Loci and regions in the Argand diagram

AS Core Pure Mathematics | Edexcel 8FM0 | 12 questions, 81 marks

Further Maths topic practice

Scan for the solutions

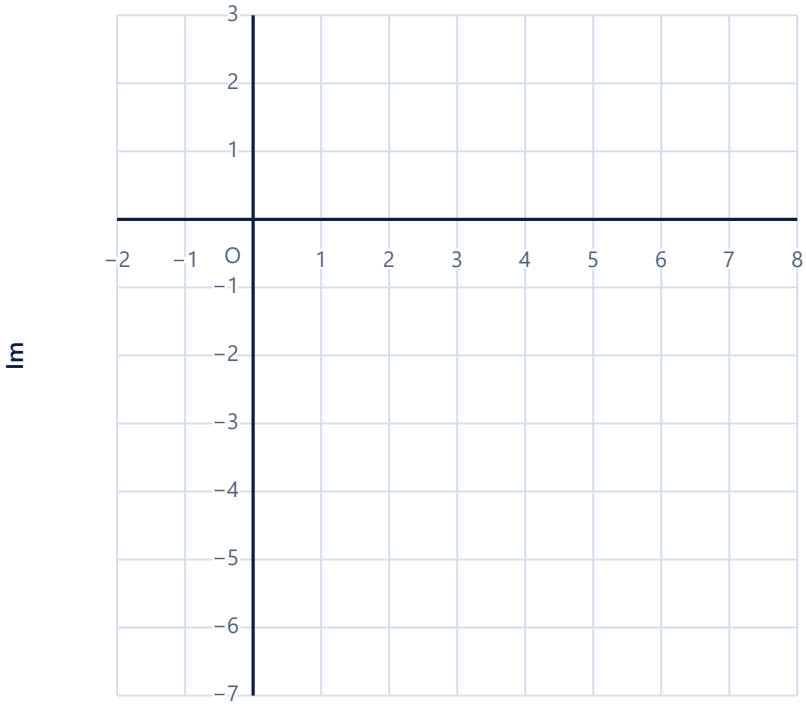
Name: _____ **Date:** _____ **Score:** _____ **/ 81**

1 The locus of points z satisfies $|z - 3 + 2i| = 4$

a. Sketch the locus on the Argand diagram.

b. Find a cartesian equation of the locus.

c. Find the exact complex numbers represented by the points where the locus meets the imaginary axis.



(4 marks)

Answer _____

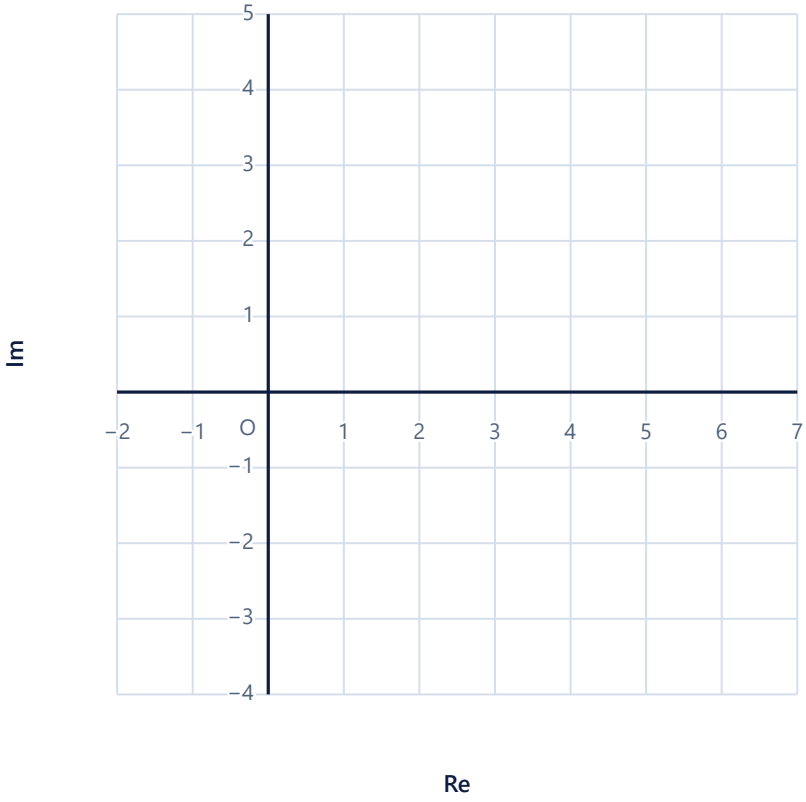
Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

