



- 1 Without using a calculator, find the exact value of $\sin 60^\circ \cos 30^\circ + \tan 45^\circ$

(2 marks)

Exact values

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 45^\circ = 1$$

$$\begin{aligned} \sin 60^\circ \cos 30^\circ + \tan 45^\circ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + 1 \\ &= \frac{3}{4} + 1 \\ &= \frac{7}{4} \end{aligned}$$

$$\boxed{\frac{7}{4}}$$

2

Without using a calculator, find the exact value of

a. $\cos 150^\circ$

b. $\tan 225^\circ$

c. $\sin 300^\circ$

(3 marks)

a) $150^\circ = 180^\circ - 30^\circ$, cos is negative there

$$\begin{aligned}\cos 150^\circ &= -\cos 30^\circ \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

$$\boxed{-\frac{\sqrt{3}}{2}}$$

b) tan has period 180°

$$\begin{aligned}\tan 225^\circ &= \tan 45^\circ \\ &= 1\end{aligned}$$

$$\boxed{1}$$

c) $300^\circ = 360^\circ - 60^\circ$, sin is negative there

$$\begin{aligned}\sin 300^\circ &= -\sin 60^\circ \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

$$\boxed{-\frac{\sqrt{3}}{2}}$$

3

The point $P(90^\circ, 1)$ lies on the curve with equation $y = \sin x$

Write down the coordinates of the point to which P is mapped on the curve with equation

a. $y = 3 \sin x$

b. $y = \sin(x - 60^\circ)$

c. $y = \sin 2x$

d. $y = \sin x - 2$

(4 marks)

a) Vertical stretch, scale factor 3

$$(90^\circ, 3)$$

b) Translate 60° to the right

$$90^\circ + 60^\circ = 150^\circ$$

$$(150^\circ, 1)$$

c) Horizontal stretch, scale factor $\frac{1}{2}$

$$90^\circ \div 2 = 45^\circ$$

$$(45^\circ, 1)$$

d) Translate 2 down

$$1 - 2 = -1$$

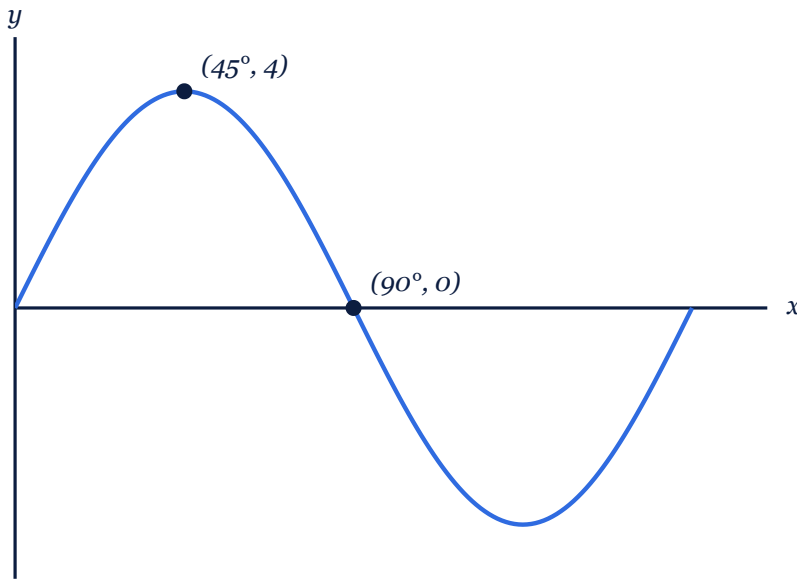
$$(90^\circ, -1)$$

4

The diagram shows part of the curve with equation $y = a \sin bx$, where a and b are positive constants and x is in degrees.

The curve has a maximum point at $(45^\circ, 4)$ and crosses the positive x -axis for the first time at $(90^\circ, 0)$.

Find the value of a and the value of b .



