and back down

see the sketch: $(-3, 0)$, $(0, 0)$, $(2, 0)$

5

The curve with equation $y = f(x)$ has a maximum point at $(3, 8)$. Write down the coordinates of the turning point on the curve with equation

a. $y = f(x - 2)$

b. $y = f(x) + 3$

c. $y = f(3x)$

d. $y = -f(x)$

e. $y = 2f(x + 1) - 4$

(5 marks)

a) $f(x - 2)$: translate 2 right, $x + 2$

$(5, 8)$

b) $f(x) + 3$: translate 3 up, $y + 3$

$(3, 11)$

c) $f(3x)$: stretch in x , scale factor $\frac{1}{3}$, so $x \div 3$

$(1, 8)$

d) $-f(x)$: reflect in the x -axis, $y \times -1$

$(3, -8)$, a minimum point

e) $x + 1$ inside: 1 left

$x = 3 - 1 = 2$

$2f(\dots) - 4$: double the y value then subtract 4

$y = 2(8) - 4 = 12$

$(2, 12)$

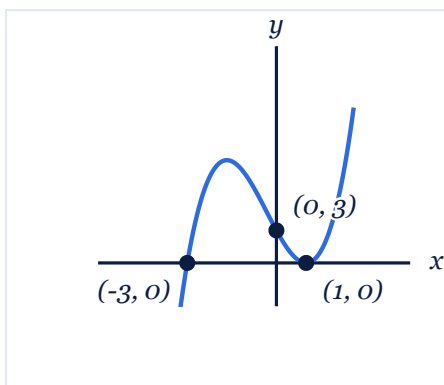
6

The diagram shows a sketch of the cubic curve with equation $y = f(x)$. The curve crosses the x -axis at $(-3, 0)$, touches it at $(1, 0)$ and crosses the y -axis at $(0, 3)$. On separate diagrams, sketch the curves with the following equations, showing the coordinates of the points where each curve meets the coordinate axes.

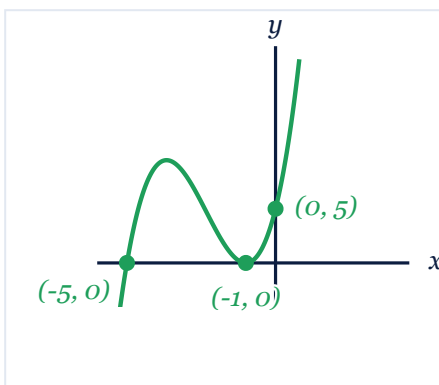
a. $y = f(x + 2)$

b. $y = 2f(x)$

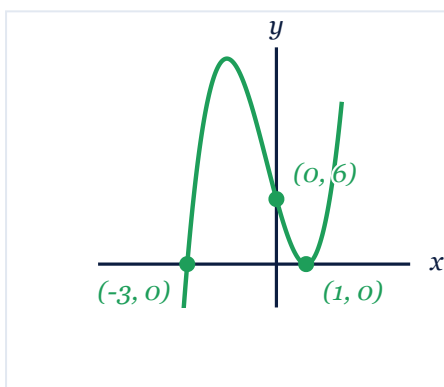
c. $y = f(-x)$



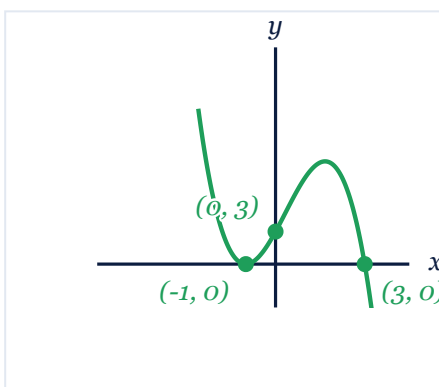
$y = f(x)$



(a)



(b)



(c)

(7 marks)

