Proof by induction

AS Core Pure Mathematics | Edexcel 8FM0 | 12 questions, 72 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 72**

1 Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} (4r - 1) = n(2n + 1)$$

(4 marks)

Answer _____

2 Prove by induction that, for $n \in \mathbb{Z}^+$, $11^n - 6$ is divisible by 5

(4 marks)

Answer _____

3 A sequence is defined by

$u_1 = 3, \quad u_{n+1} = 2u_n - 1$

Prove by induction that, for $n \in \mathbb{Z}^+$, $u_n = 2^n + 1$

(4 marks)

Answer _____

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4 Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(2n+7)$$

(5 marks)

Answer

5 Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix}^n = \begin{vmatrix} 1 & 0 \\ 4^n - 1 & 4^n \end{vmatrix}$$

(5 marks)

Answer

6 Prove by induction that, for $n \in \mathbb{Z}^+$,

$f(n) = 7^{2n-1} + 5$

is divisible by 12

(6 marks)

Answer

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7 Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{1}{(2r+1)(2r+3)} = \frac{n}{3(2n+3)}$$

(6 marks)

Answer

8 Prove by induction that, for all positive **odd** integers n,

$f(n) = 3^n + 5^n$

is divisible by 8

(6 marks)

Answer

9 A sequence of positive integers is defined by

$u_1 = 6, \quad u_{n+1} = 3u_n + 4n$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$u_n = 3^{n+1} - 2n - 1$

(6 marks)

Answer

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10 The matrix $\mathbf{M} = \begin{vmatrix} 3 & -2 \\ 2 & -1 \end{vmatrix}$

a. Prove by induction that, for $n \in \mathbb{Z}^+$,
$\begin{vmatrix} 3 & -2 \\ 2 & -1 \end{vmatrix}^n = \begin{vmatrix} 1+2n & -2n \\ 2n & 1-2n \end{vmatrix}$

b. The transformation represented by $\mathbf{M}^n$ maps the point $A(3, 1)$ to the point $A'(43, 41)$. Find the value of n.

c. A shape S of area 12 cm^2 is transformed by $\mathbf{M}^n$. Show that the area of the image does not depend on n, and state its value.

(9 marks)

Answer

11 Kwame makes a sequence of tile patterns. Pattern 1 uses 5 tiles, and to make pattern $(n + 1)$ from pattern n he adds $4n + 5$ more tiles. So the number of tiles T_n in pattern n satisfies

$T_1 = 5, \quad T_{n+1} = T_n + 4n + 5$

a. Prove by induction that, for $n \in \mathbb{Z}^+$, $T_n = 2n^2 + 3n$

b. One of Kwame's patterns uses exactly 1034 tiles. Find which pattern this is.

(7 marks)

Answer

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12 The matrix $\mathbf{A} = \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix}$

a. Find $\mathbf{A}^2$ and $\mathbf{A}^3$

b. Show that the results of part (a) fit the formula

$$\mathbf{A}^n = \begin{vmatrix} 2^n & n \times 2^{n-1} \\ 0 & 2^n \end{vmatrix}$$

and prove by induction that this formula holds for all $n \in \mathbb{Z}^+$.

c. A triangle of area 3 cm^2 is transformed by $\mathbf{A}^n$ to a triangle of area 3072 cm^2. Find the value of n, and hence write down $\mathbf{A}^n$ for this value.

(10 marks)

Answer

End of questions | Total 72 marks

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