2 The locus L is given by $|z - 1 - 3i| = |z - 5 + i|$

a. Sketch L on the Argand diagram.

b. Show that a cartesian equation of L is $y = x - 2$

c. Find the complex number on L with the least modulus, and state this least value of $|z|$



(6 marks)

Answer _____

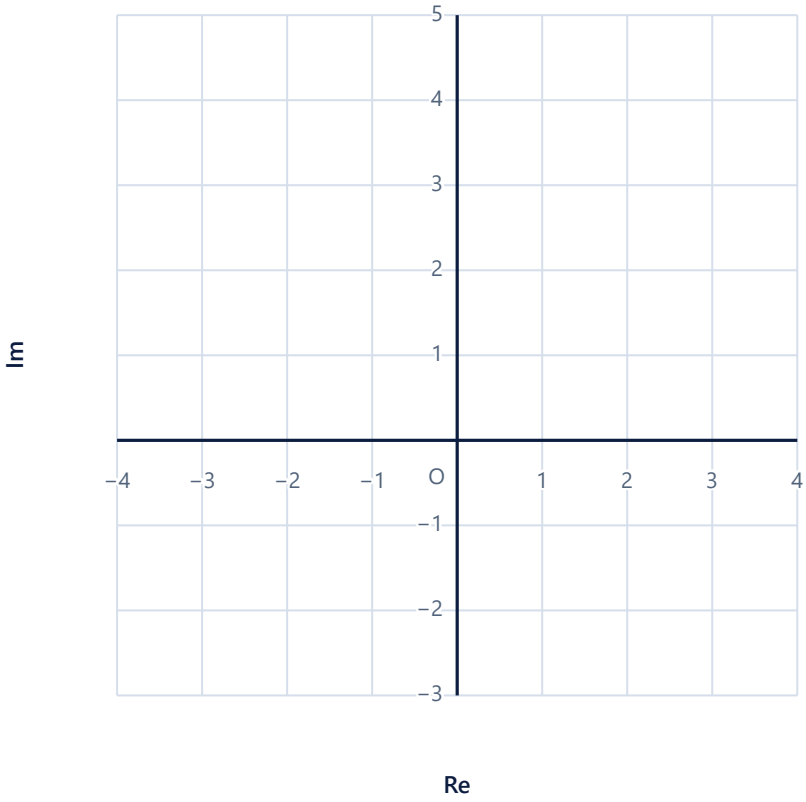
Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

3 The locus of points z satisfies $\arg(z - 2 + i) = \frac{3\pi}{4}$

a. Sketch the locus on the Argand diagram.

b. Find a cartesian equation of the locus, stating any restriction on x

c. Write down the complex number represented by the point where the locus crosses the imaginary axis.



(4 marks)

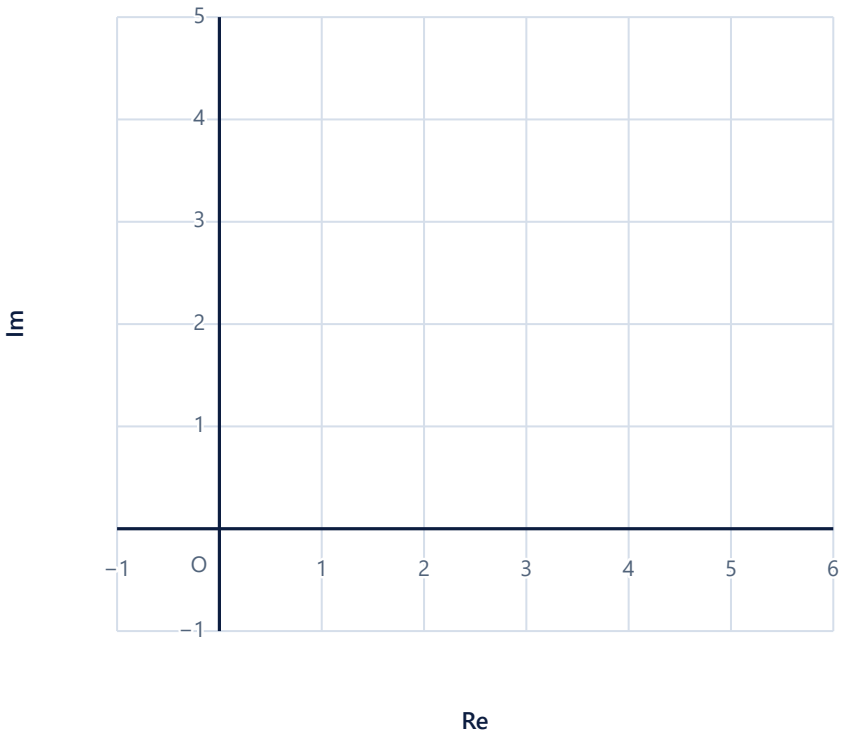
Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