a) $f(x + 2)$: translation 2 units left, $x - 2$

$$(-3, 0) \rightarrow (-5, 0)$$

$$(1, 0) \rightarrow (-1, 0)$$

New y -intercept is the old height at $x = 2$

From the sketch, $f(x) = k(x + 3)(x - 1)^2$ and $f(0) = 3k = 3$, so $k = 1$

$$f(2) = (5)(1)^2 = 5$$

$$(-5, 0), (-1, 0), (0, 5)$$

b) $2f(x)$: stretch parallel to the y -axis, scale factor 2, $y \times 2$

$$(0, 3) \rightarrow (0, 6)$$

x -intercepts do not move

$$(-3, 0), (1, 0), (0, 6)$$

c) $f(-x)$: reflection in the y-axis, $x \times -1$

$$(-3, 0) \rightarrow (3, 0)$$

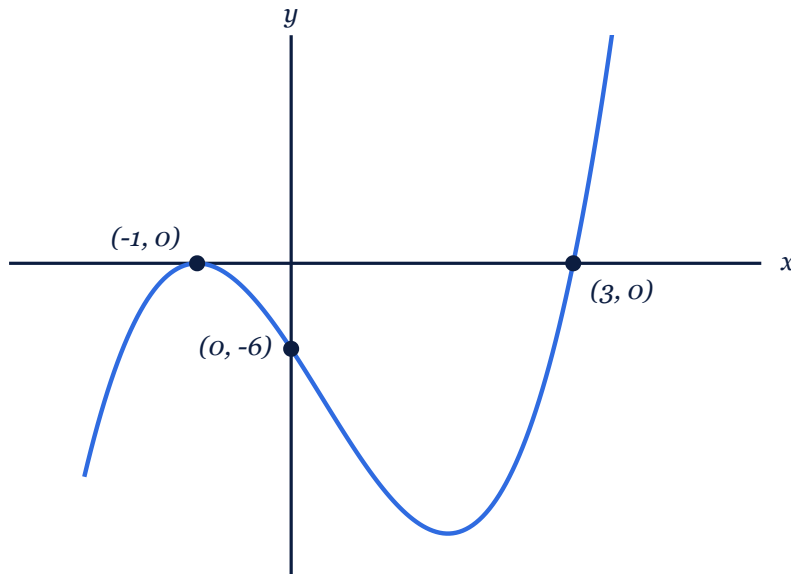
$$(1, 0) \rightarrow (-1, 0)$$

$(0, 3)$ stays on the y-axis

$$(3, 0), (-1, 0), (0, 3)$$

7

The diagram shows a sketch of a cubic curve. The curve touches the x-axis at $(-1, 0)$, crosses the x-axis at $(3, 0)$ and crosses the y-axis at $(0, -6)$. Find an equation for the curve.



(3 marks)

Touches at -1: repeated factor $(x + 1)^2$

Crosses at 3: factor $(x - 3)$

$$y = k(x + 1)^2(x - 3)$$

Use the y-intercept $(0, -6)$

$$-6 = k(0 + 1)^2(0 - 3)$$

$$-6 = -3k$$

$$k = 2$$

$$y = 2(x + 1)^2(x - 3)$$

8

The curve C has equation $y = x^2 - 6x + 13$

- Write $x^2 - 6x + 13$ in the form $(x + a)^2 + b$, where a and b are integers.
- Hence describe fully the transformation that maps the curve with equation $y = x^2$ onto C .
- C is translated k units down so that the new curve touches the x -axis. State the value of k .
- C is instead stretched parallel to the y -axis with scale factor 2. Write down the equation of the new curve.

(6 marks)

a) Complete the square

$$\begin{aligned} x^2 - 6x + 13 &= (x - 3)^2 - 9 + 13 \\ &= (x - 3)^2 + 4 \end{aligned}$$

$$(x - 3)^2 + 4$$

b) $y = (x - 3)^2 + 4$ is $f(x - 3) + 4$ with $f(x) = x^2$

translation by vector $(3, 4)$: 3 right, 4 up

c) The minimum point of C is $(3, 4)$

moving it down 4 puts it on the x -axis

$$k = 4$$

d) Stretch parallel to the y -axis, scale factor 2: $y = 2f(x)$

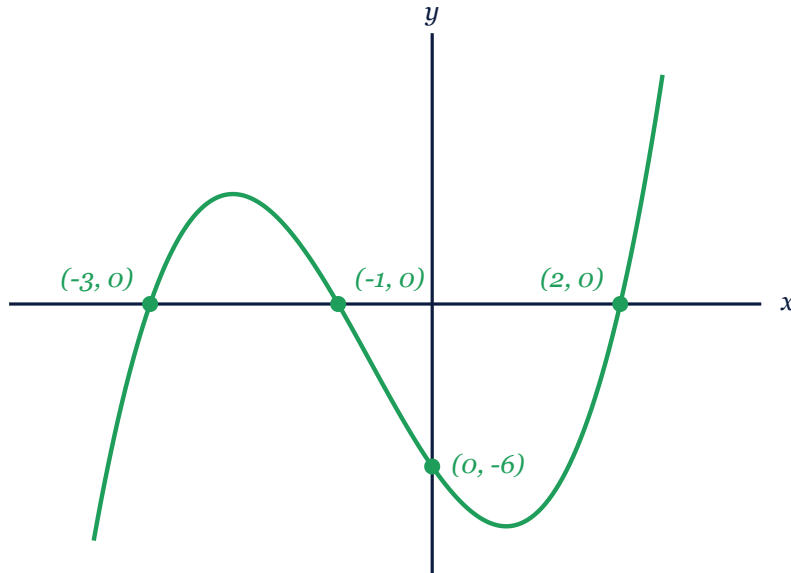
$$y = 2(x^2 - 6x + 13)$$

$$y = 2x^2 - 12x + 26$$

9

$$f(x) = (x - 2)(x + 1)(x + 3)$$

- Sketch the curve with equation $y = f(x)$, showing the coordinates of the points where it meets the coordinate axes.
- The curve with equation $y = f(x + k)$, where k is a constant, passes through the origin. Find the three possible values of k .
- Write down the coordinates of the points where the curve with equation $y = f(2x)$ meets the x -axis.



