Roots of polynomials

AS Core Pure Mathematics | Edexcel 8FM0 | 12 questions, 79 marks

Further Maths topic practice

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Name: _____ **Date:** _____ **Score:** _____ **/ 79**

1 The roots of the equation $3x^2 - 7x + 5 = 0$ are α and β.

Without solving the equation, find the exact value of

a. $\alpha^2 + \beta^2$

b. $\frac{1}{\alpha} + \frac{1}{\beta}$

(4 marks)

Answer _____

2 The cubic equation $2x^3 - 5x^2 + 8x - 3 = 0$ has roots α, β and γ.

a. Write down the value of $\alpha + \beta + \gamma$, the value of $\alpha\beta + \beta\gamma + \gamma\alpha$ and the value of $\alpha\beta\gamma$.

b. Find the value of $\alpha^2 + \beta^2 + \gamma^2$

c. Explain what your answer to part (b) tells you about the roots of the equation.

(5 marks)

Answer _____

3 The roots of the equation $x^2 - 4x + 9 = 0$ are α and β.

Without solving the equation, find a quadratic equation with integer coefficients whose roots are $(2\alpha - 1)$ and $(2\beta - 1)$.

(4 marks)

Answer _____

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4 $f(z) = z^3 + pz^2 + qz - 51$

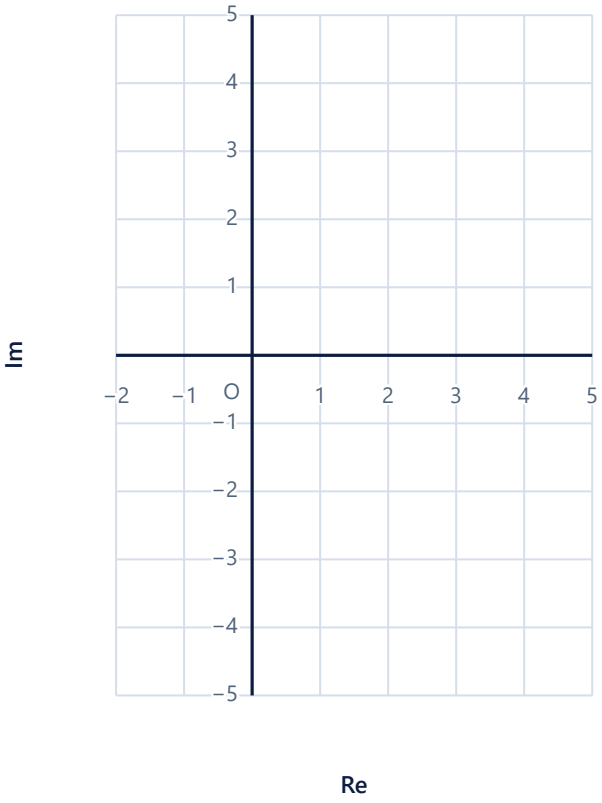
where p and q are real constants.

Given that $1 + 4i$ is a root of the equation $f(z) = 0$,

a. write down another complex root of the equation,

b. find the third root of the equation and the values of p and q,

c. show all three roots of $f(z) = 0$ on a single Argand diagram.



(6 marks)

Answer

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5 The cubic equation $3x^3 - 6x^2 + 2x + 5 = 0$ has roots α, β and γ.

Without solving the equation, find the exact value of

a. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

b. $(\alpha + 3)(\beta + 3)(\gamma + 3)$

c. $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$

(7 marks)

Answer _____

6 The cubic equation $x^3 + 2x^2 - 5x - 3 = 0$ has roots α, β and γ.

Without solving the equation, find the value of

a. $\alpha^2 + \beta^2 + \gamma^2$

b. $\alpha^3 + \beta^3 + \gamma^3$

c. $\frac{\alpha}{\beta\gamma} + \frac{\beta}{\gamma\alpha} + \frac{\gamma}{\alpha\beta}$

(6 marks)

Answer _____

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7 The cubic equation $2x^3 + 5x^2 - 4x + 1 = 0$ has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(2\alpha + 1)$, $(2\beta + 1)$ and $(2\gamma + 1)$, giving your answer in the form $w^3 + aw^2 + bw + c = 0$, where a, b and c are integers to be found.

