



- 1 The events A and B are mutually exclusive, with $P(A) = 0.4$ and $P(B) = 0.35$

Find

- $P(A \cap B)$
- $P(A \cup B)$
- $P(A' \cap B')$

(3 marks)

a) Mutually exclusive: they cannot happen together

$$P(A \cap B) = 0$$

b) Add the probabilities

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B) = 0.4 + 0.35$$

$$P(A \cup B) = 0.75$$

c) $A' \cap B'$ means neither A nor B

$$P(A' \cap B') = 1 - P(A \cup B)$$

$$P(A' \cap B') = 1 - 0.75$$

$$P(A' \cap B') = 0.25$$

2

The events A and B are independent, with $P(A) = 0.3$ and $P(B) = 0.6$

Find

a. $P(A \cap B)$

b. $P(A \cup B)$

c. $P(A \cap B')$

(4 marks)

a) Independent: multiply

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B) = 0.3 \times 0.6$$

$$P(A \cap B) = 0.18$$

b) Addition rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.3 + 0.6 - 0.18$$

$$P(A \cup B) = 0.72$$

c) A but not B

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$P(A \cap B') = 0.3 - 0.18$$

$$P(A \cap B') = 0.12$$

3

A security system has three alarms that work independently. Each alarm fails to sound with probability 0.05

An intruder sets off the system. Find the probability that

- at least one of the alarms fails to sound,
- exactly one of the alarms fails to sound.

(4 marks)

a) At least one = $1 - P(\text{none})$

$$P(\text{all three sound}) = 0.95^3$$

$$P(\text{all three sound}) = 0.857375$$

$$P(\text{at least one fails}) = 1 - 0.857375$$

$$P(\text{at least one fails}) = 0.142625$$

$$\boxed{0.143 \text{ (3 s.f.)}}$$

b) Fail, sound, sound in any order

3 orders: FSS, SFS, SSF

$$P = 3 \times 0.05 \times 0.95^2$$

$$P = 3 \times 0.05 \times 0.9025$$

$$P = 0.135375$$

$$\boxed{0.135 \text{ (3 s.f.)}}$$

4

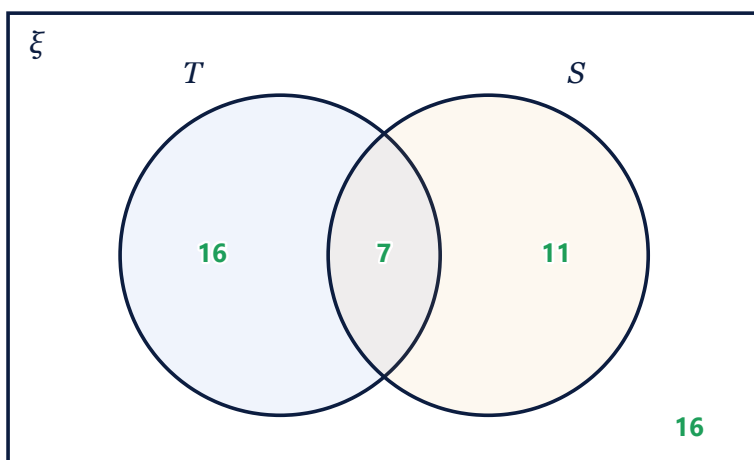
A sports club has 50 members. 23 members play tennis, 18 play squash and 7 play both.

a. Draw a Venn diagram to represent this information.

A member is chosen at random. Find the probability that the member plays

b. exactly one of these sports,

c. neither sport.



(5 marks)

