Complex numbers: arithmetic and quadratics

AS Core Pure Mathematics | Edexcel 8FM0 | 12 questions, 65 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 65**

1 The complex numbers z and w are given by $z = 3 - 4i$ and $w = -2 + 5i$

Find, in the form $a + bi$, where a and b are real,

a. $z + 2w$

b. zw

c. z^2

(4 marks)

Answer _____

2 Express $\frac{5 + i}{2 - 3i}$ in the form $a + bi$, where a and b are real.

(3 marks)

Answer _____

3 Solve the equation $2z^2 + 6z + 5 = 0$, giving your answers in the form $p + qi$, where p and q are real.

(3 marks)

Answer _____

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4 Find the complex number z that satisfies the equation

$$3z - 2iz^* = 14 - 11i$$

Give your answer in the form $a + bi$, where a and b are real.

(5 marks)

Answer

5 The equation $z^2 + pz + q = 0$, where p and q are real constants, has a root $2 - i$

a. Write down the other root of the equation.

b. Find the value of p and the value of q

(4 marks)

Answer

6 The complex number w is given by

$$w = \frac{a + 4i}{1 - 2i}$$

where a is a real constant.

a. Express w in the form $x + yi$, where x and y are given in terms of a

b. Given that w is purely imaginary, find the value of a and the value of w

c. Find the value of a for which w is real.

(5 marks)

Answer

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7 Given that $z = 2 + i$

a. find $\frac{1}{z}$ in the form $a + bi$, where a and b are real.

The complex number w is defined by $w = z + \frac{k}{z}$, where k is a real constant.

b. Given that w is real, find the value of k and the value of w

(5 marks)

Answer

8 Find the two complex numbers z that satisfy the equation

$$zz^* + 2iz = 12 + 4i$$

Give your answers in the form $a + bi$, where a and b are real.

(6 marks)

Answer

9 a. Find the two square roots of $21 - 20i$, giving your answers in the form $a + bi$, where a and b are real.

b. Hence solve the equation

$$z^2 - 3z - 3 + 5i = 0$$

(7 marks)

Answer

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10 The complex numbers z and w satisfy the simultaneous equations

$$z + 2w = 7$$
$$iz - w = -4 + 4i$$

Find z and w, giving each in the form $a + bi$, where a and b are real. (6 marks)

Answer

11 The quadratic equation

$$x^2 + 2kx + 3k + 4 = 0$$

where k is a real constant, has roots that are not real.

a. Find the range of possible values of k

Given now that $k = 2$

b. solve the equation, giving your answers in the form $p + qi$, where p and q are real and exact,

c. show that the sum of the squares of the two roots is a real number, and find it. (8 marks)

Answer

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12 a. Find the square roots of $-5 + 12i$, giving your answers in the form $a + bi$, where a and b are real.

b. Hence, showing all your working, find the four roots of the equation

$$z^4 + 10z^2 + 169 = 0$$

c. Use your answer to part (b) to write $z^4 + 10z^2 + 169$ as the product of two quadratic factors with real coefficients.

(9 marks)

Answer

End of questions | Total 65 marks

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