



- 1 Solve, for $0 \leq x < 360^\circ$, the equation

$$\sin x = 0.4$$

Give your answers to 1 decimal place.

(2 marks)

$$\begin{aligned} x &= \sin^{-1}(0.4) \\ &= 23.57...^\circ \end{aligned}$$

Second solution: $\sin x = \sin(180^\circ - x)$

$$\begin{aligned} x &= 180^\circ - 23.57...^\circ \\ &= 156.42...^\circ \end{aligned}$$

$$\boxed{x = 23.6^\circ, 156.4^\circ}$$

- 2 Solve, for $0 \leq x < 360^\circ$, the equation

$$\tan x = -2$$

Give your answers to 1 decimal place.

(3 marks)

$$\tan^{-1}(-2) = -63.43...^\circ$$

Not in the interval, so add 180° (period of tan)

$$x = -63.43...^\circ + 180^\circ = 116.56...^\circ$$

$$x = 116.56...^\circ + 180^\circ = 296.56...^\circ$$

$$\boxed{x = 116.6^\circ, 296.6^\circ}$$

3 Show that

$$1 - \frac{\cos^2 x}{\cos x} \equiv \sin x \tan x, \quad \cos x \neq 0$$

(3 marks)

Start with the left-hand side

Use $\sin^2 x + \cos^2 x = 1$

$$\begin{aligned} \frac{1 - \cos^2 x}{\cos x} &= \frac{\sin^2 x}{\cos x} \\ &= \sin x \times \frac{\sin x}{\cos x} \end{aligned}$$

Use $\tan x = \frac{\sin x}{\cos x}$

$$= \sin x \tan x$$

$$\boxed{\text{LHS} = \sin x \tan x = \text{RHS}}$$

4 Solve, for $0 \leq x < 360^\circ$, the equation

$$2 \cos(x + 30^\circ) = 1$$

(3 marks)

$$\cos(x + 30^\circ) = \frac{1}{2}$$

Change the interval

$$30^\circ \leq x + 30^\circ < 390^\circ$$

$$\cos^{-1}\left(\frac{1}{2}\right) = 60^\circ, \text{ and } 360^\circ - 60^\circ = 300^\circ$$

$$x + 30^\circ = 60^\circ, 300^\circ$$

$$x = 30^\circ, 270^\circ$$

$$\boxed{x = 30^\circ, 270^\circ}$$

5

Solve, for $0 \leq x < 360^\circ$, the equation

$$\sin 2x = 0.6$$

Give your answers to 1 decimal place.

(4 marks)

Double the interval

$$0 \leq 2x < 720^\circ$$

$$2x = \sin^{-1}(0.6) = 36.86...^\circ$$

$$2x = 180^\circ - 36.86...^\circ = 143.13...^\circ$$

Add 360° to each

$$2x = 396.86...^\circ, 503.13...^\circ$$

Halve every value

$$x = 198.43...^\circ, 251.56...^\circ, 198.43...^\circ, 251.56...^\circ$$

$$x = 18.4^\circ, 71.6^\circ, 198.4^\circ, 251.6^\circ$$

6

Solve, for $0 \leq x < 360^\circ$, the equation

$$3 \sin x = 2 \cos x$$

Give your answers to 1 decimal place.

(3 marks)

Divide both sides by $3 \cos x$

$$\frac{\sin x}{\cos x} = \frac{2}{3}$$

$$\tan x = \frac{2}{3}$$

$$x = \tan^{-1}\left(\frac{2}{3}\right) = 33.69...^\circ$$

tan has period 180°

$$x = 33.69...^\circ + 180^\circ = 213.69...^\circ$$

$$x = 33.7^\circ, 213.7^\circ$$

7

Given that $\sin \theta = \frac{2}{5}$ and that θ is obtuse, find the exact value of

- $\cos \theta$
- $\tan \theta$, giving your answer with a rational denominator.

(4 marks)

a) Use $\sin^2 \theta + \cos^2 \theta = 1$

$$\begin{aligned}\cos^2 \theta &= 1 - \left(\frac{2}{5}\right)^2 \\ &= 1 - \frac{4}{25} = \frac{21}{25}\end{aligned}$$

θ is obtuse, so $\cos \theta < 0$

$$\cos \theta = -\frac{\sqrt{21}}{5}$$

$$\boxed{-\frac{\sqrt{21}}{5}}$$

b) $\tan \theta = \frac{\sin \theta}{\cos \theta}$

$$\begin{aligned}\tan \theta &= \frac{2}{5} \div \left(-\frac{\sqrt{21}}{5}\right) \\ &= -\frac{2}{\sqrt{21}}\end{aligned}$$

Rationalise the denominator

$$= -\frac{2\sqrt{21}}{21}$$

$$\boxed{-\frac{2\sqrt{21}}{21}}$$

8

Kofi is asked to solve, for $0 \leq x < 360^\circ$, the equation

$$4 \sin x \cos x = 3 \sin x$$

His working is

$$4 \cos x = 3 \quad \text{so} \quad \cos x = 0.75 \quad \text{so} \quad x = 41.4^\circ \text{ or } 318.6^\circ$$

- Identify the error in Kofi's working.
- Find all the solutions of the equation in the interval, giving non-exact answers to 1 decimal place. (4 marks)

a)

