Argand diagrams, modulus and argument

AS Core Pure Mathematics | Edexcel 8FM0 | 12 questions, 68 marks

Further Maths topic practice

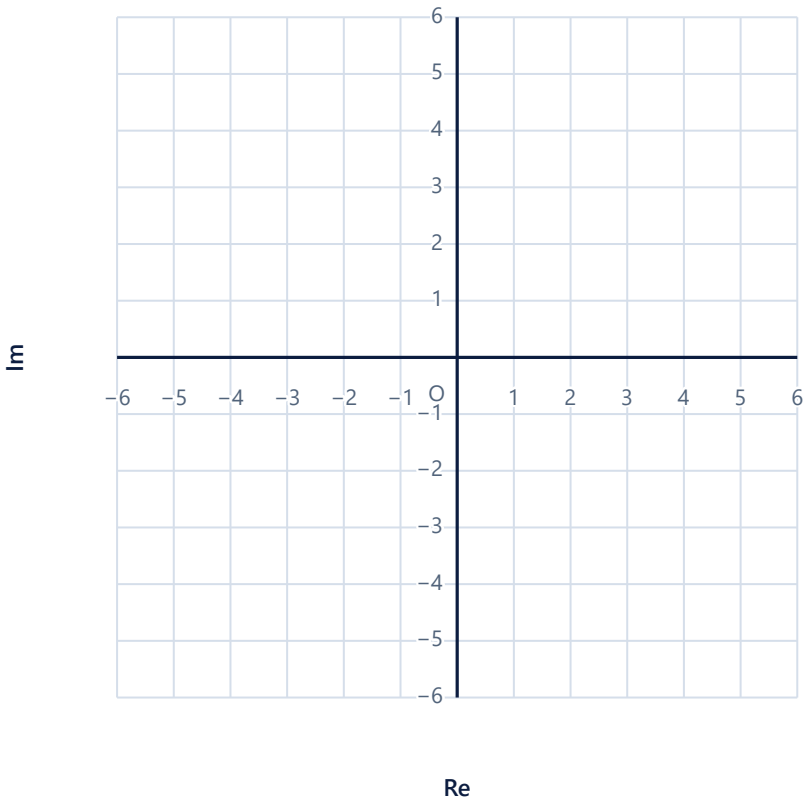
Scan for the solutions

Name: _____ **Date:** _____ **Score:** _____ **/ 68**

1 The complex numbers z_1 and z_2 are given by $z_1 = 4 + 3i$ and $z_2 = -2 - 5i$

a. Show z_1, z_2 and $z_1 + z_2$ on the Argand diagram.

b. Find the exact value of $|z_1 + z_2|$



(4 marks)

Answer _____

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2 Find the modulus and the argument of each of the following complex numbers. Give each argument in radians to 3 significant figures, in the range $-\pi < \theta \leq \pi$

a. $z = -3 + 4i$

b. $w = -1 - 2i$

(4 marks)

Answer

3 Express each complex number in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$, giving r and θ as exact values.

a. $-\sqrt{3} + i$

b. $2 - 2i$

(4 marks)

Answer

4 The complex number $z = 6\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

a. Express z in the form $a + bi$, where a and b are exact real numbers.

b. Write down z^* in the form $r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \leq \pi$

c. Find, in the form $a + bi$, the complex number $z + z^*$ and explain why it is real.

(4 marks)

Answer

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5 The complex number z is given by $z = \dfrac{5 + 5i}{1 + 2i}$

a. Express z in the form $a + bi$, where a and b are real.

b. Find the exact value of $|z|$ and find arg z in radians to 3 significant figures.

c. Hence find $|z^2|$ and arg(z^2), giving the argument in radians to 3 significant figures.

(6 marks)

Answer _____

6 $z_1 = 4(\cos \dfrac{\pi}{3} + i \sin \dfrac{\pi}{3})$ and $z_2 = 2(\cos \dfrac{3\pi}{4} + i \sin \dfrac{3\pi}{4})$

Find, in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$,

a. $z_1 z_2$

b. $\dfrac{z_1}{z_2}$

(5 marks)

Answer _____

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7 The complex number $z = p + 2i$, where p is a real constant, has argument $\frac{2\pi}{3}$

a. Find the exact value of p

b. Find the exact value of $|z|$

The complex number $w = 1 - i$

c. Find the exact values of $|zw|$ and $\arg(zw)$.

