Matrices, determinants and inverses

AS Core Pure Mathematics | Edexcel 8FM0 | 12 questions, 66 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 66**

1 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{vmatrix} 3 & -1 \\ 2 & 4 \end{vmatrix} \quad \mathbf{B} = \begin{vmatrix} 1 & 5 \\ -2 & 0 \end{vmatrix}$$

a. Find $2\mathbf{A} - 3\mathbf{B}$

b. Find $\mathbf{AB}$

c. Find $\mathbf{BA}$ and comment on your answers to parts (b) and (c).

(4 marks)

Answer _____

2 The matrices **P** and **Q** are given by

$$\mathbf{P} = \begin{vmatrix} 1 & 0 & -2 \\ 3 & 1 & 4 \end{vmatrix} \quad \mathbf{Q} = \begin{vmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 5 \end{vmatrix}$$

a. State the order of the matrix $\mathbf{PQ}$ and the order of the matrix $\mathbf{QP}$

b. Find $\mathbf{PQ}$

(3 marks)

Answer _____

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3 $\mathbf{M} = \begin{vmatrix} 5 & 3 \\ 4 & 2 \end{vmatrix}$

a. Find det **M**

b. Find $\mathbf{M}^{-1}$

(3 marks)

Answer _____

4 The matrix $\mathbf{A} = \begin{vmatrix} k-3 & 4 \\ 5 & k+5 \end{vmatrix}$, where k is a constant.

a. Find the values of k for which **A** is singular.

b. Given that $k = 1$, find $\mathbf{A}^{-1}$

(5 marks)

Answer _____

5 The matrix $\mathbf{B} = \begin{vmatrix} 2k & 5 \\ k-1 & k+3 \end{vmatrix}$, where k is a real constant.

Show that **B** is non-singular for all real values of k

(4 marks)

Answer _____

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6 $\mathbf{A} = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$ $\mathbf{B} = \begin{vmatrix} 9 & -1 \\ 8 & 3 \end{vmatrix}$

a. Find $\mathbf{A}^{-1}$

b. Find the matrix $\mathbf{X}$ such that $\mathbf{AX} = \mathbf{B}$

c. Find the matrix $\mathbf{Y}$ such that $\mathbf{YA} = \mathbf{B}$

(6 marks)

Answer

7 $\mathbf{C} = \begin{vmatrix} k & 3 & 0 \\ 1 & k & 2 \\ -2 & 1 & 1 \end{vmatrix}$, where k is a constant.

a. Show that det $\mathbf{C} = k^2 - 2k - 15$

b. Find the values of k for which $\mathbf{C}$ is singular.

c. Given that $k = 2$, find $\mathbf{C}^{-1}$, showing your working.

(7 marks)

Answer

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8 The 2 × 2 matrices **P** and **Q** are such that

$\mathbf{P}^{-1} = \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}$ $\mathbf{Q} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$

a. Find $(\mathbf{QP})^{-1}$

b. Find **P**

c. Hence find the column vector **r** such that $\mathbf{QPr} = \begin{vmatrix} 3 \\ 1 \end{vmatrix}$

(6 marks)

Answer _____

9 $\mathbf{A} = \begin{vmatrix} 3 & 2 \\ -1 & 1 \end{vmatrix}$ and **I** is the 2 × 2 identity matrix.

a. Find $\mathbf{A}^2$

b. Find the constants p and q such that $\mathbf{A}^2 = p\mathbf{A} + q\mathbf{I}$

c. Hence, by multiplying your equation by $\mathbf{A}^{-1}$, find $\mathbf{A}^{-1}$

(6 marks)

Answer _____

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10 $\mathbf{M} = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & a \\ 0 & 3 & 1 \end{vmatrix}$, where a is a constant.

a. Find det **M** in terms of a

b. State the value of a for which **M** is singular.

c. Given that $a = 4$, use algebra to find $\mathbf{M}^{-1}$

d. Hence write down $(2\mathbf{M})^{-1}$

(8 marks)

Answer

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11 A conservation group models the numbers of young and adult otters on a stretch of river. The numbers of young otters, y_n, and adult otters, a_n, n years after the survey began satisfy

$$\begin{vmatrix} y_{n+1} \\ a_{n+1} \end{vmatrix} = \mathbf{M} \begin{vmatrix} y_n \\ a_n \end{vmatrix} \quad \text{where} \quad \mathbf{M} = \begin{vmatrix} 0 & 1.2 \\ 0.5 & 0.6 \end{vmatrix}$$

This year there are 300 young otters and 200 adult otters.

a. Use the model to predict the number of young otters and the number of adult otters next year.

b. Find $\mathbf{M}^{-1}$

c. Hence find the numbers of young and adult otters last year, according to the model.

d. The group's records show 247 adult otters last year. Comment on the model in the light of this.

(7 marks)

Answer _____

12 $\mathbf{Q} = \begin{vmatrix} k & 4 \\ -2 & -k \end{vmatrix}$, where k is a real constant, and $\mathbf{I}$ is the 2 × 2 identity matrix.

a. Show that $\mathbf{Q}^2 = (k^2 - 8)\mathbf{I}$

b. Find the exact values of k for which $\mathbf{Q}$ is singular.

c. Find the values of k for which $\mathbf{Q}$ is self-inverse, that is $\mathbf{Q}^2 = \mathbf{I}$

d. Given that $k = 1$, use part (a) to find $\mathbf{Q}^{-1}$

(7 marks)

Answer _____

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End of questions | Total 66 marks

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