a) Start with the overlap

$$\text{Both} = 7$$

$$\text{Tennis only} = 23 - 7 = 16$$

$$\text{Squash only} = 18 - 7 = 11$$

$$\text{Neither} = 50 - (16 + 7 + 11) = 16$$

Put 16, 7, 11 in the circles and 16 outside

b) Exactly one sport

$$16 + 11 = 27$$

$$p = \frac{27}{50}$$

c) Outside both circles

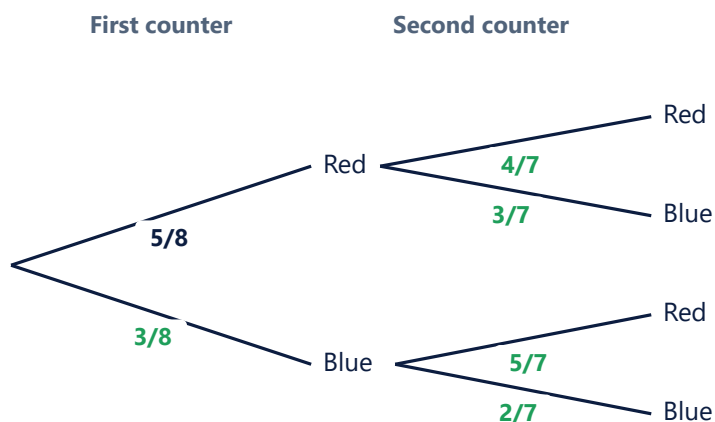
$$p = \frac{16}{50}$$

$$p = \frac{8}{25}$$

5

A bag contains 5 red counters and 3 blue counters. Two counters are taken from the bag at random, one after the other, without replacement.

- Complete the tree diagram.
- Find the probability that the two counters are the same colour.
- Find the probability that at least one of the counters is blue.



(5 marks)

a) Without replacement: 7 counters left for the second pick

After red: 4 red, 3 blue left

After blue: 5 red, 2 blue left

Branches: $\frac{5}{8}, \frac{3}{8}$; then $\frac{4}{7}, \frac{3}{7}$ and $\frac{5}{7}, \frac{2}{7}$

b) Red-red or blue-blue

$$P = \frac{5}{8} \times \frac{4}{7} + \frac{3}{8} \times \frac{2}{7}$$

$$P = \frac{20}{56} + \frac{6}{56}$$

$$P = \frac{26}{56}$$

$$P = \frac{13}{28}$$

c) At least one blue = $1 - P(\text{both red})$

$$P = 1 - \frac{20}{56}$$

$$P = \frac{36}{56}$$

$$P = \frac{9}{14}$$

6

The table shows how 120 sixth-form students usually travel to school.

	Year 12	Year 13
Walk	18	22
Bus	30	26
Car	12	12

A student is chosen at random. Find the probability that the student

- travels by bus,
- is in Year 13 and travels by car,
- walks or is in Year 12, or both.
- Determine whether the events "the student travels by car" and "the student is in Year 12" are independent.

(6 marks)

Find the totals first

Year 12: 60, Year 13: 60

Walk: 40, Bus: 56, Car: 24

a) Bus

$$P(\text{bus}) = \frac{56}{120}$$

$$P = \frac{7}{15}$$

b) One cell of the table

$$P(Y_{13} \cap \text{car}) = \frac{12}{120}$$

$$P = \frac{1}{10}$$

c) Addition rule, so the 18 are not counted twice

$$P(W \cup Y_{12}) = \frac{40 + 60 - 18}{120}$$

$$P(W \cup Y_{12}) = \frac{82}{120}$$

$$P = \frac{41}{60}$$

d) Independent if $P(C \cap Y_{12}) = P(C) \times P(Y_{12})$

$$P(C) = \frac{24}{120} = 0.2$$

$$P(Y_{12}) = \frac{60}{120} = 0.5$$

$$P(C) \times P(Y_{12}) = 0.1$$

$$P(C \cap Y_{12}) = \frac{12}{120} = 0.1$$

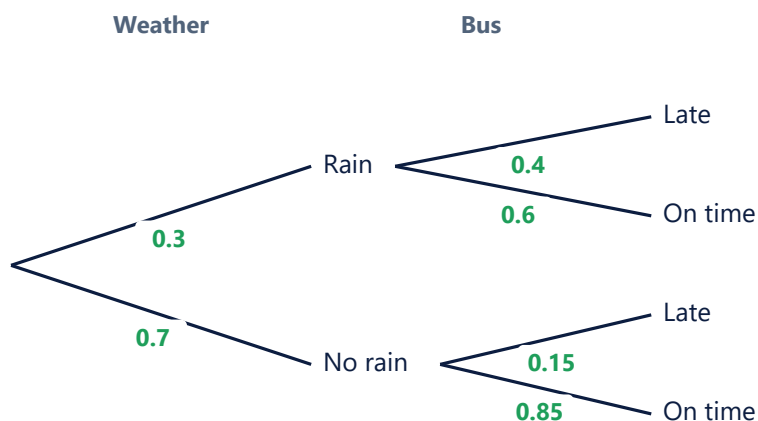
These are equal

Independent

7

On any school day the probability that it rains is 0.3. If it rains, the probability that Zara's bus is late is 0.4. If it does not rain, the probability that her bus is late is 0.15

- Complete the tree diagram to represent this information.
- Find the probability that Zara's bus is late on a randomly chosen school day.
- Two school days are chosen at random. Assuming that the days are independent, find the probability that her bus is late on exactly one of the two days.