4 a. On the same Argand diagram, sketch the locus $|z - 2 - 2i| = 2$ and the locus $|z| = |z - 4|$

b. Shade on your diagram the region satisfying both $|z - 2 - 2i| \leq 2$ and $|z| \leq |z - 4|$

c. Find the exact area of the shaded region.



(5 marks)

Answer _____

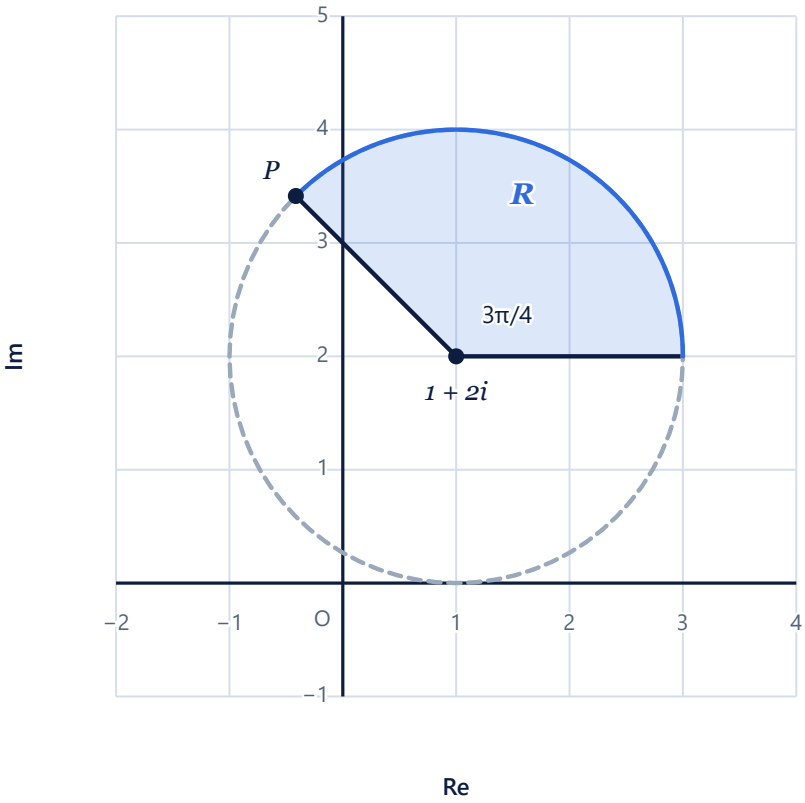
Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

5 The Argand diagram shows a shaded region R. R is bounded by an arc of the circle with centre $1 + 2i$ and radius 2, and by two straight edges that meet at the centre. One edge is parallel to the real axis. The other edge makes an angle of $\frac{3\pi}{4}$ with the positive real direction.

a. Write down, in terms of z, the two inequalities that define R

b. Find the exact area of R

c. Find, in the form $a + bi$, where a and b are exact, the complex number represented by the vertex P at the end of the sloping edge.