He divided by $\sin x$, but $\sin x$ can be 0,
so the solutions of $\sin x = 0$ are lost

b) Bring everything to one side and factorise

$$4 \sin x \cos x - 3 \sin x = 0$$

$$\sin x(4 \cos x - 3) = 0$$

$$\sin x = 0$$

$$x = 0^\circ, 180^\circ$$

$$\cos x = 0.75$$

$$x = 41.40\dots^\circ$$

$$x = 360^\circ - 41.40\dots^\circ = 318.59\dots^\circ$$

$$x = 0^\circ, 41.4^\circ, 180^\circ, 318.6^\circ$$

9

a. Show that the equation $2 \cos^2 x + 3 \sin x = 3$ can be written as $2 \sin^2 x - 3 \sin x + 1 = 0$

b. Hence solve, for $0 \leq x < 360^\circ$, the equation $2 \cos^2 x + 3 \sin x = 3$

(6 marks)

a) Use $\cos^2 x = 1 - \sin^2 x$

$$2(1 - \sin^2 x) + 3 \sin x = 3$$

$$2 - 2 \sin^2 x + 3 \sin x = 3$$

$$0 = 2 \sin^2 x - 3 \sin x + 1$$

$$2 \sin^2 x - 3 \sin x + 1 = 0 \text{ as required}$$

b) A quadratic in $\sin x$: factorise

$$(2 \sin x - 1)(\sin x - 1) = 0$$

$$\sin x = \frac{1}{2}$$

$$x = 30^\circ, 180^\circ - 30^\circ = 150^\circ$$

$$\sin x = 1$$

$$x = 90^\circ$$

$$x = 30^\circ, 90^\circ, 150^\circ$$

10

Solve, for $-180^\circ \leq x \leq 180^\circ$, the equation

$$\tan(2x - 15^\circ) = \sqrt{3}$$

(5 marks)

Change the interval

$$-360^\circ \leq 2x \leq 360^\circ$$

$$-375^\circ \leq 2x - 15^\circ \leq 345^\circ$$

$$\tan^{-1}(\sqrt{3}) = 60^\circ, \text{ then add or take } 180^\circ$$

$$2x - 15^\circ = -300^\circ, -120^\circ, 60^\circ, 240^\circ$$

Add 15°

$$2x = -285^\circ, -105^\circ, 75^\circ, 255^\circ$$

Halve

$$x = -142.5^\circ, -52.5^\circ, 37.5^\circ, 127.5^\circ$$

$$x = -142.5^\circ, -52.5^\circ, 37.5^\circ, 127.5^\circ$$

- a. Show that the equation $3 \tan x \sin x = 8$ can be written as $3 \cos^2 x + 8 \cos x - 3 = 0$
- b. Hence solve, for $0 \leq x < 360^\circ$, the equation $3 \tan x \sin x = 8$, giving your answers to 1 decimal place.
- c. Hence find the solutions of $3 \tan 2\theta \sin 2\theta = 8$ for $0 \leq \theta < 180^\circ$, giving your answers to 1 decimal place.

(8 marks)

a) Use $\tan x = \frac{\sin x}{\cos x}$

$$3 \times \frac{\sin x}{\cos x} \times \sin x = 8$$

$$3 \sin^2 x = 8 \cos x$$

Use $\sin^2 x = 1 - \cos^2 x$

$$3(1 - \cos^2 x) = 8 \cos x$$

$$3 - 3 \cos^2 x = 8 \cos x$$

$$0 = 3 \cos^2 x + 8 \cos x - 3$$

$$3 \cos^2 x + 8 \cos x - 3 = 0 \text{ as required}$$

b) Factorise the quadratic in $\cos x$

$$(3 \cos x - 1)(\cos x + 3) = 0$$

$\cos x = -3$ is impossible, since $-1 \leq \cos x \leq 1$

$$\cos x = \frac{1}{3}$$

$$x = \cos^{-1}\left(\frac{1}{3}\right) = 70.52\dots^\circ$$

$$x = 360^\circ - 70.52\dots^\circ = 289.47\dots^\circ$$

$$x = 70.5^\circ, 289.5^\circ$$

c) Same equation with $x = 2\theta$

$$0 \leq \theta < 180^\circ \text{ so } 0 \leq 2\theta < 360^\circ$$

$$2\theta = 70.52\dots^\circ, 289.47\dots^\circ$$

$$\theta = 35.26\dots^\circ, 144.73\dots^\circ$$

$$\theta = 35.3^\circ, 144.7^\circ$$

12

A passenger on a big wheel in Blackpool is at height h metres above the ground, t minutes after the ride starts, where

$$h = 20 - 18 \cos(12t)^\circ, \quad t \geq 0$$

- Find the height of the passenger when the ride starts.
- Find the maximum height of the passenger and the time taken for one full turn of the wheel.
- Find the times during the first turn when the passenger is 29 m above the ground.

(6 marks)

a) Put $t = 0$

$$\begin{aligned} h &= 20 - 18 \cos 0^\circ \\ &= 20 - 18 = 2 \end{aligned}$$

2 m

b) Largest h when $\cos(12t)^\circ = -1$

$$h = 20 - 18(-1) = 38$$

One cycle of $\cos$ is 360°

$$12t = 360$$

$$t = 30$$

38 m; 30 minutes

c) Set $h = 29$

$$20 - 18 \cos(12t)^\circ = 29$$

$$-18 \cos(12t)^\circ = 9$$

$$\cos(12t)^\circ = -\frac{1}{2}$$

First turn: $0 \leq 12t < 360$

$$12t = 120, 360 - 120 = 240$$

$$t = 10, 20$$

after 10 minutes and after 20 minutes