# Linear transformations

AS Core Pure Mathematics | Edexcel 8FM0 | 12 questions, 65 marks

**Further Maths topic practice**

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**Name:** _____ **Date:** _____ **Score:** _____ **/ 65**

**1** Write down the 2 × 2 matrix that represents

a. a rotation of 90° anticlockwise about the origin

b. a reflection in the line $y = -x$

c. a stretch parallel to the $y$-axis, scale factor 4

(3 marks)

Answer _____

**2** Describe fully the single geometrical transformation represented by each matrix.

a. $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

b. $\begin{vmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{vmatrix}$

c. $\begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}$

(4 marks)

Answer _____


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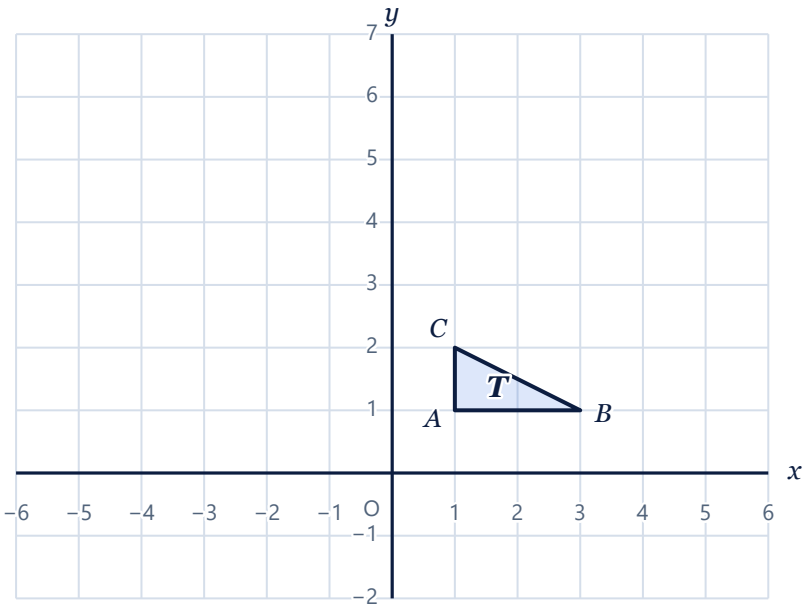
**3** The triangle $T$ has vertices $A(1, 1)$, $B(3, 1)$ and $C(1, 2)$, as shown on the grid.

$T$ is transformed to the triangle $T'$ by the matrix $\mathbf{M} = \begin{vmatrix} 0 & -2 \\ 2 & 0 \end{vmatrix}$

a. Find the coordinates of the vertices of $T'$ and draw $T'$ on the grid.

b. Find the area of $T'$ using det **M**

c. Describe the transformation represented by **M** as a combination of two transformations.




(5 marks)

Answer

**4** The transformation P is a rotation of 90° clockwise about the origin. The transformation Q is a reflection in the line $y = -x$

a. Write down the matrices **P** and **Q**

b. Find the matrix representing the transformation P followed by Q.

c. Describe fully the single transformation equivalent to P followed by Q.

(4 marks)

Answer


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**5** The matrix $\mathbf{M} = \begin{vmatrix} k & 3 \\ 2 & k+1 \end{vmatrix}$, where $k$ is a positive constant.

A region $R$ of area 5 cm$^2$ is transformed by **M** to a region $R'$ of area 70 cm$^2$

a. Find the value of $k$

b. Explain how you know that the transformation preserves orientation.

c. Using your value of $k$, find the point that **M** maps to (13, 17).

(6 marks)

Answer _____

**6** The transformation T is represented by the matrix $\begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix}$

Show that T has a line of invariant points, and find its equation.

(4 marks)

Answer _____

**7** $\mathbf{M} = \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix}$

Find the equations of the lines through the origin, of the form $y = mx$, that are invariant under the transformation represented by **M**

(5 marks)

Answer _____


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**8** The transformation S is a rotation of 45° anticlockwise about the origin followed by an enlargement, scale factor $2\sqrt{2}$, centre the origin.

a. Show that S is represented by the matrix $\begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix}$

b. Find the image of the point $(3, -1)$ under S.

c. The point $P$ is mapped by S to the point $(2, 10)$. Find the coordinates of $P$

(6 marks)

Answer

**9** The transformation U is a rotation of 90° anticlockwise about the $x$-axis. The transformation V is a reflection in the plane $z = 0$

a. Write down the $3 \times 3$ matrices **U** and **V**

b. Find the matrix representing U followed by V.

c. Find the image of the point $(2, -1, 3)$ under U followed by V.

d. Find the matrix representing V followed by U, and state whether the order of U and V matters.

(6 marks)

Answer


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**10**

$$\mathbf{R} = \begin{vmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

a. Describe fully the transformation represented by **R**

b. Find the image of the point (2, 0, 5) under this transformation.

c. Write down $\mathbf{R}^{-1}$ and describe the transformation it represents.

d. Find the smallest positive integer $n$ for which $\mathbf{R}^n = \mathbf{I}$

(6 marks)

Answer


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**11** The parallelogram $Q$ has vertices $O(0, 0)$, $A(2, 0)$, $B(3, 2)$ and $C(1, 2)$, as shown on the grid.

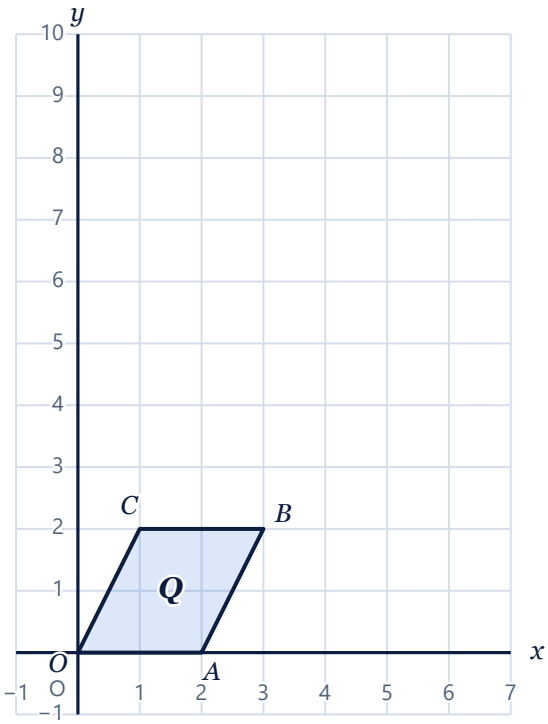
$Q$ is mapped to $Q'$ by a stretch parallel to the $x$-axis, scale factor 3, followed by a reflection in the line $y = x$

a. Find the single matrix **T** that represents this combined transformation.

b. Find the coordinates of the vertices of $Q'$ and draw $Q'$ on the grid.

c. Find the area of $Q'$

d. Find $\mathbf{T}^{-1}$ and describe the transformation it represents as a combination of two transformations.




(8 marks)

Answer


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**12** The transformation T is represented by $\mathbf{M} = \begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix}$

a. Show that $\mathbf{M}^2 = \mathbf{I}$, and explain what this tells you about T.

b. Show that $y = x$ is a line of invariant points of T.

c. Find the invariant lines of T of the form $y = mx + c$, showing that one family of them does not pass through the origin.

(8 marks)

Answer _____

End of questions | Total 65 marks


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