(3 marks)

The maximum of a $\sin bx$ is a

$$a = 4$$

$\sin x$ first crosses at 180° , this crosses at 90°

$$b \times 90^\circ = 180^\circ$$

$$b = 2$$

Check the maximum

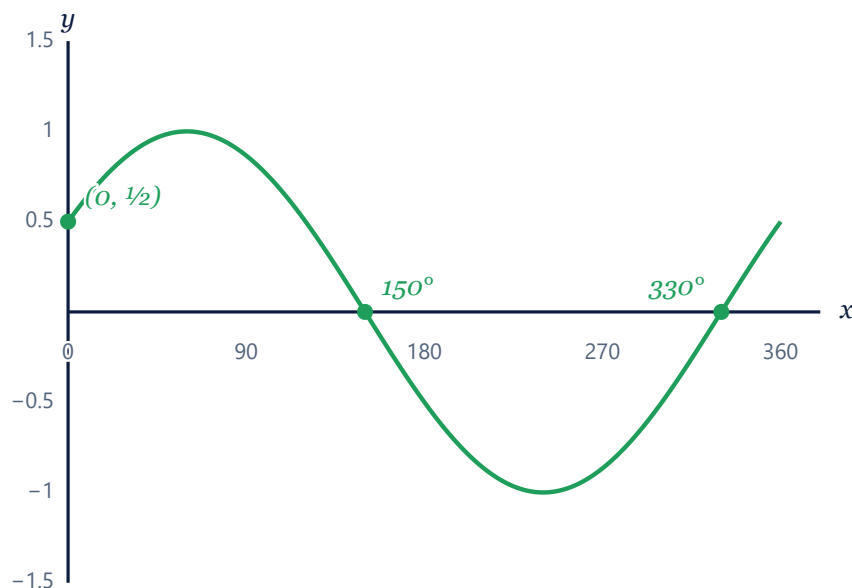
$$4 \sin(2 \times 45^\circ) = 4 \sin 90^\circ = 4$$

$$a = 4, b = 2$$

5

On the axes provided, sketch the graph of $y = \sin(x + 30^\circ)$ for $0 \leq x \leq 360^\circ$

Give the coordinates of each point where the graph meets the coordinate axes.



(4 marks)

$y = \sin x$ translated 30° to the left

y-axis: put $x = 0$

$$y = \sin 30^\circ = \frac{1}{2}$$

x-axis: $\sin(x + 30^\circ) = 0$

$$x + 30^\circ = 180^\circ, 360^\circ$$

$$x = 150^\circ, 330^\circ$$

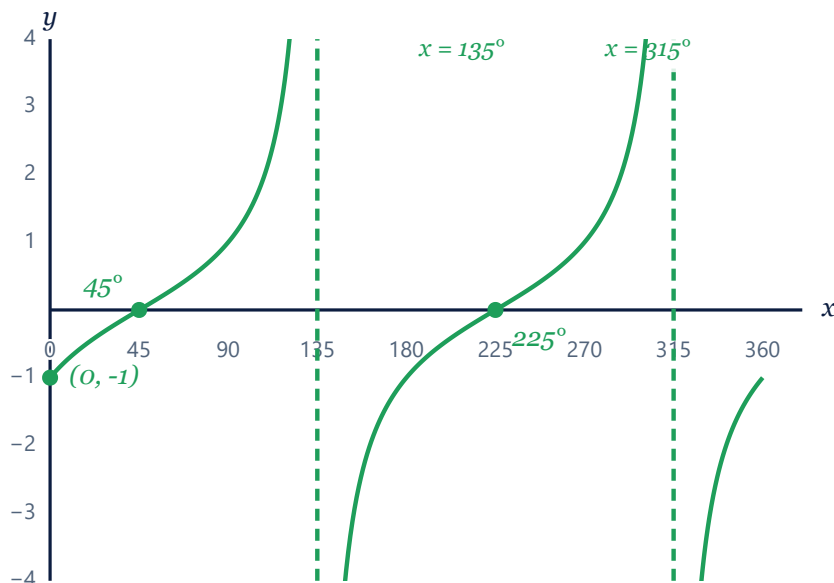
Turning points move 30° left too

max $(60^\circ, 1)$, min $(240^\circ, -1)$

$$(0, \frac{1}{2}), (150^\circ, 0), (330^\circ, 0)$$

6

- a. Describe fully the single transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x - 45^\circ)$
- b. Sketch the graph of $y = \tan(x - 45^\circ)$ for $0 \leq x \leq 360^\circ$, showing the equations of any asymptotes and the coordinates of the points where the graph meets the coordinate axes.