(5 marks)

Answer

8 The cubic equation $2x^3 + px^2 + qx + 6 = 0$, where p and q are constants, has roots α, β and γ.

The cubic equation $w^3 + 4w^2 - 11w + c = 0$, where c is a constant, has roots $(\alpha - 3)$, $(\beta - 3)$ and $(\gamma - 3)$.

Without solving either equation, determine the value of p, the value of q and the value of c.

(6 marks)

Answer

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9 $f(z) = z^4 + 2z^3 + az^2 + bz + 26$

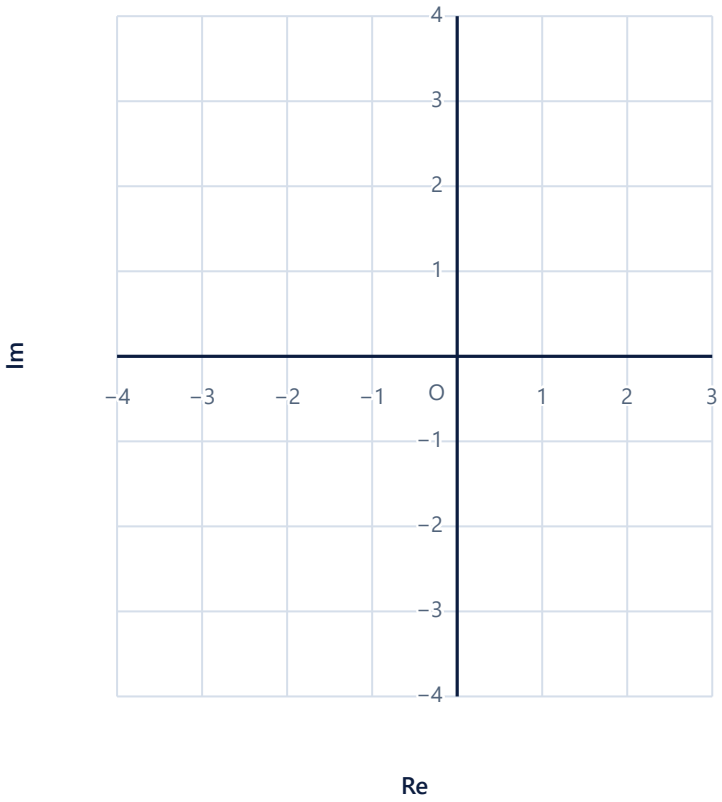
where a and b are real constants.

Given that $-2 + 3i$ is a root of the equation $f(z) = 0$,

a. show that $z^2 + 4z + 13$ is a factor of $f(z)$,

b. find the other three roots of $f(z) = 0$ and the values of a and b,

c. show all four roots of $f(z) = 0$ on a single Argand diagram.



(9 marks)

Answer

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10 The quartic equation $x^4 - 5x^3 + 2x^2 + 3x - 4 = 0$ has roots α, β, γ and δ.

Without solving the equation, find the value of

a. $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$

b. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$

c. $(\alpha + 1)(\beta + 1)(\gamma + 1)(\delta + 1)$

d. By considering the value of $\alpha\beta\gamma\delta$, explain why the equation must have at least two real roots.

(9 marks)

Answer

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11 A storage box is a cuboid. Its length, width and height, in centimetres, are the three roots of the equation

$x^3 - 12x^2 + kx - 28 = 0$

where k is a constant.

a. Explain why the volume of the box is 28 cm^3 and find the total length of all twelve edges of the box.

b. Given that the three dimensions of the box form an arithmetic sequence, find the dimensions of the box and the value of k.

c. Hence write down the total surface area of the box.

d. Chen makes a second box by adding 2 cm to each dimension of the first box. Use a substitution to find the cubic equation, in the form $w^3 + aw^2 + bw + c = 0$, whose roots are the dimensions of the second box, and state the volume of the second box.

(8 marks)

Answer

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12 $f(z) = 2z^3 + pz^2 + qz - 30$

where p and q are real constants.

The roots of $f(z) = 0$ are α, β and γ, where $\alpha = 1 - 3i$

a. Find β and γ, where γ is real.

b. Find the value of p and the value of q.

c. Find the cubic equation, in the form $w^3 + aw^2 + bw + c = 0$, whose roots are $(2\alpha - 1)$, $(2\beta - 1)$ and $(2\gamma - 1)$.

d. The roots of the equation in part (c) are represented by the points A, B and C on an Argand diagram. Find the area of triangle ABC.

(10 marks)

Answer _____

End of questions | Total 79 marks