(6 marks)

a) Each pair of branches adds to 1

Rain 0.3, No rain 0.7

After rain: Late 0.4, On time 0.6

After no rain: Late 0.15, On time 0.85

b) Two routes end in late

$$P(\text{Late}) = 0.3 \times 0.4 + 0.7 \times 0.15$$

$$P(\text{Late}) = 0.12 + 0.105$$

$$P(\text{Late}) = 0.225$$

c) $P(\text{on time}) = 1 - 0.225 = 0.775$

Late then on time, or on time then late

$$P = 2 \times 0.225 \times 0.775$$

$$P = 0.34875$$

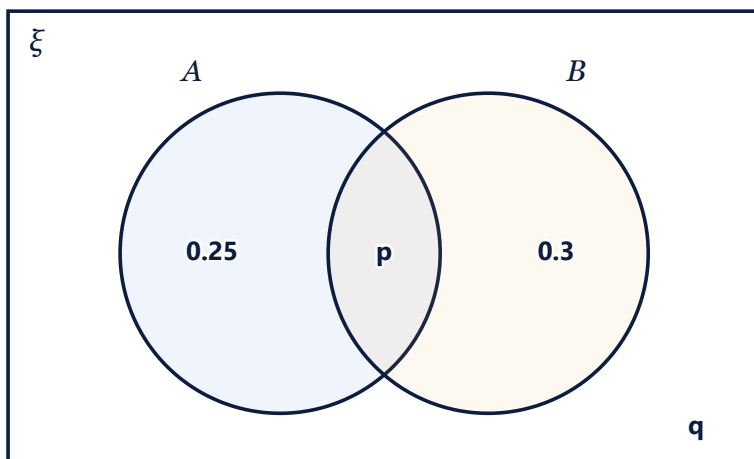
$$P = 0.349 \text{ (3 s.f.)}$$

8

The Venn diagram shows the probabilities for two events A and B , where p and q are probabilities.

Given that $P(A) = 0.4$

- find the value of p and the value of q ,
- determine whether A and B are independent,
- find $P(A \cup B')$.



(5 marks)

a) $P(A)$ is the whole A circle

$$0.25 + p = 0.4$$

$$p = 0.15$$

All probabilities add to 1

$$0.25 + 0.15 + 0.3 + q = 1$$

$$q = 0.3$$

b) Independent if $P(A \cap B) = P(A) \times P(B)$

$$P(B) = 0.15 + 0.3 = 0.45$$

$$P(A) \times P(B) = 0.4 \times 0.45 = 0.18$$

$$P(A \cap B) = 0.15$$

$$0.18 \neq 0.15$$

Not independent

c) $A \cup B'$ is every region except B only

$$P(A \cup B') = 1 - 0.3$$

$$P(A \cup B') = 0.7$$

9

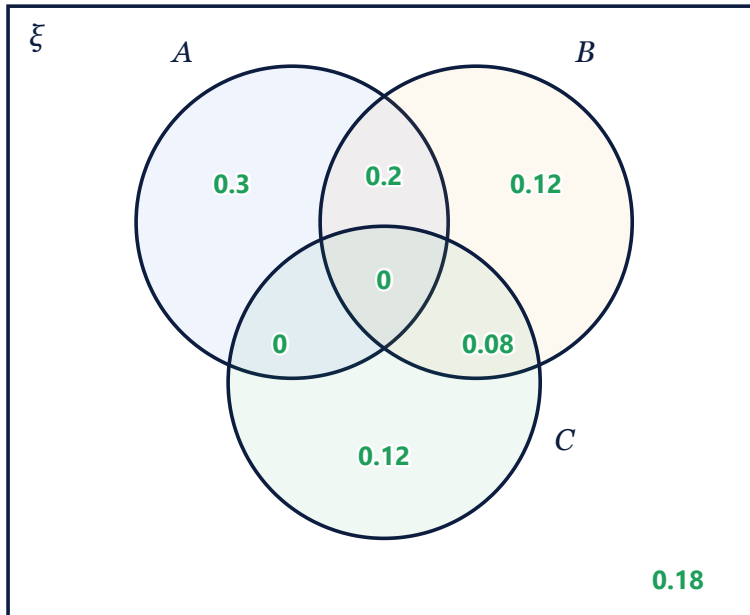
The events A , B and C have $P(A) = 0.5$, $P(B) = 0.4$ and $P(C) = 0.2$