(6 marks)

Answer

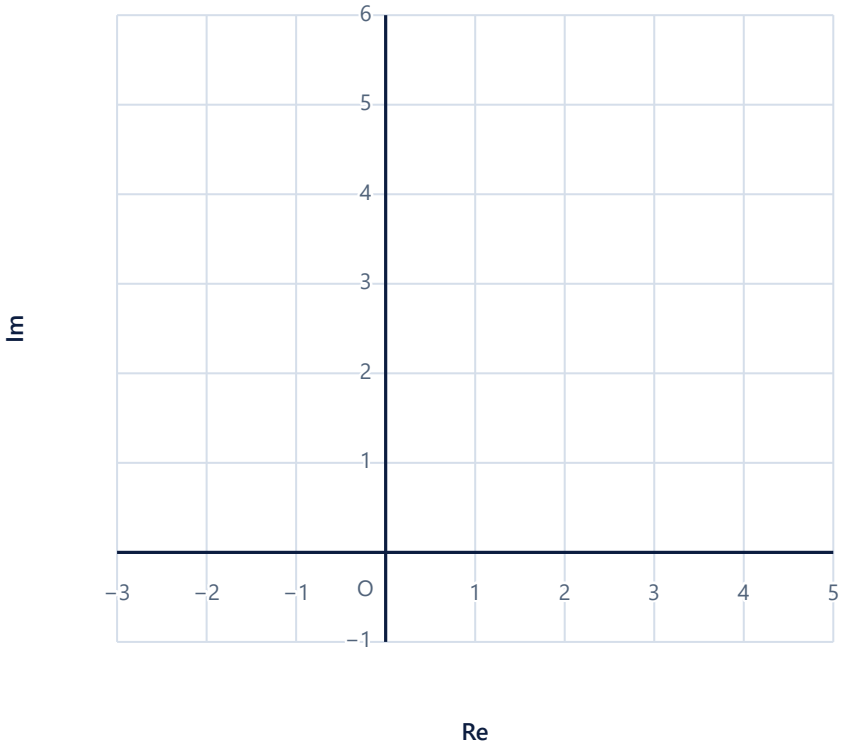
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8 The complex numbers z and w are given by $z = 3 + 2i$ and $w = -1 + 3i$

a. On the Argand diagram, show the points representing z, w and $z + w$

b. Name the quadrilateral whose vertices are the origin and the points representing z, $z + w$ and w, giving a reason.

c. Find iz in the form $a + bi$ and describe fully the single transformation that maps the point representing z to the point representing iz



(6 marks)

Answer

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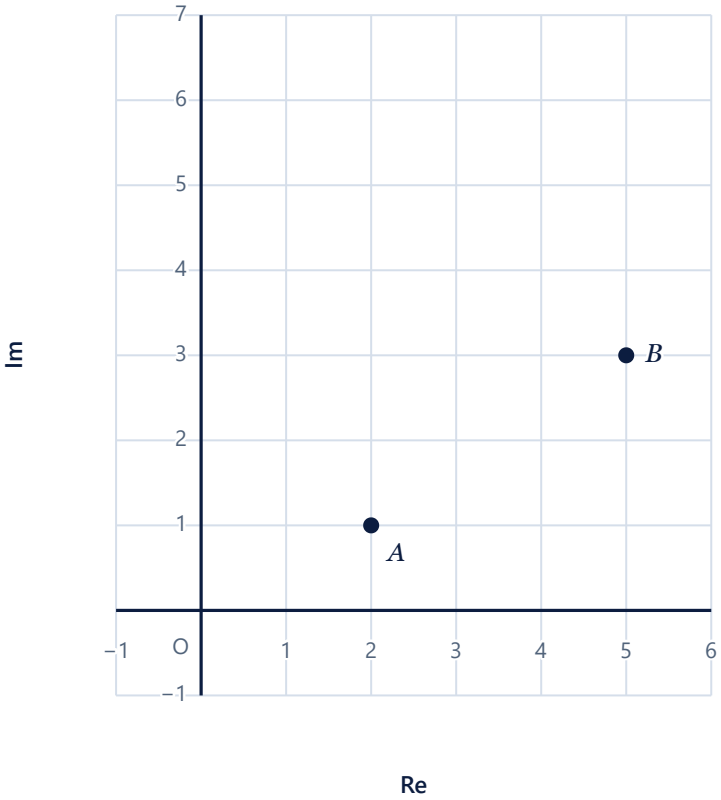
9 In an Argand diagram, the points A and B represent the complex numbers $2 + i$ and $5 + 3i$ respectively.