(6 marks)

a) Roots

$$x = 2, x = -1, x = -3$$

y-intercept

$$f(0) = (-2)(1)(3) = -6$$

see the sketch

b) Through the origin: put $x = 0, y = 0$

$$0 = f(0 + k)$$

$$f(k) = 0$$

So k must be a root of f

$$k = 2, k = -1, k = -3$$

c) $f(2x)$: stretch parallel to the x -axis, scale factor $\frac{1}{2}$, so $x \div 2$

$$2 \div 2 = 1$$

$$-1 \div 2 = -\frac{1}{2}$$

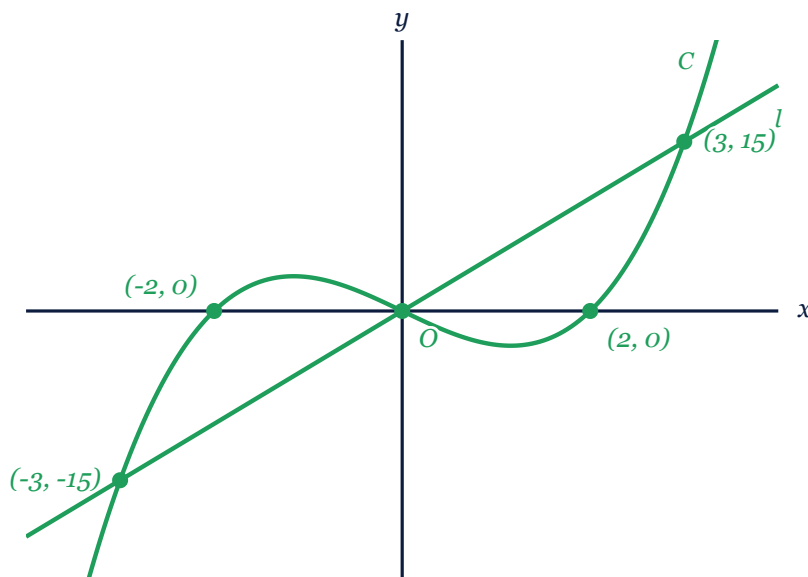
$$-3 \div 2 = -\frac{3}{2}$$

$$\left(-\frac{3}{2}, 0\right), \left(-\frac{1}{2}, 0\right), (1, 0)$$

10

The curve C has equation $y = x^3 - 4x$ and the line l has equation $y = 5x$

- Factorise $x^3 - 4x$ completely.
- On the same axes, sketch C and l , showing the coordinates of the points where C meets the x -axis.
- Find the coordinates of all the points where C and l meet.



(7 marks)

