

Differentiation

A-level Pure Mathematics (Year 2) | Edexcel 9MA0 | 12 questions, 79 marks

A-level Maths topic practice



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Name: _____ **Date:** _____ **Score:** _____ / 79

1 Find $\frac{dy}{dx}$ for each of the following.

a. $y = (4x^3 + 1)^5$

b. $y = \ln(5x^2 + 2)$

c. $y = \sec 4x$

(5 marks)

Answer _____

2 a. Given $y = x^3 e^{2x}$, show that $\frac{dy}{dx} = x^2 e^{2x}(a + bx)$, where a and b are integers to be found.

b. Given $y = \frac{3x - 1}{x^2 + 2}$, find $\frac{dy}{dx}$, giving your answer as a single fraction in its simplest form.

(5 marks)

Answer _____

3

- a. The curve with equation $y = 3^{2x}$ passes through the point P where $x = 1$. Find the exact gradient of the curve at P , giving your answer in the form $k \ln 3$
- b. Given that $x = \cos y$, where $0 < y < \pi$, show that

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

(6 marks)

Answer _____

4

Given that x is measured in radians, prove, from first principles, that the derivative of $\sin 3x$ is $3 \cos 3x$

You may assume the formula for $\sin(A \pm B)$ and that, as $h \rightarrow 0$, $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$

(5 marks)

Answer _____

5

The curve C has equation $y = (x^2 - 3)e^x$

- Find the exact coordinates of the stationary points of C .
- Use $\frac{d^2y}{dx^2}$ to determine the nature of each stationary point.

(7 marks)

Answer _____

6

The curve C has equation

$$x^2 + 3xy - y^2 = 27$$

- Find $\frac{dy}{dx}$ in terms of x and y .
- The point $P(3, 3)$ lies on C . Find an equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(6 marks)

Answer _____

7 The curve C has equation

$$y = \frac{\sin x}{1 + \cos x}, \quad 0 < x < \pi$$

a. Show that $\frac{dy}{dx} = \frac{1}{1 + \cos x}$

b. Find the exact x coordinate of the point on C where the gradient is $\frac{2}{3}$

(5 marks)

Answer _____

8 $f(x) = x^4 - 6x^3 + 12x^2 + 5$

a. Find $f''(x)$

b. Find the set of values of x for which $f(x)$ is concave.

c. Find the coordinates of the points of inflection on the curve $y = f(x)$, and show that neither of them is a stationary point.

(6 marks)

Answer _____

9

Water is poured into a container in the shape of an inverted right circular cone at a constant rate of $40 \text{ cm}^3 \text{ s}^{-1}$. When the depth of water is h cm, the radius of the water surface is $\frac{1}{2}h$ cm.

- Show that the volume of water, $V \text{ cm}^3$, is given by $V = \frac{\pi h^3}{12}$
- Find the rate at which the depth of the water is increasing at the instant when $h = 8$, giving your answer to 3 significant figures.
- Find the exact rate at which the area of the water surface is increasing at this instant.

(7 marks)

Answer _____

10

The curve C has equation

$$x^2 + xy + y^2 = 12$$

- Find $\frac{dy}{dx}$ in terms of x and y .
- Find the coordinates of the points on C where the tangent is parallel to the x -axis.
- Find the coordinates of the points on C where the tangent is parallel to the y -axis.

(8 marks)

Answer _____

11

The number of people, N thousand, who have downloaded a new revision app t weeks after its launch is modelled by

$$N = \frac{40}{1 + 7e^{-0.5t}}, \quad t \geq 0$$

- State the number of downloads at launch and the long-term limit of the number of downloads according to the model.
- Show that $\frac{dN}{dt} = \frac{N(40 - N)}{80}$
- Hence find the greatest rate of downloads, in thousands per week, according to the model, and the value of t at which it occurs. Give t to 3 significant figures.
- Explain why the graph of N against t has a point of inflection at this value of t .

(9 marks)

Answer

12 The curve C has equation $y = f(x)$, where

$$f(x) = \frac{\ln x}{x^2}, \quad x > 0$$

- Show that $f'(x) = 1 - \frac{2 \ln x}{x^3}$
- Find the exact coordinates of the stationary point of C , and use $f''(x)$ to show that it is a maximum.
- Find the set of values of x for which C is convex, giving the boundary in the form e^k
- Given that $f(x) \rightarrow 0$ as $x \rightarrow \infty$, state, with a reason, the number of solutions of $f(x) = k$ for $0 < k < \frac{1}{2e}$

(10 marks)

Answer

End of questions | Total 79 marks