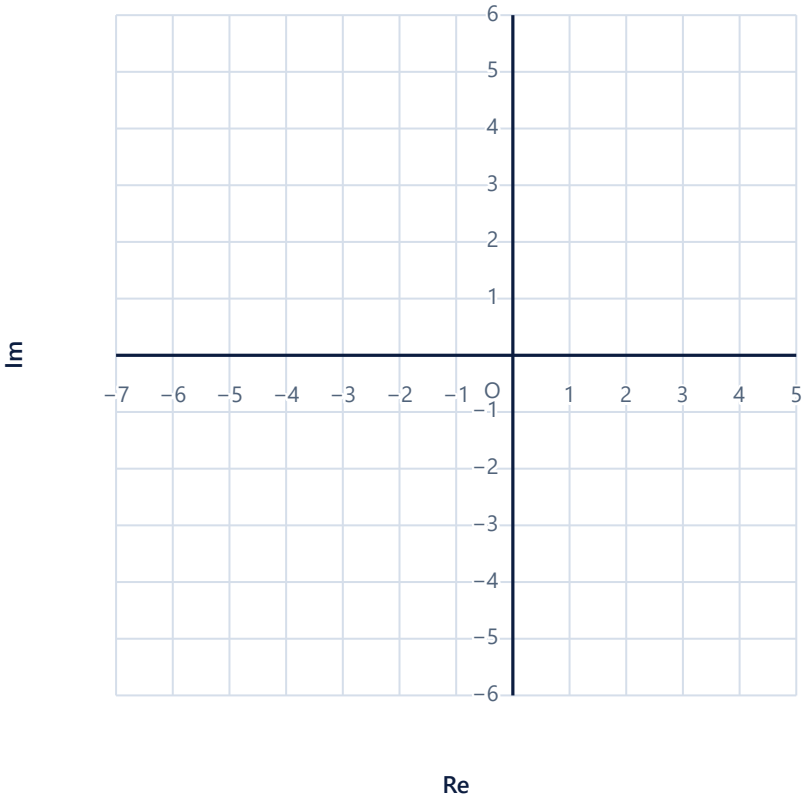
(5 marks)

Answer _____

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

6 a. On the same Argand diagram, sketch the locus $|z + 1| = 5$ and the locus $\arg(z - 1 - 2i) = \frac{\pi}{4}$

b. Find, in the form $a + bi$, where a and b are exact, the complex number that satisfies both $|z + 1| = 5$ and $\arg(z - 1 - 2i) = \frac{\pi}{4}$



(7 marks)

Answer _____

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

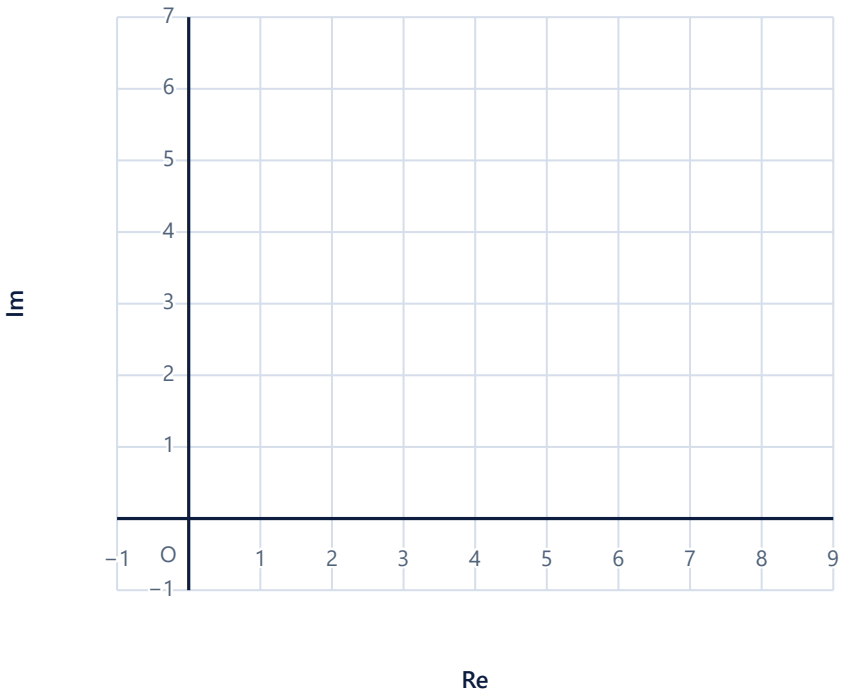
7 The complex number z satisfies $|z - 3\sqrt{3} - 3i| = 3$

a. Sketch the locus of z on the Argand diagram, and show that it touches the real axis.

b. Find the greatest and least values of $|z|$

c. Find the exact greatest value of arg z

d. Find, in the form $a + bi$, where a and b are exact, the value of z for which arg z is greatest.