(4 marks)

a)

Translation 45° in the positive x -direction

b) Move everything 45° to the rightasymptotes: $90^\circ + 45^\circ$ and $270^\circ + 45^\circ$

$$x = 135^\circ, x = 315^\circ$$

x-axis: $0^\circ + 45^\circ$ and $180^\circ + 45^\circ$

$$x = 45^\circ, 225^\circ$$

y-axis: put $x = 0$

$$y = \tan(-45^\circ) = -1$$

$$x = 135^\circ, x = 315^\circ; (0, -1), (45^\circ, 0), (225^\circ, 0)$$

7

The curve with equation $y = a + b \cos x$, where a and b are constants and $b > 0$, has a maximum value of 7 and a minimum value of -1

a. Find the value of a and the value of b .

b. Write down the coordinates of the minimum point of the curve for $0 \leq x \leq 360^\circ$

(4 marks)

a) $\cos x$ goes from -1 to 1, and $b > 0$

$$a + b = 7$$

$$a - b = -1$$

Add the equations

$$2a = 6$$

$$a = 3$$

$$b = 7 - 3 = 4$$

$$a = 3, b = 4$$

b) Minimum where $\cos x = -1$

$$x = 180^\circ$$

$$(180^\circ, -1)$$

8

Without using a calculator, show that

$$\frac{1 + \tan 60^\circ}{1 - \tan 60^\circ} = -2 - \sqrt{3}$$

(4 marks)

Exact value

$$\tan 60^\circ = \sqrt{3}$$

$$\frac{1 + \tan 60^\circ}{1 - \tan 60^\circ} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$$

Rationalise: multiply top and bottom by $1 + \sqrt{3}$

$$= \frac{(1 + \sqrt{3})^2}{(1 - \sqrt{3})(1 + \sqrt{3})}$$

$$= \frac{1 + 2\sqrt{3} + 3}{1 - 3}$$

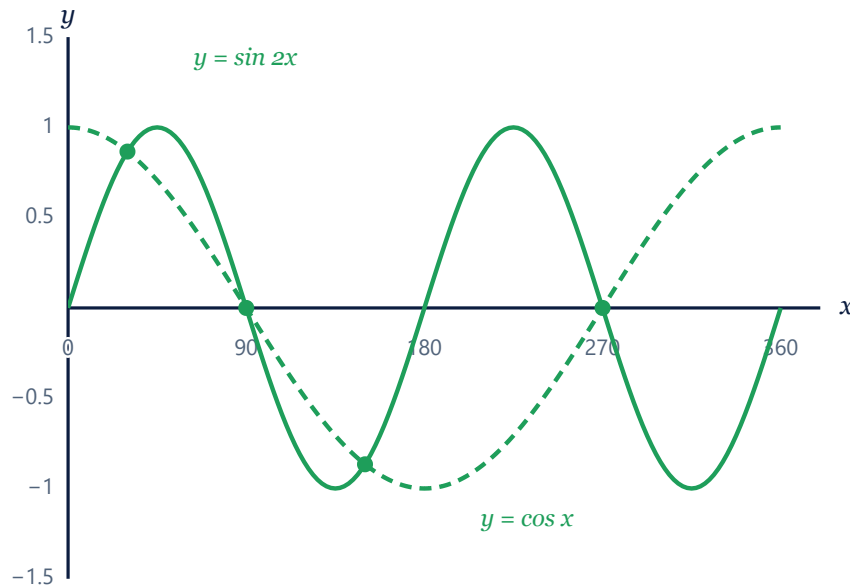
$$= \frac{4 + 2\sqrt{3}}{-2}$$

$$= -2 - \sqrt{3}$$

$$= -2 - \sqrt{3} \text{ as required}$$

9

- a. On the same axes, sketch the graphs of $y = \sin 2x$ and $y = \cos x$ for $0 \leq x \leq 360^\circ$
- b. Hence state the number of solutions of the equation $\sin 2x = \cos x$ in the interval $0 \leq x \leq 360^\circ$, giving a reason for your answer.



(4 marks)