The points A, B, C and D are the vertices of a square, labelled anticlockwise.

a. Write down the complex number that represents the vector $\overrightarrow{AB}$

b. By multiplying by i, find the complex numbers represented by C and D

c. Find the area of the square.



(6 marks)

Answer

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10 The point A on an Argand diagram represents the complex number $a = 2\sqrt{3} + 2i$

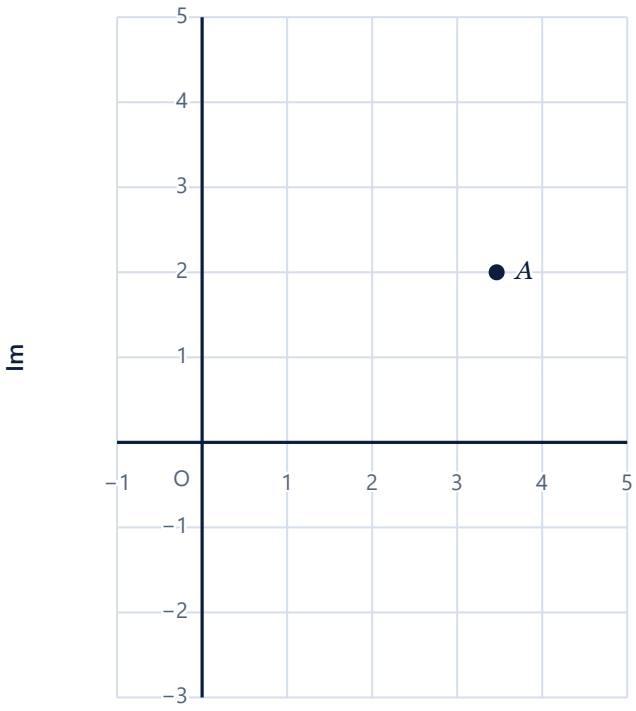
a. Express a in the form $r(\cos\theta + i\sin\theta)$, giving r and θ exactly.

The point B represents the complex number b. The origin O, A and B are the vertices of an equilateral triangle.

b. Explain why $b = a(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ or $b = a(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3})$.

c. Find the two possible values of b, giving each in the form $x + yi$

d. Find the exact area of triangle OAB



(7 marks)

Answer

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11 The complex numbers $z_1 = 1 + i$ and $z_2 = \sqrt{3} - i$

a. Write down the modulus and argument of z_1 and of z_2, giving exact values.

b. Hence find $|z_1 z_2|$, $\arg(z_1 z_2)$, $\left|\frac{z_1}{z_2}\right|$ and $\arg\left(\frac{z_1}{z_2}\right)$.

c. Express $z_1 z_2$ in the form $a + bi$

d. Hence show that $\cos \frac{\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4}$

(8 marks)

Answer

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12 The complex number z is given by

$$z = \frac{a + 2i}{1 + i}$$

where a is a real constant.

a. Show that $z = \frac{a+2}{2} + \frac{2-a}{2}i$

Given that $\arg z = \frac{\pi}{6}$

b. show that $a = 4 - 2\sqrt{3}$

c. find the exact value of $|z|$, giving your answer in the form $p\sqrt{3} + q$, where p and q are integers.

(8 marks)

Answer

End of questions | Total 68 marks

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