(9 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

8 The loci C and L are given by

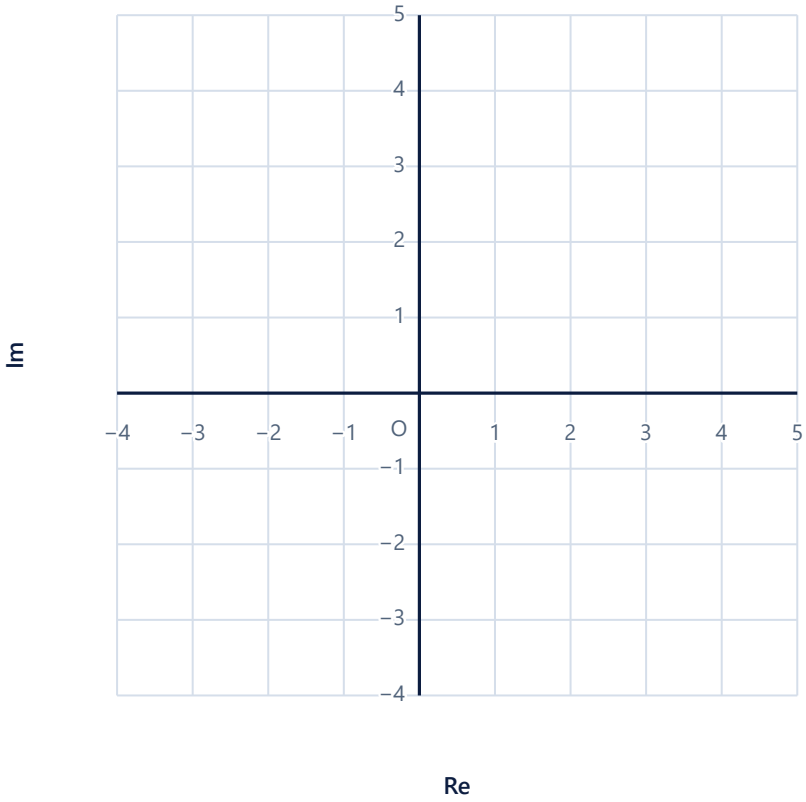
$$C: |z| = 3 \qquad L: |z - 3 - 4i| = |z + 1|$$

a. Show that a cartesian equation of L is $x + y = 3$

b. Find the complex numbers represented by the points where C and L meet.

c. On the Argand diagram, sketch C and L, and shade the region R satisfying both $|z| \leq 3$ and $|z - 3 - 4i| \leq |z + 1|$

d. Determine whether the point representing $2 + 2i$ lies in R



(8 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

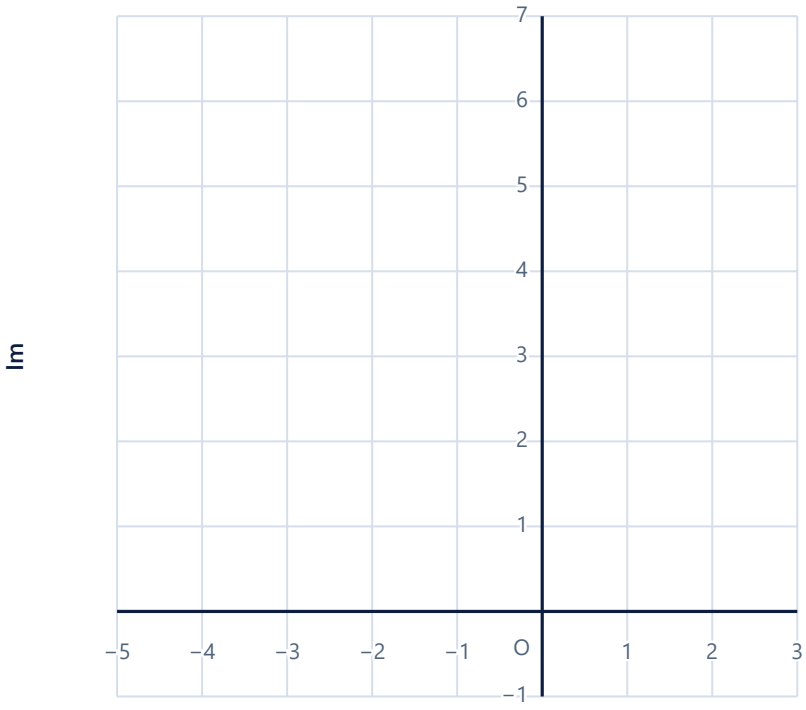
9 The region R in the Argand diagram is defined by