a) $\sin 2x$: stretch of $\sin x$, scale factor $\frac{1}{2}$

period 180° , zeros at $0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$

max at 45° and 225° , min at 135° and 315°

$\cos x$: one cycle

$(0, 1)$, zeros at 90° and 270° , min $(180^\circ, -1)$

see the sketch

b) Count where the curves cross

the graphs intersect 4 times

4 solutions

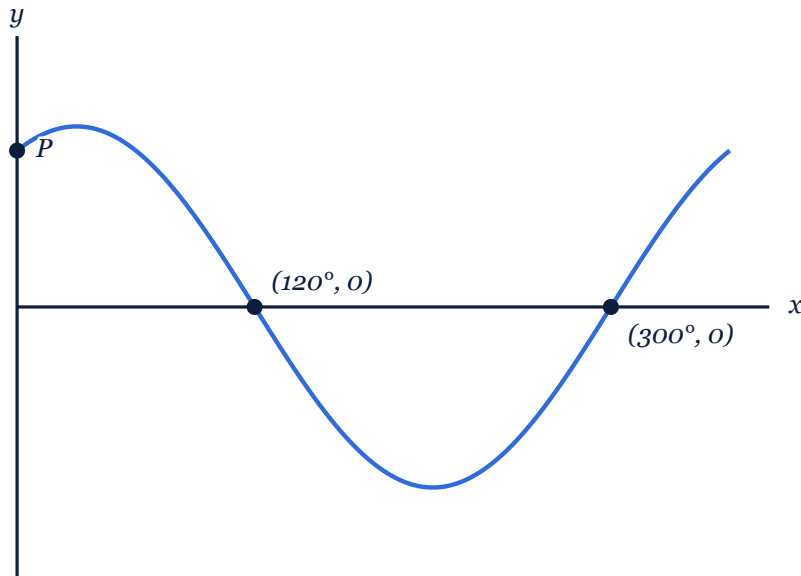
10

The diagram shows part of the curve with equation $y = \sin(x + \alpha)$, where $0 < \alpha < 90^\circ$

The curve crosses the x -axis at $(120^\circ, 0)$ and $(300^\circ, 0)$ and crosses the y -axis at the point P .

a. Find the value of α .

b. Find the exact y -coordinate of P .



(4 marks)

a) $\sin$ is zero at 180° and 360°

$$120^\circ + \alpha = 180^\circ$$

$$\alpha = 60^\circ$$

Check with the other crossing

$$300^\circ + 60^\circ = 360^\circ$$

$$\boxed{\alpha = 60^\circ}$$

b) At P , $x = 0$

$$\begin{aligned} y &= \sin(0 + 60^\circ) \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\boxed{\frac{\sqrt{3}}{2}}$$

11

Given that $\sin 20^\circ = p$, find, in terms of p

- a. $\sin 160^\circ$
- b. $\sin 340^\circ$
- c. $\cos 70^\circ$
- d. $\tan 20^\circ$

(5 marks)

a) $\sin x$ is symmetrical about $x = 90^\circ$

$$\begin{aligned}\sin 160^\circ &= \sin(180^\circ - 20^\circ) \\ &= \sin 20^\circ = p\end{aligned}$$

 p b) $\sin$ is negative between 180° and 360°

$$\begin{aligned}\sin 340^\circ &= -\sin(360^\circ - 340^\circ) \\ &= -\sin 20^\circ = -p\end{aligned}$$

 $-p$ c) In a right-angled triangle, $\cos \theta = \sin(90^\circ - \theta)$

$$\cos 70^\circ = \sin 20^\circ = p$$

 p d) Use $\sin^2 \theta + \cos^2 \theta = 1$

$$\cos^2 20^\circ = 1 - p^2$$

 $\cos 20^\circ > 0$

$$\cos 20^\circ = \sqrt{1 - p^2}$$

$$\begin{aligned}\tan 20^\circ &= \frac{\sin 20^\circ}{\cos 20^\circ} \\ &= \frac{p}{\sqrt{1 - p^2}}\end{aligned}$$

 $\frac{p}{\sqrt{1 - p^2}}$

12

The depth of water, D metres, at the entrance to a harbour in Cornwall is modelled by

$$D = 6 + 2.5 \sin(30t)^\circ, \quad 0 \leq t < 24$$

where t is the time in hours after midnight.

- Find the maximum depth of the water and the first time after midnight at which it occurs.
- State the time, in hours, between one high tide and the next.
- A fishing boat can only pass the entrance when the depth is at least 7.25 m. Find the first period of time after midnight during which the boat can pass.

(6 marks)

a) Largest value of $\sin$ is 1

$$\max D = 6 + 2.5 \times 1 = 8.5$$

First maximum when the angle is 90°

$$30t = 90$$

$$t = 3$$

8.5 m at 03:00

b) One full cycle is 360°

$$30t = 360$$

$$t = 12$$

12 hours

c) Set up the condition

$$6 + 2.5 \sin(30t)^\circ \geq 7.25$$

$$2.5 \sin(30t)^\circ \geq 1.25$$

$$\sin(30t)^\circ \geq 0.5$$

$$\sin \theta = 0.5 \text{ at } \theta = 30^\circ \text{ and } 180^\circ - 30^\circ = 150^\circ$$

$$\sin \theta \geq 0.5 \text{ for } 30^\circ \leq \theta \leq 150^\circ$$

$$30 \leq 30t \leq 150$$

$$1 \leq t \leq 5$$

from 01:00 to 05:00