Loci and regions in the Argand diagram

Answers

Further Maths topic practice

This topic online

Q	Answer	Marks
1	**a)** circle, centre (3, -2), radius 4 **b)** $(x - 3)^2 + (y + 2)^2 = 16$ **c)** $(-2 + \sqrt{7})i$ and $(-2 - \sqrt{7})i$	4
2	**a)** the perpendicular bisector of the points (1, 3) and (5, -1) **b)** $y = x - 2$ (shown) **c)** $z = 1 - i$, least $\lvert z\rvert = \sqrt{2}$	6
3	**a)** half-line from 2 - i (not included) going up and to the left at $3\pi/4$ to the positive real direction **b)** $y = 1 - x$, $x < 2$ **c)** i	4
4	**a)** circle, centre (2, 2), radius 2; the vertical line $x = 2$ **b)** the half of the disc on or to the left of $x = 2$ **c)** 2π	5
5	**a)** $\lvert z - 1 - 2i\rvert \le 2$ and $0 \le \arg(z - 1 - 2i) \le \frac{3\pi}{4}$ **b)** $\frac{3\pi}{2}$ **c)** $(1 - \sqrt{2}) + (2 + \sqrt{2})i$	5
6	**a)** circle, centre (-1, 0), radius 5; half-line from 1 + 2i (not included) at $\pi/4$, starting inside the circle **b)** $(-1 + \frac{5\sqrt{2}}{2}) + \frac{5\sqrt{2}}{2}i$	7
7	**a)** circle, centre $(3\sqrt{3}, 3)$, radius 3: the centre is 3 above the real axis, equal to the radius, so it touches it at $3\sqrt{3}$ **b)** greatest $\lvert z\rvert = 9$, least $\lvert z\rvert = 3$ **c)** $\frac{\pi}{3}$ **d)** $\frac{3\sqrt{3}}{2} + \frac{9}{2}i$	9
8	**a)** $x + y = 3$ (shown) **b)** 3 and 3i **c)** circle centre O radius 3, line $x + y = 3$; R is the part of the disc on the same side of the line as 3 + 4i **d)** yes: $\lvert 2 + 2i\rvert = \sqrt{8} \le 3$ and $2 + 2 \ge 3$	8
9	**a)** inside the circle centre (0, 3), radius 3, between the positive imaginary axis and the half-line from O at $3\pi/4$ **b)** shown: $y = -x$ meets the circle at $x = -3$, $y = 3$ **c)** $\frac{9}{4}(\pi + 2)$	8
10	**a)** half-line from 2 (not included) at $2\pi/3$ to the positive real direction **b)** $y = -\sqrt{3}x + 2\sqrt{3}$, $x < 2$ **c)** least $\lvert z\rvert = \sqrt{3}$ at $z = \frac{3}{2} + \frac{\sqrt{3}}{2}i$ **d)** every w has Re w > 2, so $\lvert w\rvert \ge$ Re w > 2; as w approaches the excluded point 2, $\lvert w\rvert$ approaches 2 but never equals it, so there is no least value	8

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11	**a)** circle centre (6, 0) radius 2, meeting the real axis at 4 and 8; circle centre (0, 8) radius 3, meeting the imaginary axis at 5i and 11i **b)** $5 \leq \lvert z - w \rvert \leq 15$ **c)** $z = 4.8 + 1.6i$, $w = 1.8 + 5.6i$	9
12	**a)** $\lvert z - 6 - 4i \rvert \leq 3$ **b)** $y = x$, $x > 1$ **c)** $2\sqrt{7}$ m **d)** $2\sqrt{13} - 3$ m ≈ 4.21 m	8

Full worked solutions, written out by hand, are on the solutions sheet for this topic.

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