$$R = \{z : |z - 3i| \leq 3\} \cap \{z : \frac{\pi}{2} \leq \arg z \leq \frac{3\pi}{4}\}$$

a. Show, by shading on the Argand diagram, the region R

b. Show that the half-line $\arg z = \frac{3\pi}{4}$ meets the circle $|z - 3i| = 3$ at the point representing $-3 + 3i$

c. Find the exact area of R, giving your answer in the form $p(\pi + q)$, where p and q are rational.



(8 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

10 The complex number z satisfies $\arg(z - 2) = \frac{2\pi}{3}$

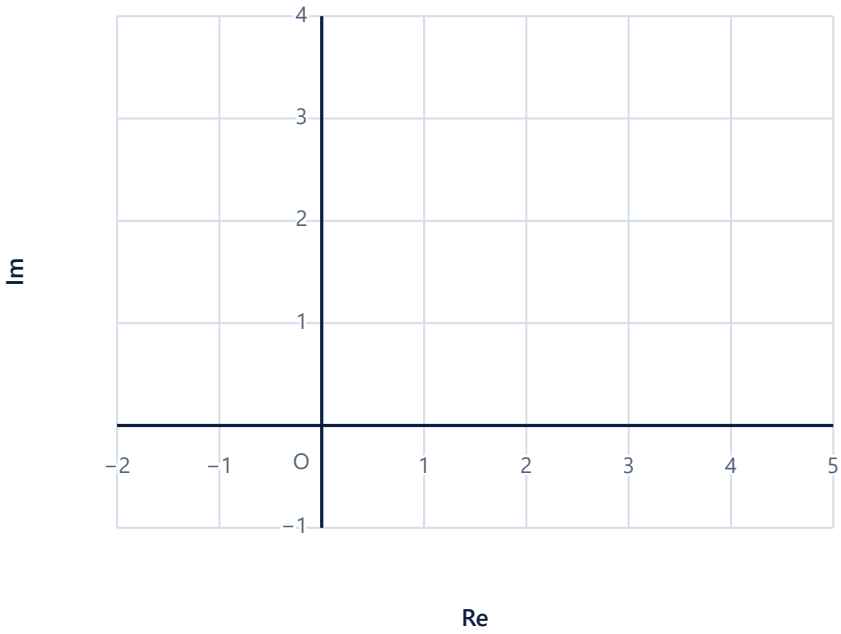
a. Sketch the locus of z on the Argand diagram.

b. Find a cartesian equation of the locus, stating any restriction on x

c. Find the exact least value of $|z|$ and the complex number z at which it occurs.

The complex number w satisfies $\arg(w - 2) = \frac{\pi}{3}$

d. Explain why $|w| > 2$ for every such w, and why $|w|$ has no least value.



(8 marks)