The events A and B are independent. The events A and C are mutually exclusive. The events B and C are independent.

a. Find $P(A \cap B)$.

b. Draw a Venn diagram to show the events A , B and C , giving the probability for each region.

c. Find $P([B \cup C]')$.



(7 marks)

a) A and B independent: multiply

$$P(A \cap B) = 0.5 \times 0.4$$

$$P(A \cap B) = 0.2$$

b) A and C mutually exclusive: no overlap

B and C independent

$$P(B \cap C) = 0.4 \times 0.2 = 0.08$$

$$A \text{ only} = 0.5 - 0.2 = 0.3$$

$$B \text{ only} = 0.4 - 0.2 - 0.08 = 0.12$$

$$C \text{ only} = 0.2 - 0.08 = 0.12$$

$$\text{Outside} = 1 - (0.3 + 0.2 + 0.12 + 0.08 + 0.12)$$

$$\text{Outside} = 1 - 0.82 = 0.18$$

c) Outside both B and C : A only and outside

$$P([B \cup C]') = 0.3 + 0.18$$

$$P([B \cup C]') = 0.48$$

10

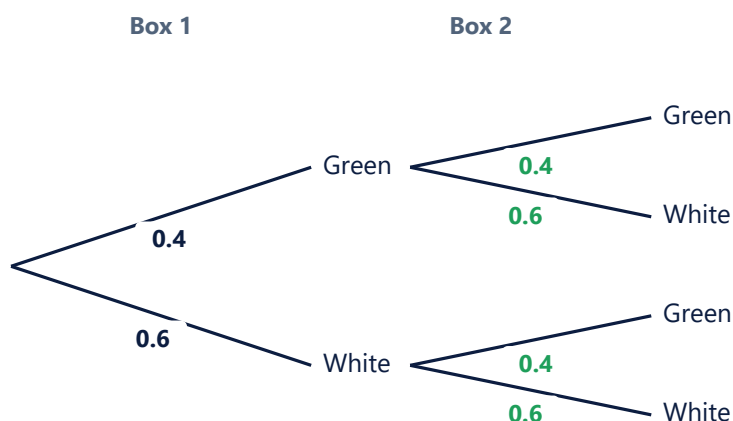
Box 1 contains 4 green counters and 6 white counters. Box 2 contains only green and white counters. One counter is taken at random from each box.

The probability that the two counters are the same colour is 0.52

- Find the probability of taking a green counter from Box 2.
- Complete the tree diagram.
- Find the probability that at least one of the counters is green.

The event G is that the counter from Box 1 is green. The event S is that the two counters are the same colour.

- Determine whether G and S are independent.



(7 marks)

