

Measures of location and spread

Worked solutions

A-level Maths topic practice



This topic online

- 1 Eleven commuters recorded the time, in minutes, of their journey to work one morning.

12 15 9 22 15 18 31 14 15 20 11

- Write down the mode.
- Find the median.
- Find the interquartile range.
- Calculate the mean journey time.

(4 marks)

Put the data in order first

9, 11, 12, 14, 15, 15, 15, 18, 20, 22, 31

a) Most common value

Mode = 15 minutes

b) $n = 11$

$$\frac{n}{2} = 5.5$$

not a whole number, so round up: 6th value

Median = 15 minutes

c) Lower quartile

$$\frac{n}{4} = 2.75$$

round up: 3rd value

$$Q_1 = 12$$

Upper quartile

$$\frac{3n}{4} = 8.25$$

round up: 9th value

$$Q_3 = 20$$

$$IQR = 20 - 12$$

IQR = 8 minutes

d) Add up all 11 times

$$\sum x = 182$$

$$\text{Mean} = \frac{182}{11}$$

$$\text{Mean} = 16.545...$$

Mean = 16.5 minutes (3 s.f.)

2

The times, t minutes, taken by 40 runners to complete a 5 km park run are summarised by

$$\Sigma t = 1260 \quad \Sigma t^2 = 41\,200$$

- Calculate the mean time.
- Calculate the standard deviation of the times.
- One more runner completes the course in exactly the mean time. State, giving a reason, the effect on the standard deviation of including this runner.

(4 marks)

$$a) \text{ Mean} = \frac{\Sigma t}{n}$$

$$\text{Mean} = \frac{1260}{40}$$

$$\text{Mean} = 31.5 \text{ minutes}$$

$$b) \text{ Variance} = \frac{\Sigma t^2}{n} - (\text{mean})^2$$

$$\sigma^2 = \frac{41\,200}{40} - 31.5^2$$

$$\sigma^2 = 1030 - 992.25$$

$$\sigma^2 = 37.75$$

$$\sigma = \sqrt{37.75}$$

$$\sigma = 6.144...$$

$$\sigma = 6.14 \text{ minutes (3 s.f.)}$$

c) Think about the deviation of the new time

$$\text{New time} - \text{mean} = 0$$

so the sum of squared deviations stays the same

but we now divide by 41 instead of 40

Standard deviation decreases

3

The table shows the number of goals, x , scored by a hockey team in each of 30 matches.

Goals, x	0	1	2	3	4	5
Frequency	4	8	9	5	3	1

- Write down the mode.
- Find the median.
- Calculate the mean number of goals.
- Calculate the standard deviation of the number of goals.

(5 marks)

a) Highest frequency is 9

$$\text{Mode} = 2$$

b) $n = 30$

$$\frac{n}{2} = 15$$

whole number, so use the 15th and 16th values

Cumulative frequency: 4, 12, 21, ...

15th and 16th values are both 2

$$\text{Median} = 2$$

c) Multiply each x by its frequency

$$\sum fx = 0 + 8 + 18 + 15 + 12 + 5$$

$$\sum fx = 58$$

$$\text{Mean} = \frac{58}{30}$$

$$\text{Mean} = 1.933...$$

$$\text{Mean} = 1.93 \text{ (3 s.f.)}$$

d) Need $\sum fx^2$

$$\sum fx^2 = 0 + 8 + 36 + 45 + 48 + 25$$

$$\sum fx^2 = 162$$

$$\text{Variance} = \frac{\sum fx^2}{n} - (\text{mean})^2$$

$$\sigma^2 = \frac{162}{30} - \left(\frac{58}{30}\right)^2$$

$$\sigma^2 = 5.4 - 3.7377...$$

$$\sigma^2 = 1.6622...$$

$$\sigma = \sqrt{1.6622...}$$

$$\sigma = 1.29 \text{ (3 s.f.)}$$

4

The masses, m grams, of 80 apples are summarised in the table.

Mass, m (grams)	Frequency
$100 \leq m < 120$	6
$120 \leq m < 130$	14
$130 \leq m < 140$	25
$140 \leq m < 150$	21
$150 \leq m < 170$	14

- Use linear interpolation to estimate the median mass.
- Estimate the mean and the standard deviation of the masses.
- Explain why your answers to part (b) are estimates.