Answer _____

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

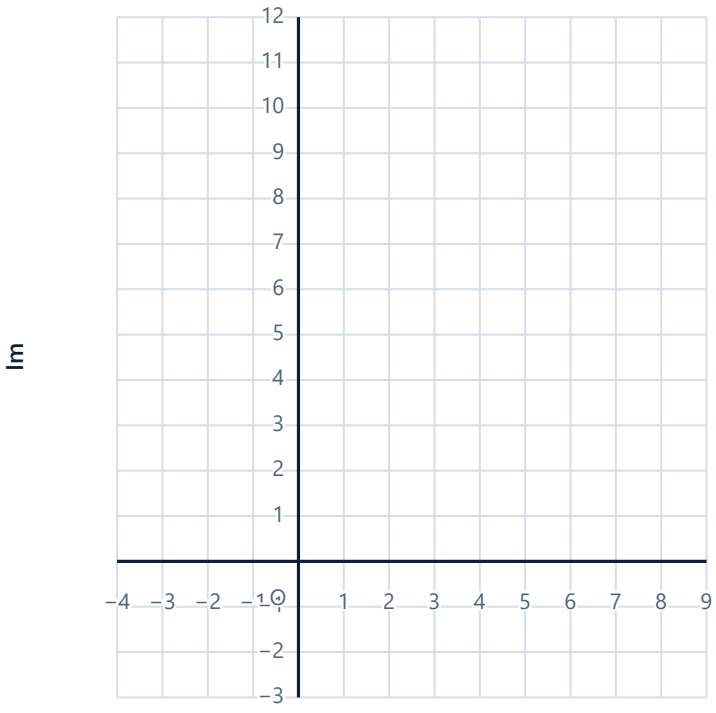
11 The complex numbers z and w satisfy

$$|z - 6| = 2 \quad \text{and} \quad |w - 8i| = 3$$

a. On the same Argand diagram, sketch the locus of z and the locus of w, showing the complex numbers where each locus meets the coordinate axes.

b. Find the range of possible values of $|z - w|$

c. Find the values of z and w for which $|z - w|$ takes its least value.



(9 marks)

Answer

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.

12 Femi draws a plan of his garden on an Argand diagram. The origin O is a corner of his house, and 1 unit represents 1 metre.

A sprinkler at the point S, representing $6 + 4i$, waters every point within 3 m of S. This is the region W.

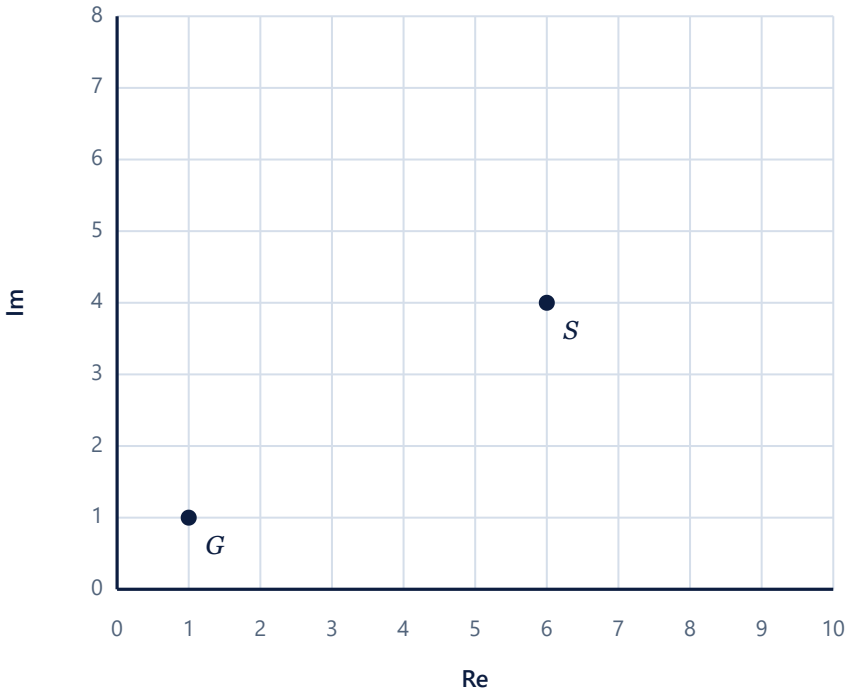
A straight path starts at the gate G, representing $1 + i$, and is modelled by the locus $\arg(z - 1 - i) = \frac{\pi}{4}$

a. Write down an inequality, in terms of z, that defines W

b. Find a cartesian equation of the path.

c. Find the exact length of path that is watered by the sprinkler.

d. Find the shortest distance from the corner of the house to the watered region, giving your answer exactly and to 3 significant figures.



(8 marks)

Answer _____

End of questions | Total 81 marks

Free for classroom and home use: print, photocopy and share. Not for resale or re-hosting.