a) Take out the common factor x

$$x^3 - 4x = x(x^2 - 4)$$

Difference of two squares

$$= x(x - 2)(x + 2)$$

$$\boxed{x(x - 2)(x + 2)}$$

b) Positive cubic through -2 , 0 and 2

l passes through 0 with gradient 5

see the sketch

c) Set the equations equal

$$x^3 - 4x = 5x$$

$$x^3 - 9x = 0$$

$$x(x - 3)(x + 3) = 0$$

$$x = 0, x = 3, x = -3$$

Find y from $y = 5x$

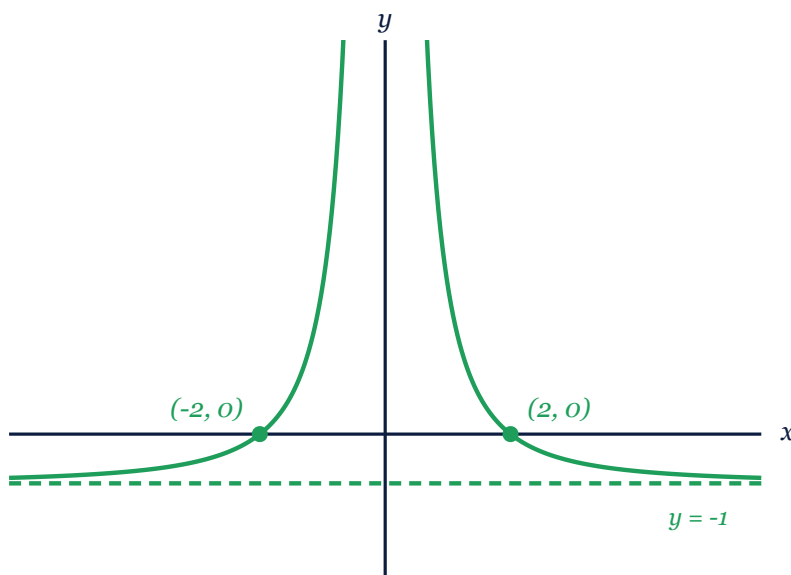
$$y = 0, y = 15, y = -15$$

$$\boxed{(-3, -15), (0, 0), (3, 15)}$$

11

$$f(x) = \frac{4}{x^2}, x \neq 0$$

- Sketch the curve with equation $y = f(x) - 1$, stating the equations of the asymptotes and the coordinates of the points where the curve crosses the x -axis.
- The curve with equation $y = f(x + a)$, where a is a constant, passes through the point $(1, 1)$. Find the possible values of a .
- For each value of a , state the equation of the vertical asymptote of $y = f(x + a)$.



(7 marks)

a) $f(x) - 1$: translation 1 unit down

asymptotes: $x = 0$ and $y = -1$

Crosses the x -axis where $y = 0$

$$\frac{4}{x^2} - 1 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$x = 0, y = -1, (-2, 0), (2, 0)$$

b) Substitute $(1, 1)$ into $y = f(x + a)$

$$\frac{4}{(1+a)^2} = 1$$

$$(1+a)^2 = 4$$

$$1+a = \pm 2$$

$$a = 1 \text{ or } a = -3$$

$$a = 1 \text{ or } a = -3$$

c) Vertical asymptote where $x + a = 0$

$$a = 1: x = -1$$

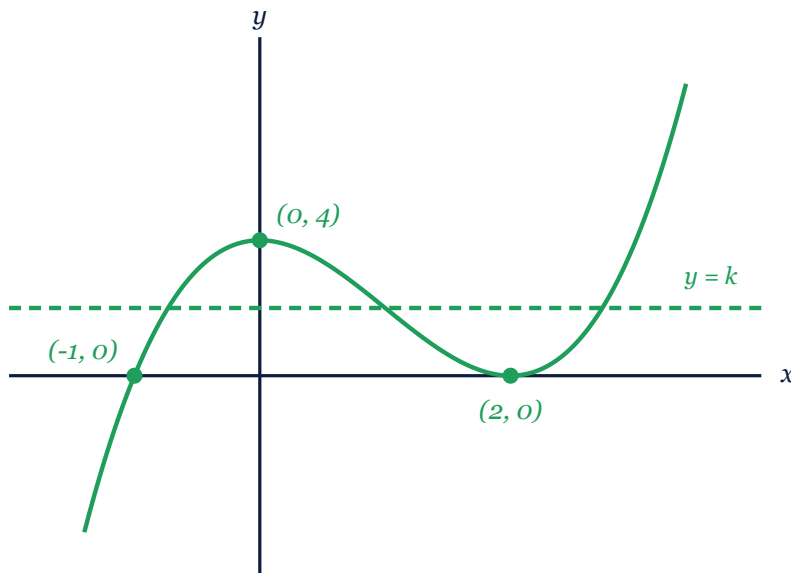
$$a = -3: x = 3$$

$$x = -1 \text{ or } x = 3$$

12

The curve C has equation $y = (x - 2)^2(x + 1)$

- Show that the equation of C can be written as $y = x^3 - 3x^2 + 4$
- Sketch C , showing the coordinates of the points where it meets the coordinate axes.
- Use differentiation to find the coordinates of the turning points of C .
- Hence find the set of values of k for which the equation $(x - 2)^2(x + 1) = k$ has three distinct real roots.



(9 marks)

a) Expand the square first

$$(x - 2)^2 = x^2 - 4x + 4$$

$$y = (x^2 - 4x + 4)(x + 1)$$

$$y = x^3 + x^2 - 4x^2 - 4x + 4x + 4$$

$$y = x^3 - 3x^2 + 4$$

shown

b) Roots: $x = 2$ (repeated, touches) and $x = -1$

y-intercept: $y = 4$

see the sketch

c) Differentiate

$$\frac{dy}{dx} = 3x^2 - 6x$$

$$3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x = 0 \text{ or } x = 2$$

$$x = 0: y = 4$$

$$x = 2: y = 0$$

$$(0, 4) \text{ and } (2, 0)$$

d) Roots of $f(x) = k$ are where the line $y = k$ meets C

three meeting points when $y = k$ lies between the turning points

$$0 < k < 4$$