(7 marks)

a) Median is the $\frac{n}{2} = 40\text{th value}$

Cumulative frequency: 6, 20, 45, 66, 80

40th value is in the $130 \leq m < 140$ class

20 apples before the class, 25 in it, width 10

$$\text{Median} = 130 + \frac{40 - 20}{25} \times 10$$

$$\text{Median} = 130 + 8$$

$$\boxed{\text{Median} = 138 \text{ g}}$$

b) Use the class midpoints

$$x = 110, 125, 135, 145, 160$$

$$\sum fx = 660 + 1750 + 3375 + 3045 + 2240$$

$$\sum fx = 11070$$

$$\text{Mean} = \frac{11070}{80}$$

$$\text{Mean} = 138.375$$

$$\boxed{\text{Mean} = 138 \text{ g (3 s.f.)}}$$

$$\sum fx^2 = 72600 + 218750 + 455625 + 441525 + 358400$$

$$\sum fx^2 = 1546900$$

$$\sigma^2 = \frac{1546900}{80} - 138.375^2$$

$$\sigma^2 = 19336.25 - 19147.64...$$

$$\sigma^2 = 188.609...$$

$$\sigma = \sqrt{188.609...}$$

$$\sigma = 13.7 \text{ g (3 s.f.)}$$

c) Why estimates?

We only know the class each apple is in,
not its actual mass.

Each apple is taken to be at its class midpoint.

5

A machine fills bottles with olive oil. The volume, x ml, in each of 20 bottles is measured and the data are coded using

$$y = \frac{x - 500}{2}$$

The coded data are summarised by $\Sigma y = 36$ and $\Sigma y^2 = 410$

- Find the mean of x .
- Find the standard deviation of x .
- Find the value of Σx^2 .

(6 marks)

Rearrange the code

$$x = 2y + 500$$

a) Mean of the coded data

$$\text{Mean of } y = \frac{36}{20}$$

$$\text{Mean of } y = 1.8$$

$$\text{Mean of } x = 2(1.8) + 500$$

$$\text{Mean of } x = 503.6 \text{ mL}$$

b) Variance of the coded data

$$\text{Var}(y) = \frac{410}{20} - 1.8^2$$

$$\text{Var}(y) = 20.5 - 3.24$$

$$\text{Var}(y) = 17.26$$

$$\text{sd of } y = 4.1545...$$

Adding 500 does not change the spread; $\times 2$ doubles it

$$\text{sd of } x = 2 \times 4.1545...$$

$$\text{sd of } x = 8.309...$$

$$\text{sd of } x = 8.31 \text{ mL (3 s.f.)}$$

c) Use $\text{Var}(x) = \frac{\Sigma x^2}{n} - (\text{mean})^2$

$$\text{Var}(x) = 2^2 \times 17.26$$

$$\text{Var}(x) = 69.04$$

$$\frac{\Sigma x^2}{20} = 69.04 + 503.6^2$$

$$\frac{\Sigma x^2}{20} = 69.04 + 253\,612.96$$

$$\frac{\sum x^2}{20} = 253\,682$$

$$\sum x^2 = 20 \times 253\,682$$

$$\boxed{\sum x^2 = 5\,073\,640}$$

6

A helpline records the time, t minutes, that each of 200 callers waits before their call is answered.

Waiting time, t (minutes)	Frequency
$0 \leq t < 2$	18
$2 \leq t < 4$	46
$4 \leq t < 6$	62
$6 \leq t < 10$	48
$10 \leq t < 15$	20
$15 \leq t < 25$	6

- Use linear interpolation to estimate the median waiting time.
- Use linear interpolation to estimate the 10th to 90th interpercentile range.
- Estimate the number of callers who waited more than 8 minutes.

(6 marks)

Cumulative frequencies

18, 64, 126, 174, 194, 200

a) Median: $\frac{200}{2} = 100\text{th value, in } 4 \leq t < 6$

$$\text{Median} = 4 + \frac{100 - 64}{62} \times 2$$

$$\text{Median} = 5.161\dots$$

$$\boxed{\text{Median} = 5.16 \text{ minutes (3 s.f.)}}$$

b) 10th percentile: $\frac{10}{100} \times 200 = 20\text{th value, in } 2 \leq t < 4$

$$p_{10} = 2 + \frac{20 - 18}{46} \times 2$$

$$p_{10} = 2.0869\dots$$

90th percentile: $\frac{90}{100} \times 200 = 180\text{th value, in } 10 \leq t < 15$

$$p_{90} = 10 + \frac{180 - 174}{20} \times 5$$

$$p_{90} = 11.5$$

$$p_{90} - p_{10} = 11.5 - 2.0869\dots$$

$$p_{90} - p_{10} = 9.413\dots$$

$$\boxed{\text{Range} = 9.41 \text{ minutes (3 s.f.)}}$$

c) More than 8 minutes

8 to 10 is 2 of the 4 minutes in the $6 \leq t < 10$ class

$$\frac{2}{4} \times 48 = 24$$

$$24 + 20 + 6 = 50$$

About 50 callers

7

Two Year 12 classes sit the same test.