a) Let $p = P(\text{green from Box 2})$

Same colour = both green or both white

$$0.4p + 0.6(1 - p) = 0.52$$

$$0.4p + 0.6 - 0.6p = 0.52$$

$$0.6 - 0.2p = 0.52$$

$$0.2p = 0.08$$

$$p = 0.4$$

b) Box 2 does not depend on Box 1

Both pairs of Box 2 branches: Green 0.4, White 0.6

c) At least one green = $1 - P(\text{both white})$

$$p = 1 - 0.6 \times 0.6$$

$$p = 1 - 0.36$$

$$p = 0.64$$

d) Compare $P(G \cap S)$ with $P(G) \times P(S)$

$$P(G \cap S) = P(\text{both green}) = 0.4 \times 0.4 = 0.16$$

$$P(G) \times P(S) = 0.4 \times 0.52 = 0.208$$

$$0.16 \neq 0.208$$

Not independent

11

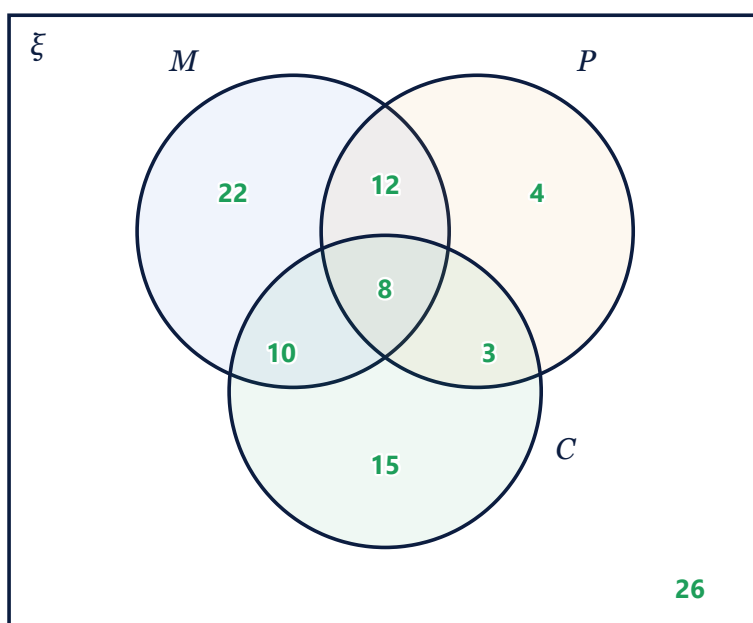
A college surveys 100 A-level students about three subjects: Maths (M), Physics (P) and Chemistry (C).

- 52 study Maths, 27 study Physics and 36 study Chemistry
- 20 study Maths and Physics, 18 study Maths and Chemistry, and 11 study Physics and Chemistry
- 8 study all three

a. Draw a Venn diagram to represent this information.

A student is chosen at random. Find the probability that the student studies

- exactly two of these subjects,
- none of these subjects.
- Determine whether studying Physics and studying Chemistry are independent.



(8 marks)

a) Work from the middle outwards

$$\text{All three} = 8$$

$$M \cap P \text{ only} = 20 - 8 = 12$$

$$M \cap C \text{ only} = 18 - 8 = 10$$

$$P \cap C \text{ only} = 11 - 8 = 3$$

$$M \text{ only} = 52 - 12 - 10 - 8 = 22$$

$$P \text{ only} = 27 - 12 - 3 - 8 = 4$$

$$C \text{ only} = 36 - 10 - 3 - 8 = 15$$

$$\text{Inside the circles} = 22 + 4 + 15 + 12 + 10 + 3 + 8 = 74$$

$$\text{None} = 100 - 74 = 26$$

b) Exactly two: the three pair-only regions

$$12 + 10 + 3 = 25$$

$$P = 0.25$$

c) Outside all three circles

$$P = 0.26$$

d) Independent if $P(P \cap C) = P(P) \times P(C)$

$$P(P) \times P(C) = 0.27 \times 0.36 = 0.0972$$

$$P(P \cap C) = \frac{11}{100} = 0.11$$

$$0.11 \neq 0.0972$$

Not independent

12

A factory has two machines, X and Y . In any week, the events " X breaks down" and " Y breaks down" are independent. Machine X is more likely to break down than machine Y .

In a week, the probability that both machines break down is 0.12 and the probability that at least one machine breaks down is 0.58

- Find the probability that machine X breaks down in a week.
- Find the probability that exactly one machine breaks down in a week.
- Assuming that different weeks are independent, find the probability that neither machine breaks down in two weeks running.

(8 marks)

a) Let $P(X) = x$ and $P(Y) = y$

Independent: multiply

$$xy = 0.12$$

Addition rule

$$x + y - xy = 0.58$$

$$x + y = 0.58 + 0.12 = 0.7$$

Substitute $y = 0.7 - x$

$$x(0.7 - x) = 0.12$$

$$0.7x - x^2 = 0.12$$

$$x^2 - 0.7x + 0.12 = 0$$

$$(x - 0.4)(x - 0.3) = 0$$

$$x = 0.4 \text{ or } x = 0.3$$

X is more likely to break down

$$P(X) = 0.4$$

b) At least one, take away both

$$P(\text{exactly one}) = 0.58 - 0.12$$

$$P(\text{exactly one}) = 0.46$$

c) Neither in one week

$$P(\text{neither}) = 1 - 0.58 = 0.42$$

Two independent weeks: multiply

$$P = 0.42^2$$

$$P = 0.1764$$

$$P = 0.176 \text{ (3 s.f.)}$$