Class	Number of students	Mean mark	Standard deviation
A	12	64	5
B	18	59	7

Find the mean and the standard deviation of the marks of all 30 students.

(6 marks)

Rebuild $\sum x$ and $\sum x^2$ for each class

$$\sigma^2 = \frac{\sum x^2}{n} - (\text{mean})^2, \text{ so } \sum x^2 = n(\sigma^2 + \text{mean}^2)$$

Class A

$$\sum x = 12 \times 64 = 768$$

$$\sum x^2 = 12(5^2 + 64^2)$$

$$\sum x^2 = 12 \times 4121 = 49\,452$$

Class B

$$\sum x = 18 \times 59 = 1062$$

$$\sum x^2 = 18(7^2 + 59^2)$$

$$\sum x^2 = 18 \times 3530 = 63\,540$$

Combine

$$\sum x = 768 + 1062 = 1830$$

$$\sum x^2 = 49\,452 + 63\,540 = 112\,992$$

$$\text{Mean} = \frac{1830}{30}$$

$$\text{Mean} = 61$$

$$\sigma^2 = \frac{112\,992}{30} - 61^2$$

$$\sigma^2 = 3766.4 - 3721$$

$$\sigma^2 = 45.4$$

$$\sigma = \sqrt{45.4}$$

$$\sigma = 6.74 \text{ (3 s.f.)}$$

8

Marek measured the lengths, in cm, of 15 fish caught in a lake. He found the mean length was 52 cm and the standard deviation was 8 cm.

He then noticed that one length had been written down as 48 cm when it should have been 84 cm.

Find the correct mean and the correct standard deviation of the lengths.

(5 marks)

Work out the original sums

$$\sum x = 15 \times 52 = 780$$

$$\sum x^2 = 15(8^2 + 52^2)$$

$$\sum x^2 = 15 \times 2768 = 41\,520$$

Take out 48 and put in 84

$$\sum x = 780 - 48 + 84 = 816$$

$$\sum x^2 = 41\,520 - 48^2 + 84^2$$

$$\sum x^2 = 41\,520 - 2304 + 7056 = 46\,272$$

New mean

$$\text{Mean} = \frac{816}{15}$$

$$\boxed{\text{Mean} = 54.4 \text{ cm}}$$

New standard deviation

$$\sigma^2 = \frac{46\,272}{15} - 54.4^2$$

$$\sigma^2 = 3084.8 - 2959.36$$

$$\sigma^2 = 125.44$$

$$\sigma = \sqrt{125.44}$$

$$\boxed{\sigma = 11.2 \text{ cm}}$$

The prices, in thousands of pounds, of the nine houses sold in a village last year were

185 210 196 224 202 1150 215 190 207

- Calculate the mean price.
- Find the median price.
- An outlier is a value greater than $Q_3 + 1.5 \times (Q_3 - Q_1)$ or less than $Q_1 - 1.5 \times (Q_3 - Q_1)$. Show that 1150 is an outlier.
- State, giving a reason, whether the mean or the median gives a better average price for a house in this village.

(6 marks)

a) Add the prices

$$\sum x = 2779$$

$$\text{Mean} = \frac{2779}{9}$$

$$\text{Mean} = 308.77...$$

$$\text{Mean} = \text{£}309 \text{ thousand (3 s.f.)}$$

b) Put in order

185, 190, 196, 202, 207, 210, 215, 224, 1150

$n = 9$, so the median is the 5th value

$$\text{Median} = \text{£}207 \text{ thousand}$$

c) Quartiles

$$\frac{n}{4} = 2.25$$

round up: 3rd value

$$Q_1 = 196$$

$$\frac{3n}{4} = 6.75$$

round up: 7th value

$$Q_3 = 215$$

$$\text{IQR} = 215 - 196 = 19$$

Upper limit for outliers

$$Q_3 + 1.5 \times \text{IQR} = 215 + 1.5 \times 19$$

$$Q_3 + 1.5 \times \text{IQR} = 243.5$$

$$1150 > 243.5$$

$$\text{So } 1150 \text{ is an outlier}$$

d) Compare with the data

Eight of the nine prices are below the mean

The mean is dragged up by the outlier

Median is better

10

A group of people were asked how many times, x , they went to the cinema last month. The results are shown in the table, where f is a positive integer.

x	0	1	2	3	4
Frequency	7	12	f	6	3

The mean number of visits is 1.6

- Show that $f = 7$
- Calculate the standard deviation of x .
- Find the median of x .

(6 marks)

a) Write the mean in terms of f

$$\sum f = 7 + 12 + f + 6 + 3 = 28 + f$$

$$\sum fx = 0 + 12 + 2f + 18 + 12 = 42 + 2f$$

$$\frac{42 + 2f}{28 + f} = 1.6$$

$$28 + f$$

$$42 + 2f = 1.6(28 + f)$$

$$42 + 2f = 44.8 + 1.6f$$

$$0.4f = 2.8$$

$$f = 7$$

b) Now $n = 35$ and $\sum fx = 56$

$$\sum fx^2 = 0 + 12 + 28 + 54 + 48$$

$$\sum fx^2 = 142$$

$$\sigma^2 = \frac{142}{35} - 1.6^2$$

$$\sigma^2 = 4.0571... - 2.56$$

$$\sigma^2 = 1.4971...$$

$$\sigma = \sqrt{1.4971...}$$

$$\sigma = 1.22 \text{ (3 s.f.)}$$

c) $n = 35$, so the median is the 18th value

Cumulative frequency: 7, 19, ...

18th value is in $x = 1$

$$\text{Median} = 1$$

11

Anya records the daily maximum temperature, x °C, at a weather station in Scotland for the 31 days of July. She codes the data using

$$y = 2x - 30$$

and finds $\Sigma y = 155$ and $\Sigma y^2 = 1520$

- Find the mean daily maximum temperature.
- Find the standard deviation of the daily maximum temperatures.

For the same 31 days at a weather station in southern England the mean daily maximum temperature was 21.3 °C and the standard deviation was 4.1 °C.

- Compare the daily maximum temperatures at the two weather stations in July.

(7 marks)

Rearrange the code

$$x = \frac{y + 30}{2}$$

a) Mean of y

$$\text{Mean of } y = \frac{155}{31} = 5$$

$$\text{Mean of } x = \frac{5 + 30}{2}$$

$$\boxed{\text{Mean} = 17.5^\circ\text{C}}$$

b) Variance of y

$$\text{Var}(y) = \frac{1520}{31} - 5^2$$

$$\text{Var}(y) = 49.032... - 25$$

$$\text{Var}(y) = 24.032...$$

$$\text{sd of } y = 4.9022...$$

Adding 30 does not change the spread; $\div 2$ halves it

$$\text{sd of } x = \frac{4.9022...}{2}$$

$$\text{sd of } x = 2.4511...$$

$$\boxed{\text{sd of } x = 2.45^\circ\text{C (3 s.f.)}}$$

c) Compare a location and a spread, in context

$$\text{Mean: } 21.3 > 17.5$$

so on average it was warmer in southern England

$$\text{Standard deviation: } 4.1 > 2.45$$

so the temperatures in southern England varied more

12

Eighteen students took a quiz. Their scores had a mean of 12 and a standard deviation of 2. Two more students, Ola and Kwame, then took the quiz. Ola scored a and Kwame scored b , where $a < b$.

For all 20 students the mean score was 13 and the standard deviation was $\sqrt{13}$.

Find the value of a and the value of b .

(8 marks)

Use $\sum x = n \times \text{mean}$ and $\sum x^2 = n(\sigma^2 + \text{mean}^2)$

First 18 students

$$\sum x = 18 \times 12 = 216$$

$$\sum x^2 = 18(2^2 + 12^2) = 2664$$

All 20 students

$$\sum x = 20 \times 13 = 260$$

$$\sum x^2 = 20(13 + 13^2) = 3640$$

The two new scores make up the difference

$$a + b = 260 - 216 = 44$$

$$a^2 + b^2 = 3640 - 2664 = 976$$

Substitute $b = 44 - a$

$$a^2 + (44 - a)^2 = 976$$

$$a^2 + 1936 - 88a + a^2 = 976$$

$$2a^2 - 88a + 960 = 0$$

$$a^2 - 44a + 480 = 0$$

$$(a - 20)(a - 24) = 0$$

$$a = 20 \text{ or } a = 24$$

$a < b$, so a is the smaller root

$$a = 20, b = 24$$